

Model risk in claims reserving within Tweedie's compound Poisson models

Dr Pavel V. Shevchenko

Principal Research Scientist, Team leader

CSIRO Division of Mathematical and Information Sciences

Quantitative Risk Management group, Sydney, Australia

E-mail: Pavel.Shevchenko@csiro.au www.cmis.csiro.au/Pavel.Shevchenko

Gareth Peters (UNSW/CSIRO), Pavel Shevchenko (CSIRO), and Mario Wüthrich (ETH). *Model risk in claims reserving within Tweedie's compound Poisson models*, preprint 2008.

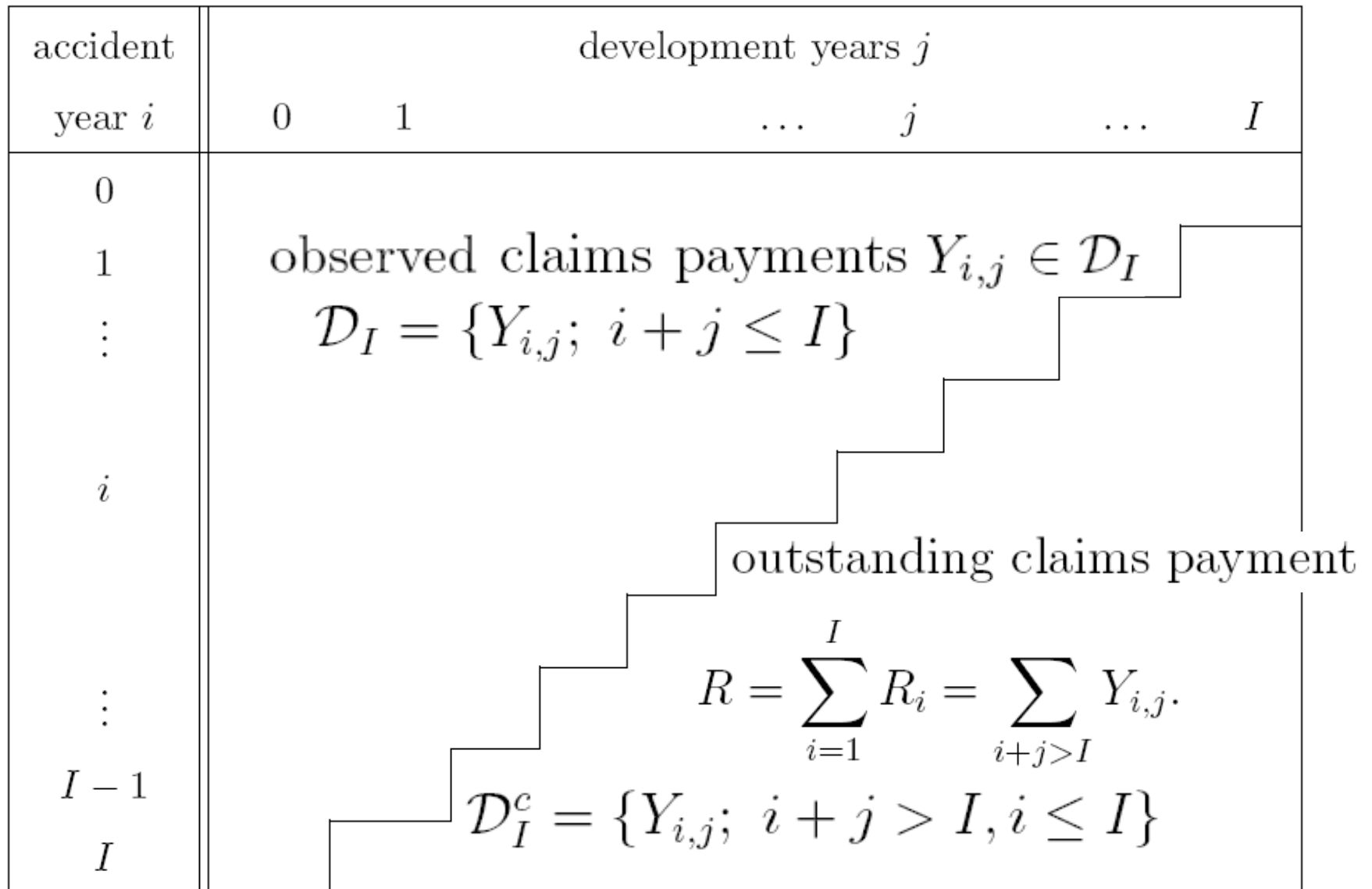
ASTIN Colloquium, Manchester, UK, July 2008

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Claims Reserving (non-life insurance), solvency requirements, claims development triangle (real data)



Content

- ◆ Tweedie's compound Poisson family to model annual claims
- ◆ Process Uncertainty, Parameter Estimation Error, Model uncertainty
- ◆ Variable selection
- ◆ Maximum likelihood and Bayesian estimation
- ◆ MCMC (random walk Metropolis-Hastings within Gibbs)
- ◆ Analysis/Conclusions

\hat{R} - predictor for R and estimator for $E [R | \mathcal{D}_I]$

$$R = \sum_{i=1}^I R_i = \sum_{i+j>I} Y_{i,j} \quad E [R | \mathcal{D}_I] = \sum_{i=1}^I E [R_i | \mathcal{D}_I]$$

$$\text{mse}_{R|\mathcal{D}_I}(\hat{R}) = E \left[(R - \hat{R})^2 \middle| \mathcal{D}_I \right] \quad \text{Mean Square Error of Prediction}$$

$$\begin{aligned} \text{mse}_{R|\mathcal{D}_I}(\hat{R}) &= \text{Var}(R | \mathcal{D}_I) + (E [R | \mathcal{D}_I] - \hat{R})^2 \\ &= \text{process variance} + \text{estimation error} \end{aligned}$$

$$\hat{R} = E [R | \mathcal{D}_I] \quad \text{“best estimate” of reserve}$$

Bayesian context – variance decomposition

$$\begin{aligned} \text{Var}(R | \mathcal{D}_I) &= E [\text{Var}(R | \boldsymbol{\theta}, \mathcal{D}_I) | \mathcal{D}_I] + \text{Var}(E [R | \boldsymbol{\theta}, \mathcal{D}_I] | \mathcal{D}_I) \\ &= \text{average process variance} + \text{parameter estimation error.} \end{aligned}$$

