Modelling the Claims Development Result for Solvency Purposes

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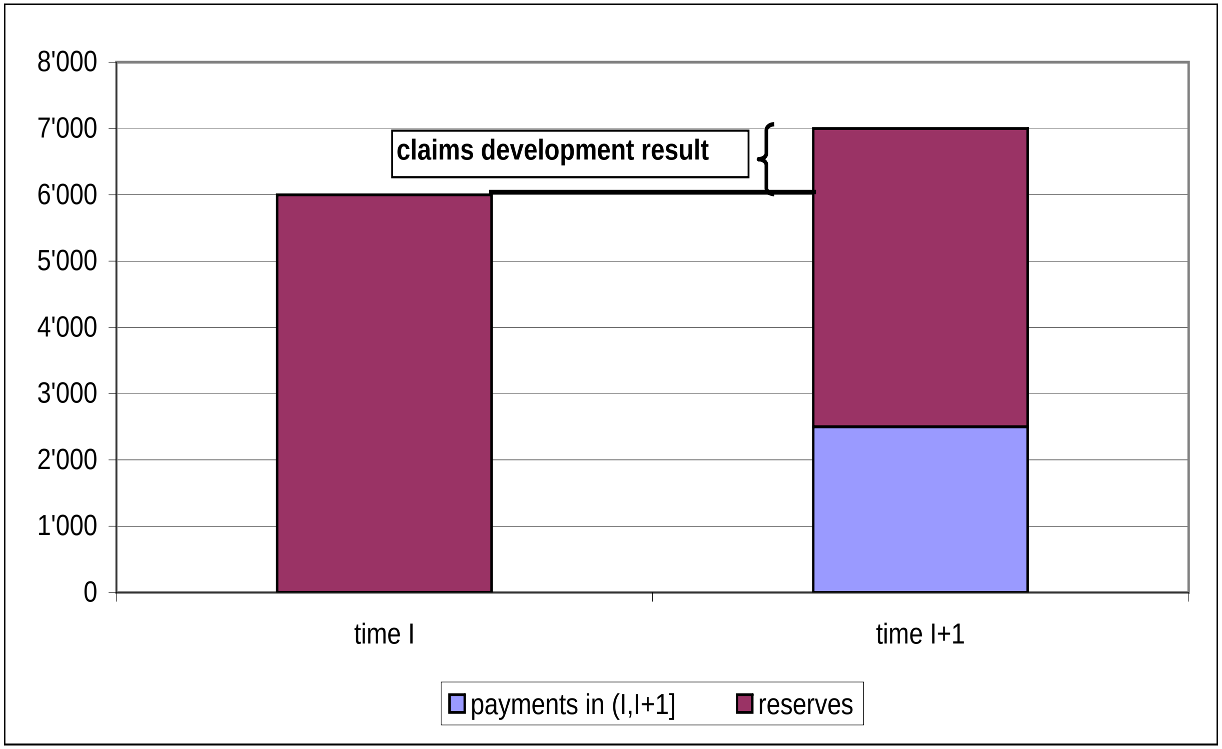
1. Introduction Non-Life Insurance

For accounting year $I + 1 = 2008$:

- **Budget statement at 1/1/2008**
- **Profit & Loss (P&L) statement at 31/12/2008**

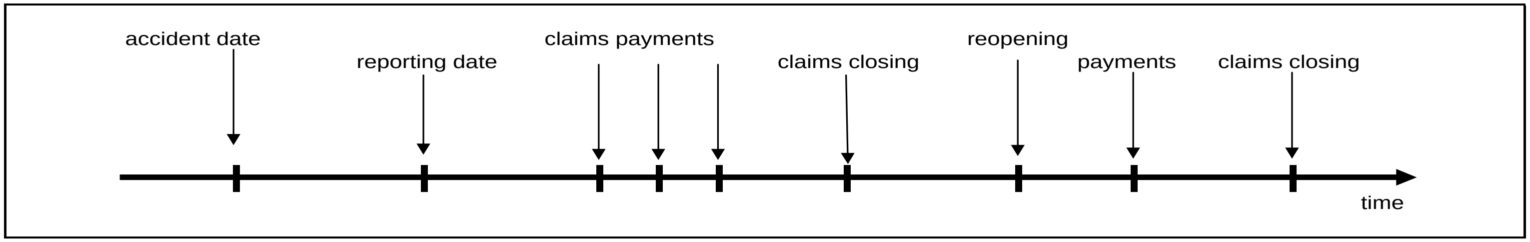
	Budget 1/1/2008	P&L 31/12/2008
premium earned	4'000'000	4'020'000
claims incurred current accident year	-3'300'000	-3'340'000
loss experience prior accident years	**0**	**-40'000**
administrative expenses	-1'000'000	-1'090'000
investment income	500'000	510'000
income before taxes	200'000	60'000

