Bornhuetter–Ferguson as a General Principle of Loss Reserving

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ASTIN Conference
Manchester, July 14–16, 2008

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Run–Off Triangles of Cumulative Losses (1)

An example from the *Claims Reserving Manual*:

Accident Year	Development Year 0	1	2	3	4	5
0	1001	1855	2423	2988	3335	3483
1	1113	2103	2774	3422	3844	
2	1265	2433	3233	3977		
3	1490	2873	3880			
4	1725	3261				
5	1889					

The enumeration of the development years represents delays with respect to the accident years.

Run–Off Triangles of Cumulative Losses (2)

Accident Year	Development Year 0	1	...	k	...	$n-i$	...	$n-1$	n
0	$S_{0,0}$	$S_{0,1}$	...	$S_{0,k}$	...	$S_{0,n-i}$	...	$S_{0,n-1}$	$S_{0,n}$
1	$S_{1,0}$	$S_{1,1}$	...	$S_{1,k}$	...	$S_{1,n-i}$	...	$S_{1,n-1}$	$S_{1,n}$
$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$		$\vdots$	$\vdots$
i	$S_{i,0}$	$S_{i,1}$	...	$S_{i,k}$	...	$S_{i,n-i}$	...	$S_{i,n-1}$	$S_{i,n}$
$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$		$\vdots$	$\vdots$
$n-k$	$S_{n-k,0}$	$S_{n-k,1}$	...	$S_{n-k,k}$	...	$S_{n-k,n-i}$	...	$S_{n-k,n-1}$	$S_{n-k,n}$
$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$		$\vdots$	$\vdots$
$n-1$	$S_{n-1,0}$	$S_{n-1,1}$	...	$S_{n-1,k}$	...	$S_{n-1,n-i}$	...	$S_{n-1,n-1}$	$S_{n-1,n}$
n	$S_{n,0}$	$S_{n,1}$	...	$S_{n,k}$	...	$S_{n,n-i}$	...	$S_{n,n-1}$	$S_{n,n}$

Run–Off Triangles of Cumulative Losses (2)

Accident Year	Development Year 0	1	…	k	…	$n-i$	…	$n-1$	n
0	$S_{0,0}$	$S_{0,1}$	…	$S_{0,k}$	…	$S_{0,n-i}$	…	$S_{0,n-1}$	$S_{0,n}$
1	$S_{1,0}$	$S_{1,1}$	…	$S_{1,k}$	…	$S_{1,n-i}$	…	$S_{1,n-1}$	$S_{1,n}$
⋮	⋮	⋮		⋮		⋮		⋮	⋮
i	$S_{i,0}$	$S_{i,1}$	…	$S_{i,k}$	…	$S_{i,n-i}$	…	$S_{i,n-1}$	$S_{i,n}$
⋮	⋮	⋮		⋮		⋮		⋮	⋮
$n-k$	$S_{n-k,0}$	$S_{n-k,1}$	…	$S_{n-k,k}$	…	$S_{n-k,n-i}$	…	$S_{n-k,n-1}$	$S_{n-k,n}$
⋮	⋮	⋮		⋮		⋮		⋮	⋮
$n-1$	$S_{n-1,0}$	$S_{n-1,1}$	…	$S_{n-1,k}$	…	$S_{n-1,n-i}$	…	$S_{n-1,n-1}$	$S_{n-1,n}$
n	$S_{n,0}$	$S_{n,1}$	…	$S_{n,k}$	…	$S_{n,n-i}$	…	$S_{n,n-1}$	$S_{n,n}$

A cumulative loss $S_{i,k}$ is said to be

Run–Off Triangles of Cumulative Losses (2)

Accident Year	**Development Year** 0	1	$\dots$	k	$\dots$	$n-i$	$\dots$	$n-1$	n
0	$S_{0,0}$	$S_{0,1}$	$\dots$	$S_{0,k}$	$\dots$	$S_{0,n-i}$	$\dots$	$S_{0,n-1}$	$S_{0,n}$
1	$S_{1,0}$	$S_{1,1}$	$\dots$	$S_{1,k}$	$\dots$	$S_{1,n-i}$	$\dots$	$S_{1,n-1}$	$S_{1,n}$
$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$		$\vdots$	$\vdots$
i	$S_{i,0}$	$S_{i,1}$	$\dots$	$S_{i,k}$	$\dots$	$S_{i,n-i}$	$\dots$	$S_{i,n-1}$	$S_{i,n}$
$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$		$\vdots$	$\vdots$
$n-k$	$S_{n-k,0}$	$S_{n-k,1}$	$\dots$	$S_{n-k,k}$	$\dots$	$S_{n-k,n-i}$	$\dots$	$S_{n-k,n-1}$	$S_{n-k,n}$
$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$		$\vdots$	$\vdots$
$n-1$	$S_{n-1,0}$	$S_{n-1,1}$	$\dots$	$S_{n-1,k}$	$\dots$	$S_{n-1,n-i}$	$\dots$	$S_{n-1,n-1}$	$S_{n-1,n}$
n	$S_{n,0}$	$S_{n,1}$	$\dots$	$S_{n,k}$	$\dots$	$S_{n,n-i}$	$\dots$	$S_{n,n-1}$	$S_{n,n}$