$\boldsymbol{\theta}$ is model parameter vector modelled as random variable

Tweedie's compound Poisson model

$Y_{i,j}$ are independent for $i, j \in \{0, \dots, I\}$

$$Y_{i,j} = 1_{\{N_{i,j} > 0\}} \sum_{k=1}^{N_{i,j}} X_{i,j}^{(k)}$$

$N_{i,j}$ and $X_{i,j}^{(k)}$ are independent for all k

$N_{i,j}$ is Poisson distributed with parameter $\lambda_{i,j}$

$X_{i,j}^{(k)}$ are independent gamma severities with
mean $\tau_{i,j} > 0$ and shape parameter $\gamma > 0$

Tweedie's compound Poisson: exponential dispersion family representation

$$P [Y_{i,j} = 0] = P [N_{i,j} = 0] = \exp \left\{ -\phi_{i,j}^{-1} \kappa_p(\theta_{i,j}) \right\}$$

$$f_{\theta_{i,j}}(y; \phi_{i,j}, p) = c(y; \phi_{i,j}, p) \exp \left\{ \frac{y \theta_{i,j} - \kappa_p(\theta_{i,j})}{\phi_{i,j}} \right\} \quad \text{for } y > 0$$

where $\theta_{i,j} < 0, \phi_{i,j} > 0, \kappa_p(\theta) \stackrel{\text{def.}}{=} \frac{1}{2-p} [(1-p)\theta]^\gamma$

$$c(y; \phi, p) = \sum_{r \geq 1} \left(\frac{(1/\phi)^{\gamma+1} y^\gamma}{(p-1)^\gamma (2-p)} \right)^r \frac{1}{r! \Gamma(r\gamma) y}$$

$$p = p(\gamma) = \frac{\gamma + 2}{\gamma + 1} \in (1, 2) \quad \phi_{i,j} = \frac{\lambda_{i,j}^{1-p} \tau_{i,j}^{2-p}}{2-p} > 0$$

$$\theta_{i,j} = \left(\frac{1}{1-p} \right) (\mu_{i,j})^{(1-p)} < 0, \quad \mu_{i,j} = \lambda_{i,j} \tau_{i,j} > 0$$

Tweedie's compound Poisson model: final representation

$$P[Y_{i,j} = 0] = P[N_{i,j} = 0] = \exp \left\{ -\phi_{i,j}^{-1} \frac{\mu_{i,j}^{2-p}}{2-p} \right\}$$
$$f_{\mu_{i,j}}(y; \phi_{i,j}, p) = c(y; \phi_{i,j}, p) \exp \left\{ \phi_{i,j}^{-1} \left[y \frac{\mu_{i,j}^{1-p}}{1-p} - \frac{\mu_{i,j}^{2-p}}{2-p} \right] \right\} \quad \text{for } y > 0$$

$$E[Y_{i,j}] = \frac{\partial}{\partial \theta_{i,j}} \kappa_p(\theta_{i,j}) = \kappa'_p(\theta_{i,j}) = [(1-p)\theta_{i,j}]^{1/(1-p)} = \mu_{i,j}$$

$$\text{Var}(Y_{i,j}) = \phi_{i,j} \kappa''_p(\theta_{i,j}) = \phi_{i,j} \mu_{i,j}^p$$

$p \in (1, 2)$, typically fixed by the modeller. *Model Risk*

$p \rightarrow 1$, overdispersed Poisson model

$p \rightarrow 2$, gamma model

Parameter estimation: estimate $\mu_{i,j}$, p and $\phi_{i,j}$ using \mathcal{D}_I

Model assumptions: multiplicative model $\mu_{i,j} = \alpha_i \beta_j$, $\alpha_0 = 1$
 $\phi_{i,j} = \phi$ and $\alpha_i > 0$, $\beta_j > 0$

<i>exposures</i> $\alpha = (\alpha_0, \dots, \alpha_I)$	accident year i	<i>development pattern</i> $\beta = (\beta_0, \dots, \beta_I)$					
		0	1	...	j	...	I
	0						
	1						
	\vdots						
	i						
	\vdots	$\mathcal{D}_I = \{Y_{i,j}; i + j \leq I\}$					
	i	outstanding claims payment $R = \sum_{i=1}^I R_i = \sum_{i+j>I} Y_{i,j}.$					
	\vdots	$\mathcal{D}_I^c = \{Y_{i,j}; i + j > I, i \leq I\}$					
	$I - 1$						
I							

Likelihood function

$$\boldsymbol{\theta} = (p, \phi, \boldsymbol{\alpha}, \boldsymbol{\beta})$$

$$L_{\mathcal{D}_I}(\boldsymbol{\theta}) = \prod_{i+j \leq I} c(Y_{i,j}; \phi, p) \exp \left\{ \phi^{-1} \left[Y_{i,j} \frac{(\alpha_i \beta_j)^{1-p}}{1-p} - \frac{(\alpha_i \beta_j)^{2-p}}{2-p} \right] \right\}$$

$$c(y; \phi, p) = \sum_{r \geq 1} \left(\frac{(1/\phi)^{\gamma+1} y^\gamma}{(p-1)^\gamma (2-p)} \right)^r \frac{1}{r! \Gamma(r\gamma) y} = \frac{1}{y} \sum_{r \geq 1} W_r$$

$$\log W_r = r \log z - \log \Gamma(1+r) - \log \Gamma(\gamma r), \quad z = \frac{(1/\phi)^{\gamma+1} y^\gamma}{(p-1)^\gamma (2-p)}$$

$$W_r \text{ is unimodal in } r, \quad R_0 = R_0(\phi, p) = \frac{y^{2-p}}{(2-p)\phi}$$

find $R_L < R_0 < R_U$ such that

$$W_{R_L} \leq e^{-37} W_{R_0} \text{ (or } R_L = 1) \text{ and } W_{R_U} \leq e^{-37} W_{R_0}$$

$$c(y; \phi, p) \approx \tilde{c}(y; \phi, p) = \frac{1}{y} \sum_{r=R_L}^{R_U} W_r$$