One-Year Claims Development Result (CDR)



Loss experience prior accident years (claims development result) describes the **changes of the insurance (runoff) liabilities** over the next accounting year $I + 1 = 2008$.

Best Estimate Claims Reserves



- Best estimate claims reserve at time I is an (unbiased) prediction for the outstanding loss liabilities at time I based on the available information at time I.

- The CDR describes how we update this prediction at time $I + 1$.

- The CDR is a short term view / one-year view.

- The traditional view is a long term view.

Claims Reserving is a Prediction Problem

- X future cash flow (**random variable**) to be **predicted**.
- $\mathcal{D}_I$ information available at time I.
- Assume $\widehat{X}$ is a $\mathcal{D}_I$-measurable predictor for X.

The (conditional) **mean square error of prediction (MSEP)** is defined b

$$\begin{aligned} \mathrm{msep}_{X|\mathcal{D}_I}\left(\widehat{X}\right) &= E\left[\left(X - \widehat{X}\right)^2 \middle| \mathcal{D}_I\right] \\ &= \mathrm{Var}\left(X \middle| \mathcal{D}_I\right) + \left(E\left[X \middle| \mathcal{D}_I\right] - \widehat{X}\right)^2. \end{aligned}$$

Hence, $\widehat{X}$ is a **predictor** for X and an **estimator** for $E\left[X \middle| \mathcal{D}_I\right]$.

2. Distribution-Free Chain Ladder Model

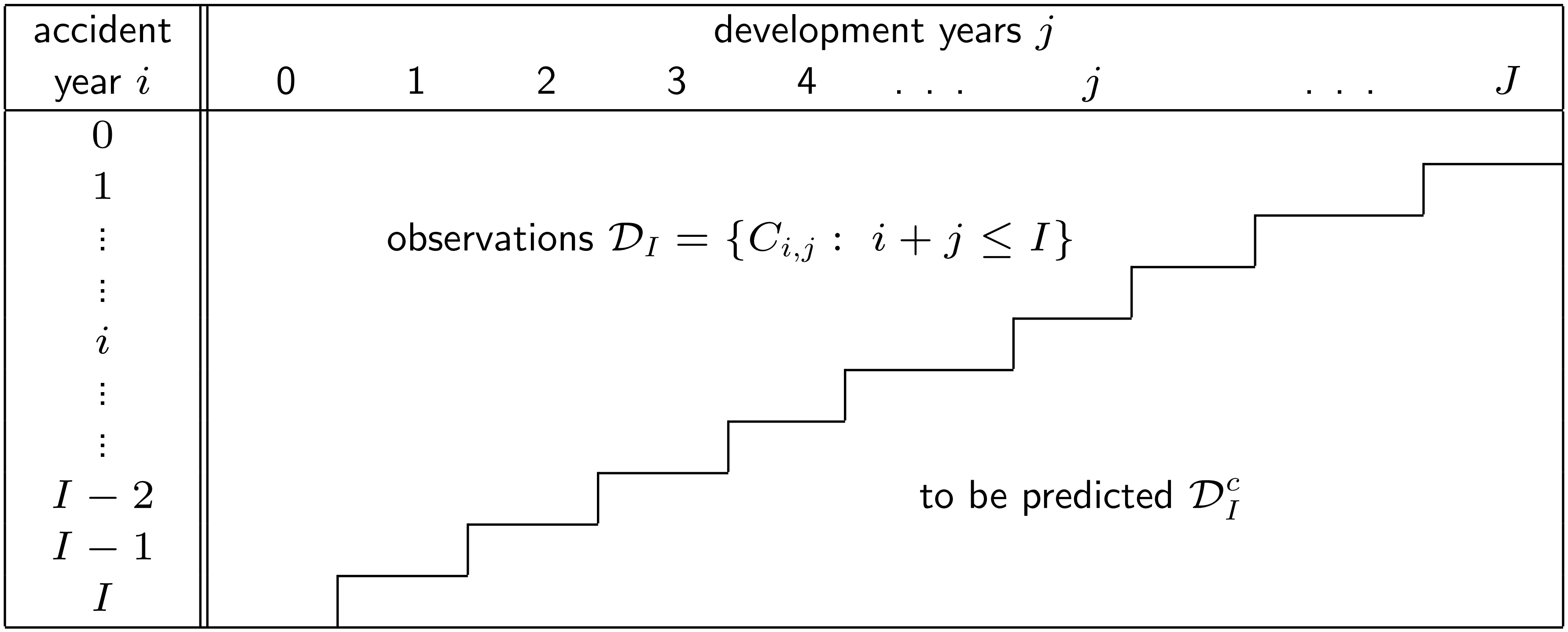
We use the following notations:

- **accident years** are denoted by $i \in \{0, \ldots, I\}$
- **development years** are denoted by $j \in \{0, \ldots, J\}$
- **incremental claims** are denoted by $X_{i,j}$
- **cumulative claims** are denoted by

$$C_{i,j} = \sum_{k=0}^{j} X_{i,k}.$$

Here: $X_{i,j}$ denote incremental payments.

Loss Development Triangle at Time I



- **observations** $\mathcal{D}_I = \{C_{i,j}: \; i+j \le I\}$,

- **to be predicted** $\mathcal{D}_I^c = \{C_{i,j}: \; i+j > I, \; i \le I\}$.

Distribution-Free CL Model

CL assumptions:

- Different accident years i are independent.
- $\{C_{i,j}\}_{j \geq 0}$ is a Markov chain with

$$E\left[C_{i,j} \middle| C_{i,j-1}\right] = f_{j-1}\, C_{i,j-1}, \qquad \text{for all } i, j.$$

$$\mathrm{Var}\left(C_{i,j} \middle| C_{i,j-1}\right) = \sigma^2_{j-1}\, C_{i,j-1}, \qquad \text{for all } i, j.$$

Expected ultimate claim $C_{i,J}$, given $\mathcal{D}_I$, is

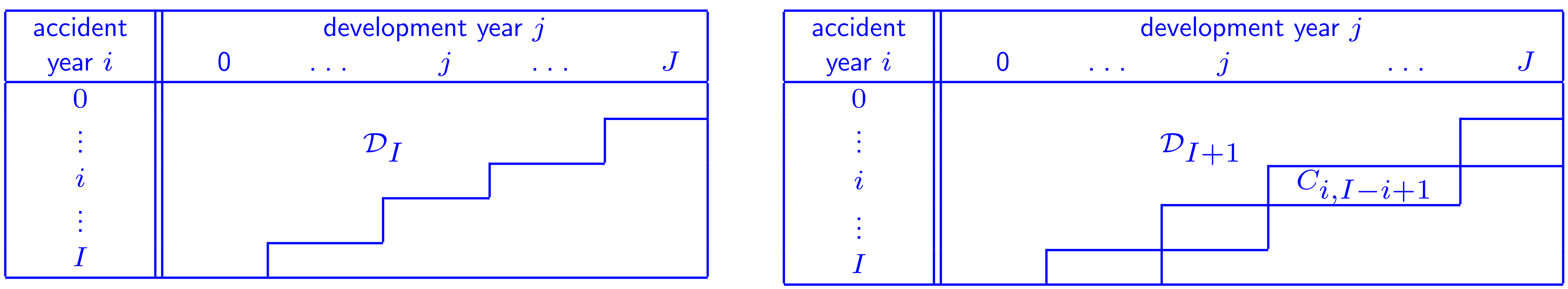
$$E\left[C_{i,J} \middle| \mathcal{D}_I\right] = C_{i,I-i} \prod_{j=I-i}^{J-1} f_j.$$

3. Claims Development Result

The CDR is mainly concerned with **updating the information**.

$$\mathcal{D}_I = \{C_{i,j} :\ i+j \le I \text{ and } i \le I\},$$

$$\mathcal{D}_{I+1} = \{C_{i,j} :\ i+j \le I+1 \text{ and } i \le I\} \supset \mathcal{D}_I.$$



Loss development triangle at time I and at time $I+1$.