A cumulative loss $S_{i,k}$ is said to be

- observable if $i + k \leq n$.

Run–Off Triangles of Cumulative Losses (2)

Accident Year	Development Year								
	0	1	...	k	...	$n-i$	...	$n-1$	n
0	$S_{0,0}$	$S_{0,1}$	...	$S_{0,k}$	...	$S_{0,n-i}$	...	$S_{0,n-1}$	$S_{0,n}$
1	$S_{1,0}$	$S_{1,1}$	...	$S_{1,k}$	...	$S_{1,n-i}$	...	$S_{1,n-1}$	$S_{1,n}$
$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$		$\vdots$	$\vdots$
i	$S_{i,0}$	$S_{i,1}$	...	$S_{i,k}$	...	$S_{i,n-i}$	...	$S_{i,n-1}$	$S_{i,n}$
$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$		$\vdots$	$\vdots$
$n-k$	$S_{n-k,0}$	$S_{n-k,1}$	...	$S_{n-k,k}$	...	$S_{n-k,n-i}$	...	$S_{n-k,n-1}$	$S_{n-k,n}$
$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$		$\vdots$	$\vdots$
$n-1$	$S_{n-1,0}$	$S_{n-1,1}$	...	$S_{n-1,k}$	...	$S_{n-1,n-i}$	...	$S_{n-1,n-1}$	$S_{n-1,n}$
n	$S_{n,0}$	$S_{n,1}$	...	$S_{n,k}$	...	$S_{n,n-i}$	...	$S_{n,n-1}$	$S_{n,n}$

A cumulative loss $S_{i,k}$ is said to be

- observable if $i+k \leq n$.
- non–observable or future if $i+k > n$.

Run–Off Triangles of Cumulative Losses (2)

Accident Year	Development Year 0	1	...	k	...	$n-i$	...	$n-1$	n
0	$S_{0,0}$	$S_{0,1}$	...	$S_{0,k}$	...	$S_{0,n-i}$	...	$S_{0,n-1}$	$S_{0,n}$
1	$S_{1,0}$	$S_{1,1}$	...	$S_{1,k}$	...	$S_{1,n-i}$	...	$S_{1,n-1}$	$S_{1,n}$
$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$		$\vdots$	$\vdots$
i	$S_{i,0}$	$S_{i,1}$	...	$S_{i,k}$	...	$S_{i,n-i}$	...	$S_{i,n-1}$	$S_{i,n}$
$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$		$\vdots$	$\vdots$
$n-k$	$S_{n-k,0}$	$S_{n-k,1}$	...	$S_{n-k,k}$	...	$S_{n-k,n-i}$	...	$S_{n-k,n-1}$	$S_{n-k,n}$
$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$		$\vdots$	$\vdots$
$n-1$	$S_{n-1,0}$	$S_{n-1,1}$	...	$S_{n-1,k}$	...	$S_{n-1,n-i}$	...	$S_{n-1,n-1}$	$S_{n-1,n}$
n	$S_{n,0}$	$S_{n,1}$	...	$S_{n,k}$	...	$S_{n,n-i}$	...	$S_{n,n-1}$	$S_{n,n}$

A cumulative loss $S_{i,k}$ is said to be

- observable if $i + k \leq n$.
- non–observable or future if $i + k > n$.
- current if $i + k = n$.