Maximum likelihood estimation

maximizing $L_{\mathcal{D}_I}(\boldsymbol{\theta})$ in $\boldsymbol{\theta} = (p, \phi, \boldsymbol{\alpha}, \boldsymbol{\beta})$ leads to

$$\hat{\boldsymbol{\theta}}^{\text{MLE}} = (\hat{p}^{\text{MLE}}, \hat{\phi}^{\text{MLE}}, \hat{\boldsymbol{\alpha}}^{\text{MLE}}, \hat{\boldsymbol{\beta}}^{\text{MLE}})$$

typically MLE is done for fixed p (expert choice)

$$\hat{R}^{\text{MLE}} = \sum_{i+j>I} \hat{\alpha}_i^{\text{MLE}} \hat{\beta}_j^{\text{MLE}} \quad \text{the best estimate reserves for } R$$

$$\text{cov}(\hat{\theta}_i^{\text{MLE}}, \hat{\theta}_j^{\text{MLE}}) = (\mathbf{I}^{-1})_{i,j}, \quad (\mathbf{I})_{i,j} = - \left. \frac{\partial^2 \ln L_{\mathcal{D}_I}(\boldsymbol{\theta})}{\partial \theta_i \partial \theta_j} \right|_{\boldsymbol{\theta} = \hat{\boldsymbol{\theta}}^{\text{MLE}}}$$

$$\hat{\beta}_I^{\text{MLE}} = Y_{0,I}, \quad \text{cov}(\hat{\beta}_I^{\text{MLE}}, \hat{\theta}_i^{\text{MLE}}) = 0, \quad \hat{\theta}_i^{\text{MLE}} \neq \hat{\beta}_I^{\text{MLE}}$$

Maximum likelihood: process and estimation errors

$$\widehat{R}^{\text{MLE}} = \sum_{i+j>I} \widehat{\alpha}_i^{\text{MLE}} \widehat{\beta}_j^{\text{MLE}}$$

$$\text{stdev} \left(\widehat{R}^{\text{MLE}} \right) = \sqrt{\text{Var} \left(\widehat{R}^{\text{MLE}} \right)} \quad \text{parameter estimation error}$$

$$\begin{aligned} \widehat{\text{Var}} \left(\widehat{R}^{\text{MLE}} \right) &= \sum_{i_1+j_1>I} \sum_{i_2+j_2>I} \widehat{\alpha}_{i_1}^{\text{MLE}} \widehat{\alpha}_{i_2}^{\text{MLE}} \text{cov} \left(\widehat{\beta}_{j_1}^{\text{MLE}}, \widehat{\beta}_{j_2}^{\text{MLE}} \right) \\ &+ \sum_{i_1+j_1>I} \sum_{i_2+j_2>I} \widehat{\beta}_{j_1}^{\text{MLE}} \widehat{\beta}_{j_2}^{\text{MLE}} \text{cov} \left(\widehat{\alpha}_{i_1}^{\text{MLE}}, \widehat{\alpha}_{i_2}^{\text{MLE}} \right) \\ &+ 2 \sum_{i_1+j_1>I} \sum_{i_2+j_2>I} \widehat{\alpha}_{i_1}^{\text{MLE}} \widehat{\beta}_{j_2}^{\text{MLE}} \text{cov} \left(\widehat{\alpha}_{i_2}^{\text{MLE}}, \widehat{\beta}_{j_1}^{\text{MLE}} \right) \end{aligned}$$

$$\widehat{\text{Var}} (R) = \sum_{i+j>I} \left(\widehat{\alpha}_i^{\text{MLE}} \widehat{\beta}_j^{\text{MLE}} \right)^{\widehat{p}^{\text{MLE}}} \widehat{\phi}^{\text{MLE}} \quad \text{process variance}$$

$$\widehat{\text{mse}}_{R|\mathcal{D}_I} \left(\widehat{R}^{\text{MLE}} \right) = \widehat{\text{Var}} (R) + \widehat{\text{Var}} \left(\widehat{R}^{\text{MLE}} \right)$$

= MLE process variance + MLE estimation error

Bayesian inference

$\boldsymbol{\theta} = (p, \phi, \boldsymbol{\alpha}, \boldsymbol{\beta})$ are treated as random

$\pi(\boldsymbol{\theta} \mid \mathcal{D}_I) \propto L_{\mathcal{D}_I}(\boldsymbol{\theta}) \pi(\boldsymbol{\theta})$ Markov Chain Monte Carlo (MCMC)

$$MAP : \quad \hat{\boldsymbol{\theta}}^{MAP} = \arg \max_{\boldsymbol{\theta}} [\pi(\boldsymbol{\theta} \mid \mathcal{D}_I)],$$

$$MMSE : \quad \hat{\boldsymbol{\theta}}^{MMSE} = E[\boldsymbol{\theta} \mid \mathcal{D}_I].$$

if the prior $\pi(\boldsymbol{\theta})$ is constant and the parameter range includes the MLE, then the MAP of the posterior is the same as the MLE.

Gaussian approximation for $\pi(\boldsymbol{\theta} \mid \mathcal{D}_I)$

$$\ln \pi(\boldsymbol{\theta} \mid \mathcal{D}_I) \approx \ln \pi(\hat{\boldsymbol{\theta}}^{MAP} \mid \mathcal{D}_I) + \frac{\partial^2}{\partial \theta_i \partial \theta_j} \ln \pi(\boldsymbol{\theta} \mid \mathcal{D}_I) \Big|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}^{MAP}} \left(\theta_i - \hat{\theta}_i^{MAP} \right) \left(\theta_j - \hat{\theta}_j^{MAP} \right)$$

Bayesian inference estimates

$$E [R | \mathcal{D}_I] = \sum_{i+j>I} E [\alpha_i \beta_j | \mathcal{D}_I] \quad \tilde{R} = E [R | \boldsymbol{\theta}] = \sum_{i+j>I} \alpha_i \beta_j$$

the best consistent estimate of reserves (ER)

$$\hat{R}^B = E \left[\tilde{R} \mid \mathcal{D}_I \right] = \sum_{i+j>I} E [\alpha_i \beta_j | \mathcal{D}_I] = E [R | \mathcal{D}_I]$$

$$\text{mse}_{R|\mathcal{D}_I} \left(\hat{R}^B \right) = E \left[\left(R - \hat{R}^B \right)^2 \mid \mathcal{D}_I \right] = \text{Var} (R | \mathcal{D}_I)$$

$$\begin{aligned} \text{Var} (R | \mathcal{D}_I) &= \text{Var} \left(\sum_{i+j>I} Y_{i,j} \mid \mathcal{D}_I \right) \\ &= \sum_{i+j>I} E [(\alpha_i \beta_j)^p \phi | \mathcal{D}_I] + \text{Var} \left(\tilde{R} \mid \mathcal{D}_I \right) \end{aligned}$$