This provides predictors $\widehat{C}_{i,J}^{I}$ and $\widehat{C}_{i,J}^{I+1}$.

CL Factor Estimation

CL factor estimators at times I and $I+1$:

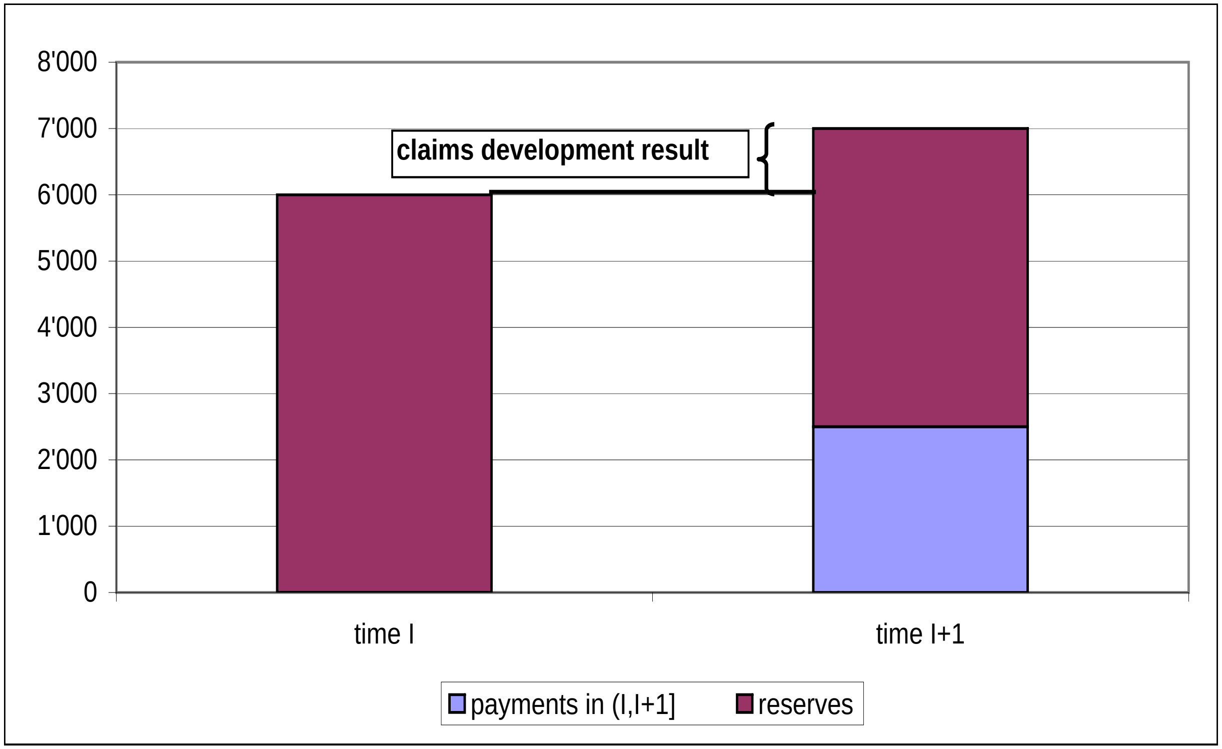
$$\widehat{f}_j^I = \frac{\sum_{i=0}^{I-j-1} C_{i,j+1}}{\sum_{i=0}^{I-j-1} C_{i,j}} \qquad \text{and} \qquad \widehat{f}_j^{I+1} = \frac{\sum_{i=0}^{I-j} C_{i,j+1}}{\sum_{i=0}^{I-j} C_{i,j}}.$$

Best estimate CL reserves at times I and $I+1$:

$$\widehat{R}_i^{\mathcal{D}_I} = \widehat{C}_{i,J}^I - C_{i,I-i} = C_{i,I-i} \prod_{j=I-i}^{J-1} \widehat{f}_j^I - C_{i,I-i},$$

$$\widehat{R}_i^{\mathcal{D}_{I+1}} = \widehat{C}_{i,J}^{I+1} - C_{i,I-i+1} = C_{i,I-i+1} \prod_{j=I-i+1}^{J-1} \widehat{f}_j^{I+1} - C_{i,I-i+1}.$$