= average process variance (PV) + parameter estimation error (EE)

Note, model error is incorporated via averaging over values of p

Random Walk Metropolis Hastings (RW-MH) within Gibbs

1. Initialize randomly or deterministically for $t = 0$
the parameter vector $\boldsymbol{\theta}^{t=0}$ to the maximum likelihood estimates.
2. For $t = 1, \dots, T$
 - a) Set $\boldsymbol{\theta}^t = \boldsymbol{\theta}^{t-1}$
 - b) For $i = 1, \dots, 2I + 3$
Sample proposal θ_i^* from Gaussian truncated density

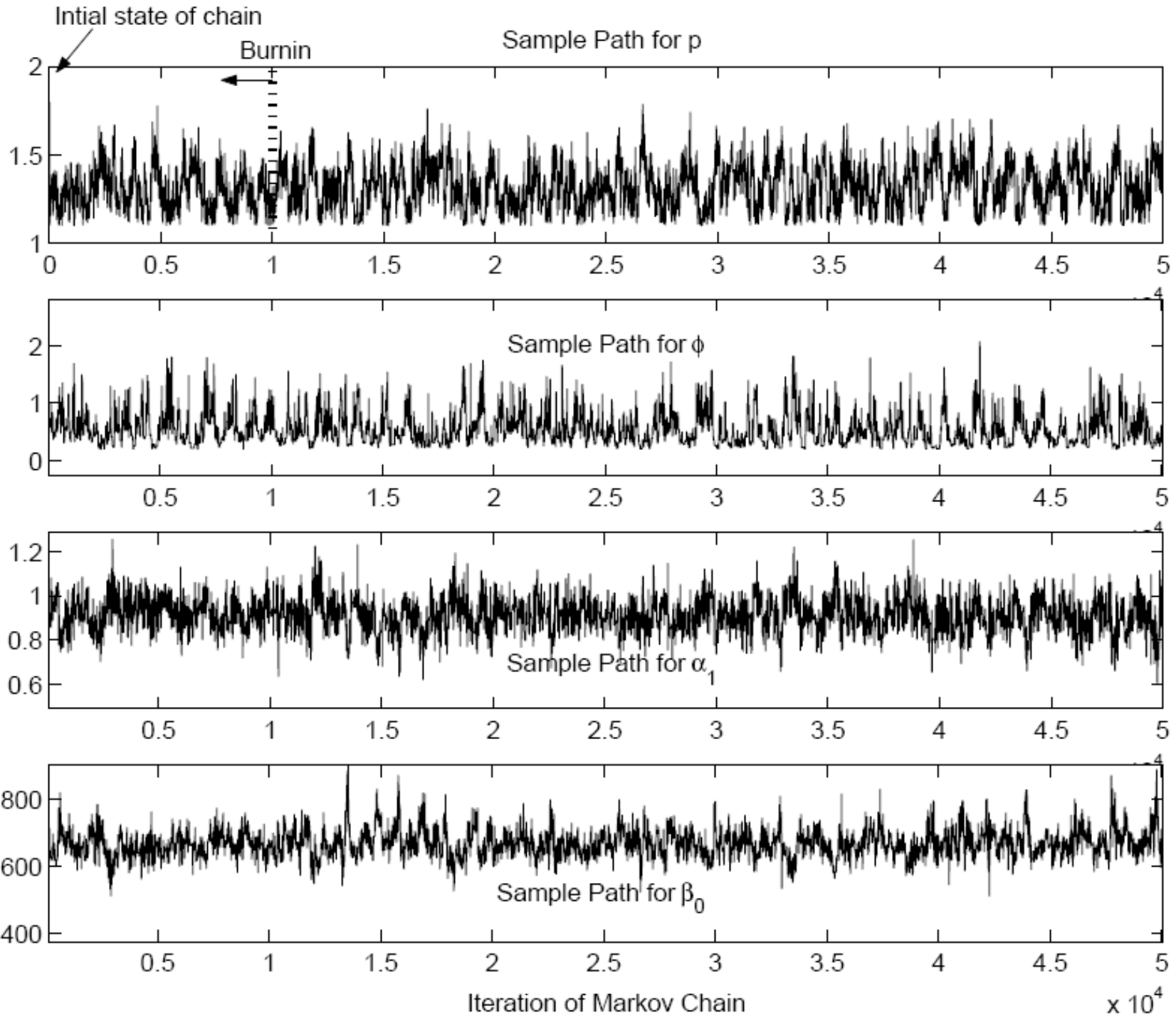
$$f_N^T(\theta_i^*; \theta_i^t, \sigma_{RWi}) = \frac{f_N(\theta_i^*; \theta_i^t, \sigma_{RWi})}{F_N(b_i; \theta_i^t, \sigma_{RWi}) - F_N(a_i; \theta_i^t, \sigma_{RWi})}$$

to obtain $\boldsymbol{\theta}^* = (\theta_1^t, \dots, \theta_{i-1}^t, \theta_i^*, \theta_{i+1}^{t-1}, \dots)$

Accept proposal with acceptance probability

$$\alpha(\boldsymbol{\theta}^t, \boldsymbol{\theta}^*) = \min \left\{ 1, \frac{\pi(\boldsymbol{\theta}^* | \mathcal{D}_I) f_N^T(\theta_i^t; \theta_i^*, \sigma_{RWi})}{\pi(\boldsymbol{\theta}^t | \mathcal{D}_I) f_N^T(\theta_i^*; \theta_i^t, \sigma_{RWi})} \right\}$$

Note: normalization constant in posterior is not needed;
optimal acceptance rate is 0.234



The prior domains

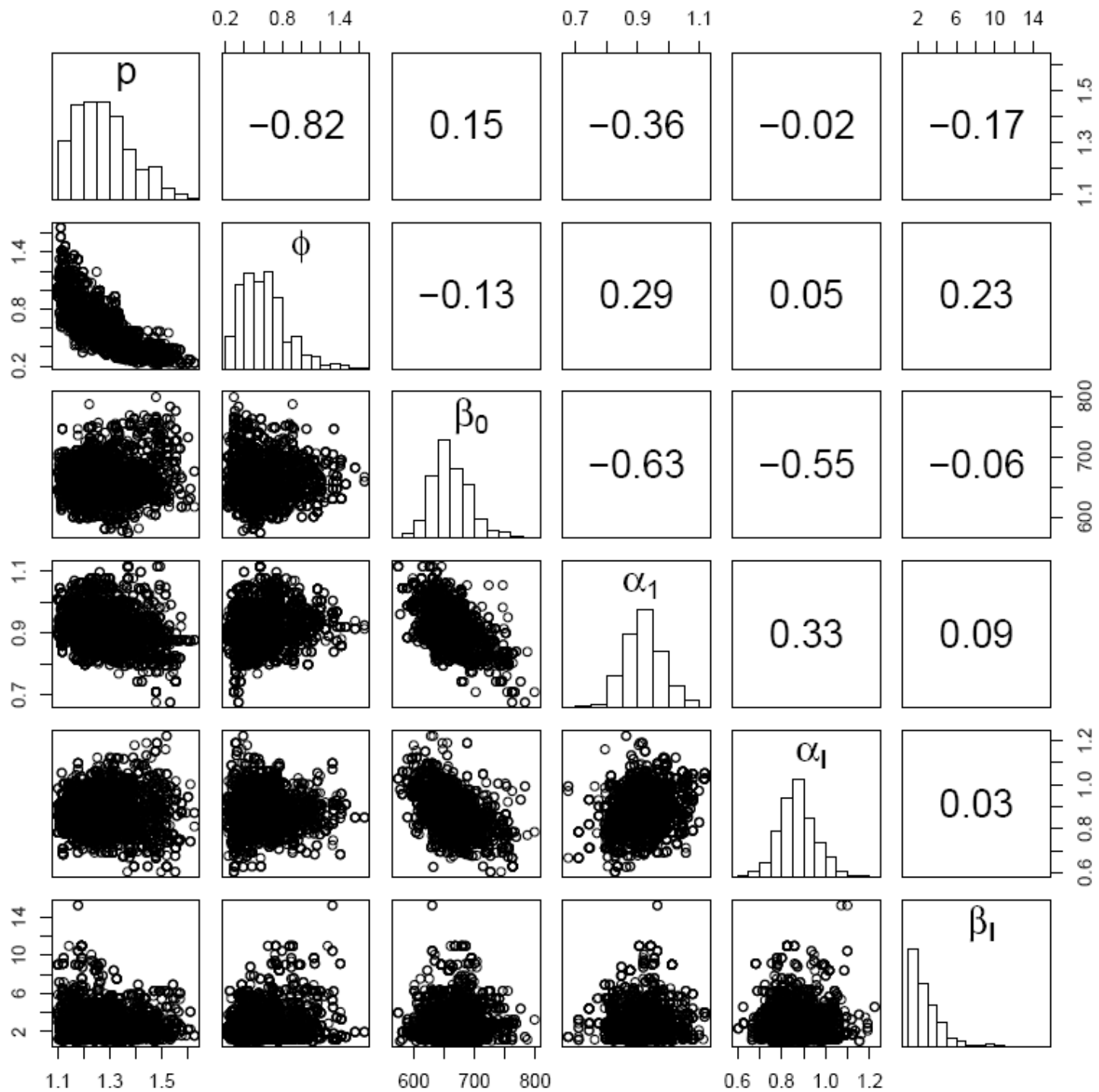
$p \in (1.1, 1.95)$, $\phi \in (0.01, 100)$, $\alpha_i \in (0.01, 100)$ and $\beta_j \in (0.01, 10^4)$

	MLE	MLE stdev	Bayesian posterior			σ_{RW}
			MMSE	stdev	$[Q_{0.05}; Q_{0.95}]$	
p	1.259	0.149	1.332 (0.007)	0.143 (0.004)	[1.127;1.590]	1.61
ϕ	0.351	0.201	0.533 (0.013)	0.289 (0.005)	[0.174;1.119]	1.94
α_1	0.918	0.056	0.901 (0.004)	0.074 (0.001)	[0.778;1.022]	0.842
α_2	0.946	0.051	0.946 (0.003)	0.073 (0.001)	[0.833;1.072]	0.907
α_3	0.861	0.048	0.861 (0.003)	0.068 (0.001)	[0.756;0.977]	0.849
α_4	0.891	0.049	0.902 (0.003)	0.072 (0.002)	[0.794;1.027]	0.893
α_5	0.879	0.051	0.876 (0.003)	0.070 (0.001)	[0.768;0.994]	0.932
α_6	0.842	0.048	0.843 (0.002)	0.069 (0.001)	[0.736;0.958]	0.751
α_7	0.762	0.046	0.762 (0.003)	0.066 (0.001)	[0.660;0.876]	0.888
α_8	0.763	0.047	0.765 (0.003)	0.067 (0.001)	[0.661;0.874]	0.897
α_9	0.848	0.059	0.856 (0.003)	0.090 (0.002)	[0.716;1.009]	1.276

Table 3: MLE and Bayesian estimators.

Table 3: MLE and Bayesian estimators.

	MLE	MLE stdev	Bayesian posterior			σ_{RW}
			MMSE	stdev	$[Q_{0.05}; Q_{0.95}]$	
β_0	669.1	27.7	672.7 (2.1)	39.7 (0.7)	[610.0;740.0]	296
β_1	329.0	14.4	331.1 (1.0)	20.6 (0.4)	[298.1;365.9]	190
β_2	77.43	4.38	78.06 (0.24)	6.10 (0.06)	[68.58;88.29]	75.4
β_3	24.59	1.96	24.95 (0.08)	2.64 (0.03)	[20.89;29.64]	40.9
β_4	16.28	1.55	16.65 (0.05)	2.09 (0.03)	[13.44;20.30]	40.6
β_5	7.773	1.028	8.068 (0.024)	1.356 (0.020)	[6.064;10.473]	26.0
β_6	5.776	0.937	6.115 (0.022)	1.261 (0.016)	[4.246;8.347]	24.1
β_7	1.219	0.396	1.494 (0.006)	0.609 (0.013)	[0.739;2.609]	13.1
β_8	1.188	0.476	1.622 (0.008)	0.802 (0.016)	[0.674;3.070]	15.1
β_9	1.581	0.790	2.439 (0.021)	1.496 (0.026)	[0.829;5.250]	32.1