One-Year Claims Development Result (CDR)



$$\widehat{R}_i^{\mathcal{D}_I} \quad \xrightarrow{\text{accounting year } I+1} \quad X_{i,I-i+1} + \widehat{R}_i^{\mathcal{D}_{I+1}}$$

Observable Claims Development Result

The **observable CDR** is given by

$$\widehat{\mathrm{CDR}}_i(I+1) = \widehat{R}_i^{\mathcal{D}_I} - \left(X_{i,I-i+1} + \widehat{R}_i^{\mathcal{D}_{I+1}} \right).$$

- Fluctuation of the observable CDR around 0

$$\begin{aligned} \mathrm{msep}_{\widehat{\mathrm{CDR}}_i(I+1)|\mathcal{D}_I}(0) &= E\left[\left(\widehat{\mathrm{CDR}}_i(I+1) - 0 \right)^2 \Big| \mathcal{D}_I \right] \\ &= E\left[\left(\widehat{C}_{i,J}^{I} - \widehat{C}_{i,J}^{I+1} \right)^2 \Big| \mathcal{D}_I \right]. \end{aligned}$$

This exactly quantifies the prediction uncertainty of the position “loss experience prior accident years” in the budget statement.

MSEP, Observable CDR (1/2)

Conditional MSEP for the **one-year runoff uncertainty** is:

$$\widehat{\mathsf{msep}}_{\widehat{\mathrm{CDR}}_i(I+1)|\mathcal{D}_I}(0)$$

$$= \left(\widehat{C}_{i,J}^I\right)^2 \left[\frac{\widehat{\sigma}_{I-i}^2 / \left(\widehat{f}_{I-i}^I\right)^2}{C_{i,I-i}} + \frac{\widehat{\sigma}_{I-i}^2 / \left(\widehat{f}_{I-i}^I\right)^2}{\sum_{k=0}^{i-1} C_{k,I-i}} + \sum_{j=I-i+1}^{J-1} \frac{C_{I-j,j}}{\sum_{k=0}^{I-j} C_{k,j}} \frac{\widehat{\sigma}_j^2 / \left(\widehat{f}_j^I\right)^2}{\sum_{k=0}^{I-j-1} C_{k,j}}\right].$$

Compare to the CL Mack (1993) formula (total runoff uncertainty, long term view)!

MSEP, Observable CDR (2/2)

The conditional MSEP for aggregated accident years is estimated by

$$\widehat{\mathsf{msep}}_{\sum_i \widehat{\mathrm{CDR}}_i(I+1)|\mathcal{D}_I}(0) = \sum_i \widehat{\mathsf{msep}}_{\widehat{\mathrm{CDR}}_i(I+1)|\mathcal{D}_I}(0)$$

$$+2\sum_{i<l} \widehat{C}_{i,J}^I \widehat{C}_{l,J}^I \left[\frac{\widehat{\sigma}_{I-i}^2 / \left(\widehat{f}_{I-i}^I\right)^2}{\sum_{k=0}^{i-1} C_{k,I-i}} + \sum_{j=I-i+1}^{J-1} \frac{C_{I-j,j}}{\sum_{k=0}^{I-j} C_{k,j}} \frac{\widehat{\sigma}_j^2 / \left(\widehat{f}_j^I\right)^2}{\sum_{k=0}^{I-j-1} C_{k,j}} \right].$$