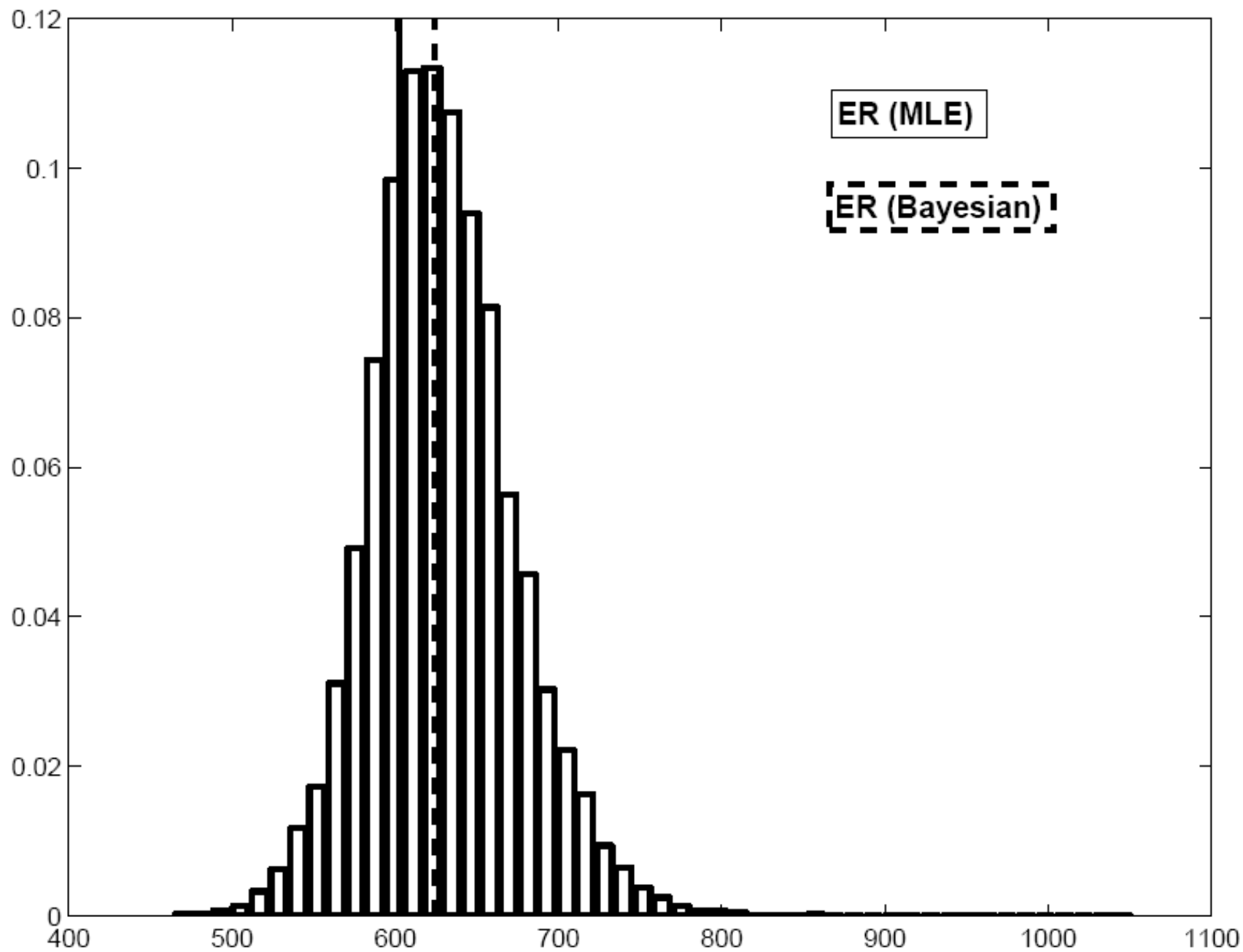


Figure 3: Predicted distribution of reserves, $\tilde{R} = \sum_{i+j>I} \alpha_i \beta_j$.

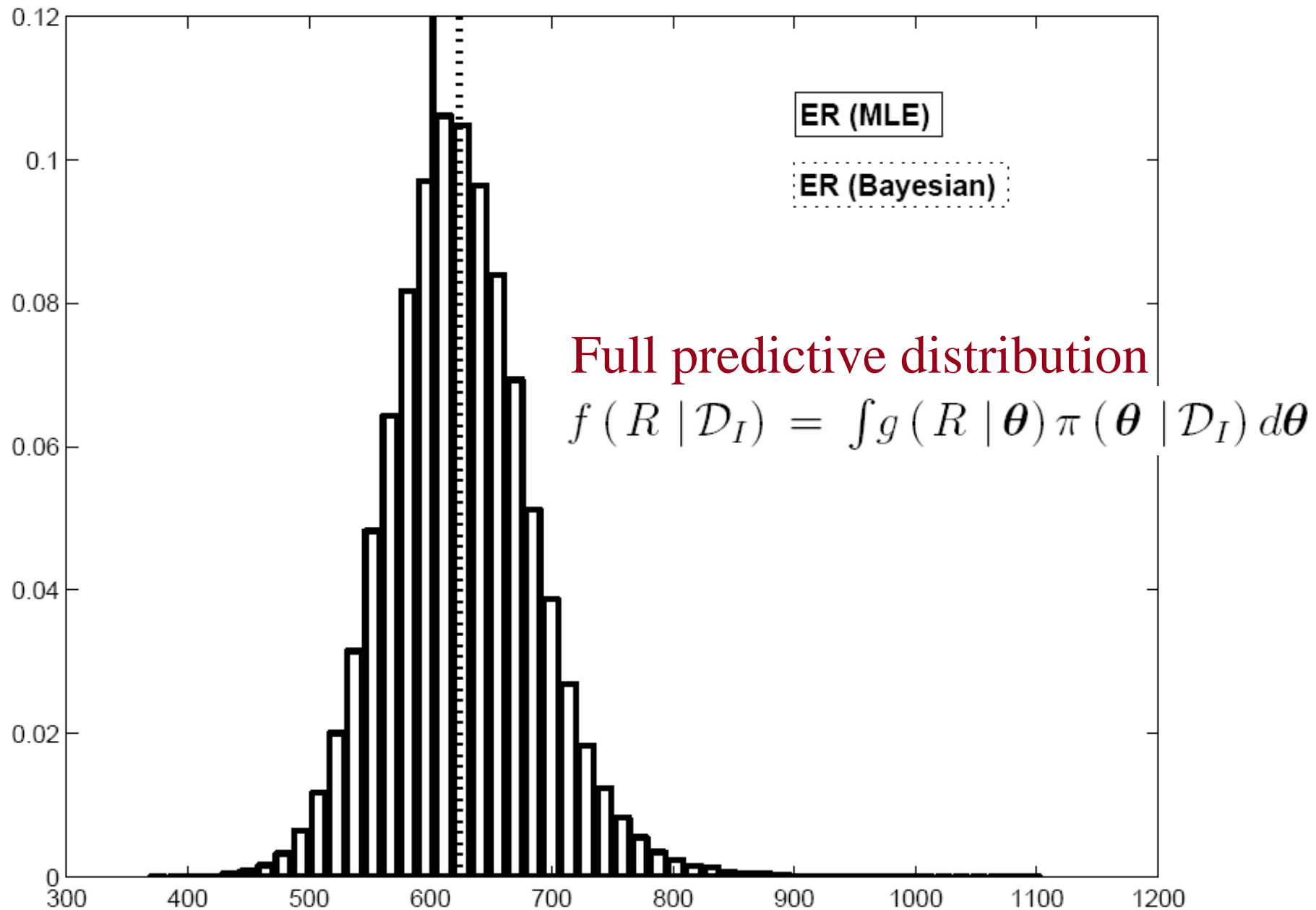


Figure 5: Distribution of total outstanding claims payment $R = \sum_{i+j>I} Y_{i,j}$, accounting for all process, estimation and model uncertainties.

Variable selection models

development pattern $\beta = (\beta_0, \dots, \beta_I)$

exposures $\alpha = (\alpha_0, \dots, \alpha_I)$	accident	development years j				
	year i	0	1	...	j	...
0	observed claims payments $Y_{i,j} \in \mathcal{D}_I$					
1						
⋮	$\mathcal{D}_I = \{Y_{i,j}; i + j \leq I\}$					
i						
⋮	outstanding claims payment					
$I - 1$						
I	$R = \sum_{i=1}^I R_i = \sum_{i+j>I} Y_{i,j}$					
	$\mathcal{D}_I^c = \{Y_{i,j}; i + j > I, i \leq I\}$					

Variable selection models

- $M_0 : \boldsymbol{\theta}_{[0]} = (p, \phi, \tilde{\alpha}_0 = \alpha_0, \dots, \tilde{\alpha}_I = \alpha_I, \tilde{\beta}_0 = \beta_0, \dots, \tilde{\beta}_I = \beta_I)$
- $M_1 : \boldsymbol{\theta}_{[1]} = (p, \phi, \tilde{\beta}_0)$ with $(\tilde{\beta}_0 = \beta_0 \dots = \beta_I), (\alpha_0 = \dots = \alpha_I = 1)$
- $M_2 : \boldsymbol{\theta}_{[2]} = (p, \phi, \tilde{\alpha}_1, \tilde{\beta}_0, \tilde{\beta}_1)$ with $(\alpha_0 = \dots = \alpha_4 = 1), (\tilde{\alpha}_1 = \alpha_5 = \dots = \alpha_I),$
 $(\tilde{\beta}_0 = \beta_0 = \dots = \beta_4), (\tilde{\beta}_1 = \beta_5 = \dots = \beta_I).$
-
-
-
- $M_6 : \boldsymbol{\theta}_{[6]} = (p, \phi, \alpha_0, \tilde{\alpha}_1, \beta_0, \beta_1, \dots, \beta_I)$ with $(\tilde{\alpha}_1 = \alpha_1 = \dots = \alpha_I)$

Variable selection models

the joint posterior distribution $\pi(M_k, \boldsymbol{\theta}_{[k]} \mid \mathcal{D}_I), \boldsymbol{\theta}_{[k]} = (\tilde{\theta}_1, \tilde{\theta}_2, \dots, \tilde{\theta}_{N_{[k]}})$

a prior distribution $\pi(M_k)$ for the model

a prior for the parameters conditional on the model $\pi(\boldsymbol{\theta}_{[k]} \mid M_k)$

$$\begin{aligned} \pi(M_k \mid \mathcal{D}_I) &= \int \pi(M_k, \boldsymbol{\theta}_{[k]} \mid \mathcal{D}_I) d\boldsymbol{\theta}_{[k]} = \int \pi(M_k \mid \boldsymbol{\theta}_{[k]}, \mathcal{D}_I) \pi(\boldsymbol{\theta}_{[k]} \mid \mathcal{D}_I) d\boldsymbol{\theta}_{[k]} \\ &\approx \frac{1}{T - T_b} \sum_{j=T_b+1}^T \frac{L_{\mathcal{D}_I}(M_k, \boldsymbol{\theta}_{j,[k]})}{\sum_{m=0}^K L_{\mathcal{D}_I}(M_m, \boldsymbol{\theta}_{j,[m]})} \end{aligned}$$

	M_0	M_1	M_2	M_3	M_4	M_5	M_6
$\pi(M_k \mid \mathcal{D}_I)$	0.71	4.19E-54	3.04E-43	1.03E-28	6.71E-20	2.17E-21	0.29
DIC	399	649	600	535	498	507	398
LHR <i>p</i> - value	1	2.76E-50	1.67E-40	3.53E-28	5.78E-21	3.03E-23	0.043

Table 4: Posterior model probabilities $\pi(M_k \mid \mathcal{D}_I)$, Deviance Information Criterion (DIC) and Likelihood Ratio (LHR) p-values for variable selection models M_0, \dots, M_6

Claims reserves

	Model Averaging	Model Selection for p
Estimated Reserves	$ER = \widehat{R}^B = E[\widetilde{R} \mathcal{D}_I]$	$ER_p = E[\widetilde{R} \mathcal{D}_I, p]$
Process Variance	$PV = E[\sum \phi(\alpha_i \beta_j)^p \mathcal{D}_I]$	$PV_p = E[\sum \phi(\alpha_i \beta_j)^p \mathcal{D}_I, p]$
Estimation Error	$EE = \text{Var}(\widetilde{R} \mathcal{D}_I)$	$EE_p = \text{Var}(\widetilde{R} \mathcal{D}_I, p)$