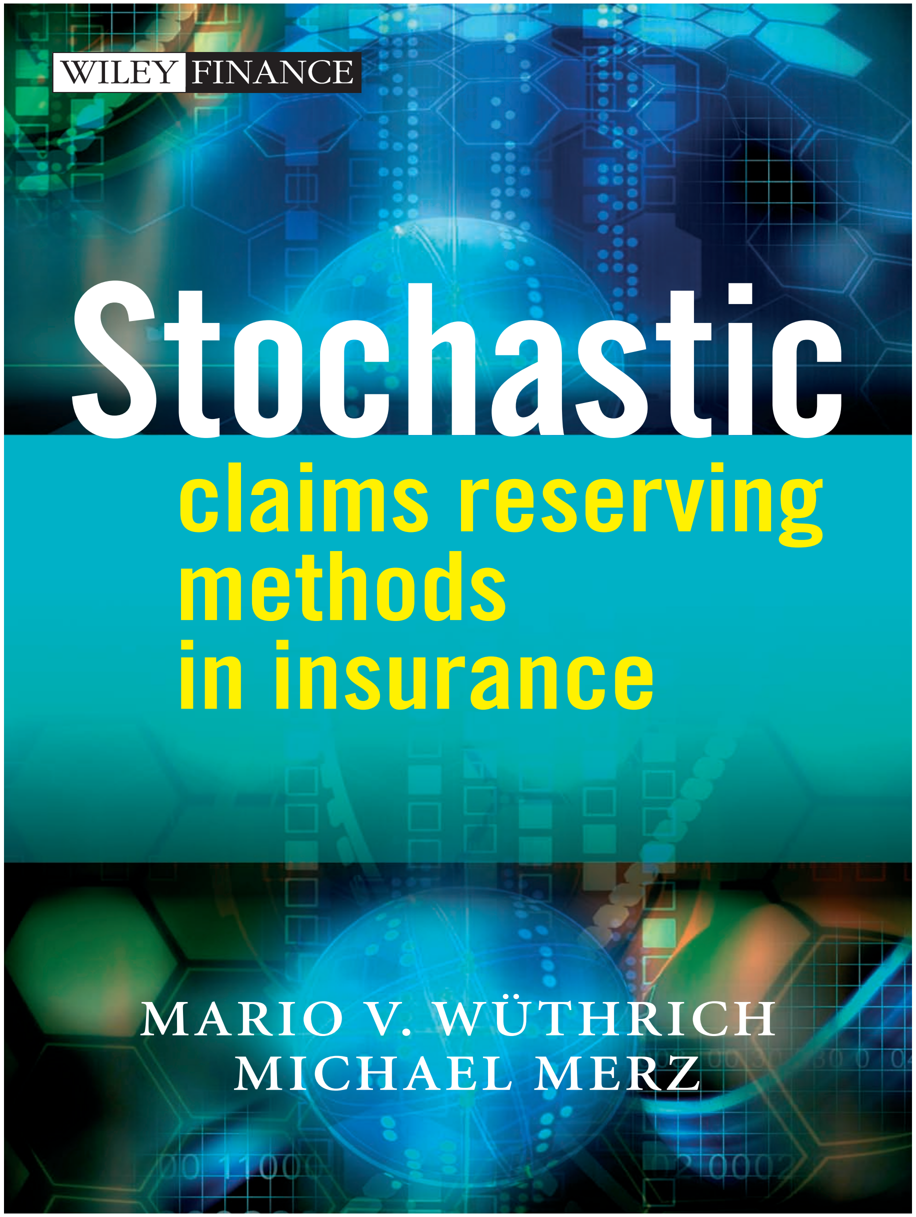
This is the one-year solvency view: Budget vs. P&L statement.

Example

i	$\widehat{R}_i^{\mathcal{D}_I}$	$\widehat{\text{msep}}_{\widehat{\text{CDR}}_i(I+1)\|\mathcal{D}_I}(0)^{1/2}$	$\widehat{\text{msep}}_{C_{i,J}^I\|\mathcal{D}_I}(\widehat{C}_{i,J})^{1/2}$ Mack '93
0	0		
1	4'378	567	567
2	9'348	1'488	1'566
3	28'392	3'923	4'157
4	51'444	9'723	10'536
5	111'811	28'443	30'319
6	187'084	20'954	35'967
7	411'864	28'119	45'090
8	1'433'505	53'320	69'552
$\text{cov}^{1/2}$		39'746	50'361
Total	2'237'826	81'080	108'401

- $\widehat{\text{msep}}_{\widehat{\text{CDR}}_i(I+1)|\mathcal{D}_I}(0)^{1/2}$ is the one-year CDR view (short term).
- $\widehat{\text{msep}}_{C_{i,J}|\mathcal{D}_I}(\widehat{C}_{i,J}^I)^{1/2}$ is the whole runoff uncertainty (long term).

We see that the ratio is around 75%.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