Table 5: Quantities used for analysis of the claims reserving problem

$$ER = E[ER_p|\mathcal{D}_I],$$

$$PV = E[PV_p|\mathcal{D}_I],$$

$$EE = E[EE_p|\mathcal{D}_I] + \text{Var}(ER_p|\mathcal{D}_I)$$

	Model Averaging	
Statistic	Bayesian Estimate	MLE Estimate
ER	624.1 (0.7)	602.630
\sqrt{PV}	37.3 (0.2)	25.937
\sqrt{EE}	44.8 (0.5)	28.336
\sqrt{MSEP}	58.3(0.5)	38.414

Table 6: Model averaged estimates

	Model Averaging	
VaR_q	R	\tilde{R}
$VaR_{75\%}$	659.8 (0.9)	650.6 (1.0)
$VaR_{90\%}$	698.4 (1.2)	680.4 (1.3)
$VaR_{95\%}$	724.0 (1.5)	701.7 (1.6)

Table 7: Bayesian model averaged estimates of Value at Risk

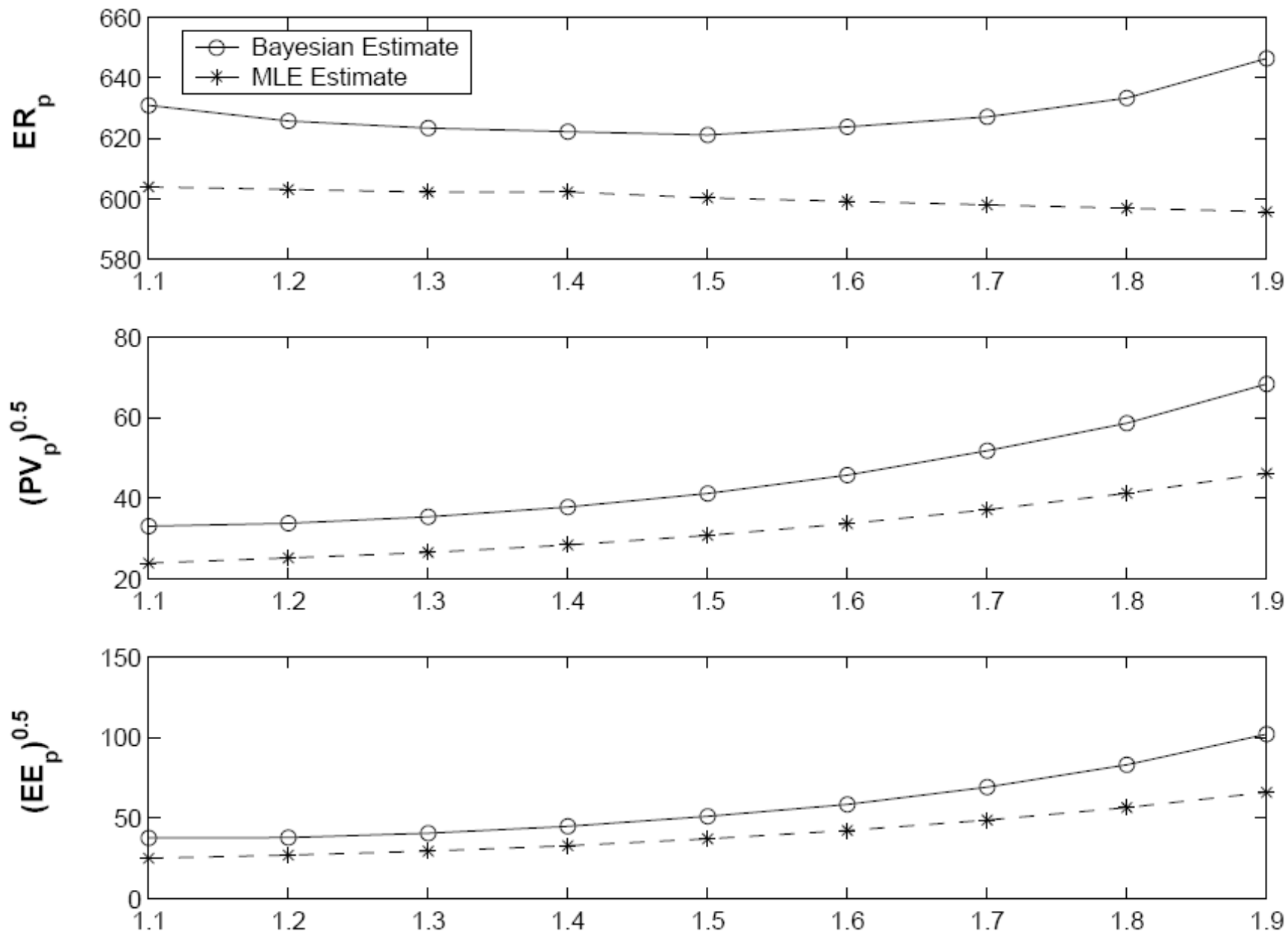


Figure 6: Estimates of quantities from Table 5 conditional on p .

Results: average over p

- Claims reserve MLE, \hat{R}^{MLE} , is less than Bayesian estimate \hat{R}^{B}
- \sqrt{EE} and \sqrt{PV} are of the same magnitude, 6-7% of total reserve
- MLEs for \sqrt{EE} and \sqrt{PV} are less than Bayesian estimates, $\approx 30\%$
- The difference between \hat{R}^{MLE} and \hat{R}^{B} is of the order of magnitude as \sqrt{EE} and \sqrt{PV}

Results: conditioning on p

- MLE of ER_p is almost constant
 - Bayesian estimates for ER_p changes as a function of p .
 - Bayesian estimators for $\sqrt{PV_p}$ and $\sqrt{EE_p}$ increase as p increases
- The MLEs for PV_p and EE_p are significantly less than Bayesian estimators.

Conclusions

- ◆ Development of a Bayesian model for claims reserving under Tweedie's compound Poisson model covering range between Poisson and Gamma models
- ◆ Quantification process, estimation and model uncertainties and variable selection using MCMC (random walk Metropolis-Hastings within Gibbs)
- ◆ MLEs are materially different from Bayesian estimators – posterior distributions are different from Gaussian.

Future work: variable selection problem – Reversible Jump MCMC; considering model parameter p outside the (1, 2) range; dynamic model.

Pavel Shevchenko

Principal Research Scientist

CSIRO Mathematical & Information Sciences

Email: Pavel.Shevchenko@csiro.au

www.cmis.csiro.au/Pavel.Shevchenko

www.csiro.au

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