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Accident Year	Development Year 0	1	…	k	…	$n-i$	…	$n-1$	n
0	$S_{0,0}$	$S_{0,1}$	…	$S_{0,k}$	…	$S_{0,n-i}$	…	$S_{0,n-1}$	$S_{0,n}$
1	$S_{1,0}$	$S_{1,1}$	…	$S_{1,k}$	…	$S_{1,n-i}$	…	$S_{1,n-1}$	$S_{1,n}$
⋮	⋮	⋮		⋮		⋮		⋮	⋮
i	$S_{i,0}$	$S_{i,1}$	…	$S_{i,k}$	…	$S_{i,n-i}$	…	$S_{i,n-1}$	$S_{i,n}$
⋮	⋮	⋮		⋮		⋮		⋮	⋮
$n-k$	$S_{n-k,0}$	$S_{n-k,1}$	…	$S_{n-k,k}$	…	$S_{n-k,n-i}$	…	$S_{n-k,n-1}$	$S_{n-k,n}$
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$n-1$	$S_{n-1,0}$	$S_{n-1,1}$	…	$S_{n-1,k}$	…	$S_{n-1,n-i}$	…	$S_{n-1,n-1}$	$S_{n-1,n}$
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- observable if $i+k \leq n$.
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1	$S_{1,0}$	$S_{1,1}$	...	$S_{1,k}$	...	$S_{1,n-i}$	...	$S_{1,n-1}$	$S_{1,n}$
$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$		$\vdots$	$\vdots$
i	$S_{i,0}$	$S_{i,1}$	...	$S_{i,k}$	...	$S_{i,n-i}$	...	$S_{i,n-1}$	$S_{i,n}$
$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$		$\vdots$	$\vdots$
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$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$		$\vdots$	$\vdots$
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with $i + k \geq n + 1$

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- the future cumulative losses $S_{i,k}$
- the future incremental losses $Z_{i,k} := S_{i,k} - S_{i,k-1}$
- the calendar year reserves $\sum_{j=p-n}^{n} Z_{j,p-j}$

with $i + k \geq n + 1$ and $p = n + 1, \ldots, 2n$.

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- the ultimate losses $S_{i,n}$ and
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- the future cumulative losses $S_{i,k}$
- the future incremental losses $Z_{i,k} := S_{i,k} - S_{i,k-1}$
- the calendar year reserves $\sum_{j=p-n}^{n} Z_{j,p-j}$
- the total reserve $\sum_{j=1}^{n} \sum_{l=n-j+1}^{n} Z_{j,l}$

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- the total reserve $\sum_{j=1}^{n} \sum_{l=n-j+1}^{n} Z_{j,l}$

with $i + k \geq n + 1$ and $p = n + 1, \ldots, 2n$.

Thus: The principal task considered here is to predict the future cumulative losses.

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Development Patterns

- A development pattern for quotas consists of parameters $\gamma_0, \gamma_1, \ldots, \gamma_n$ with

$$\gamma_k = \mathrm{E}[S_{i,k}]/\mathrm{E}[S_{i,n}]$$

for all $k = 0, 1, \ldots, n$ and for all $i = 0, 1, \ldots, n$.
These parameters are called development quotas (percentages reported).

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for all $k = 0, 1, \ldots, n$ and for all $i = 0, 1, \ldots, n$.
These parameters are called development quotas (percentages reported).

- A development pattern for factors consists of parameters $\varphi_1, \ldots, \varphi_n$ with

$$\varphi_k = \mathrm{E}[S_{i,k}]/\mathrm{E}[S_{i,k-1}]$$

for all $k = 1, \ldots, n$ and for all $i = 0, 1, \ldots, n$.
These parameters are called development factors (age–to–age factors).

Development Patterns: Cumulative Losses and Quotas

Accident Year	Development Year 0	1	2	3	4	5
0	1001	1855	2423	2988	3335	3483
1	1113	2103	2774	3422	3844	4044
2	1265	2433	3233	3977	4477	4677
3	1490	2873	3880	4880	5380	5680
4	1725	3261	4361	5461	5961	6361
5	1889	3489	4889	5889	6489	6889
0	0.287	0.533	0.696	0.858	0.958	1.000
1	0.275	0.520	0.686	0.846	0.951	1.000
2	0.270	0.520	0.691	0.850	0.957	1.000
3	0.262	0.506	0.683	0.859	0.947	1.000
4	0.271	0.513	0.686	0.859	0.937	1.000
5	0.274	0.506	0.710	0.855	0.942	1.000

Development Patterns: Cumulative Losses and Factors

Accident Year	Development Year 0	1	2	3	4	5
0	1001	1855	2423	2988	3335	3483
1	1113	2103	2774	3422	3844	4044
2	1265	2433	3233	3977	4477	4677
3	1490	2873	3880	4880	5380	5680
4	1725	3261	4361	5461	5961	6361
5	1889	3489	4889	5889	6489	6889
0		1.853	1.306	1.233	1.116	1.044
1		1.889	1.319	1.234	1.123	1.052
2		1.923	1.329	1.230	1.126	1.045
3		1.928	1.351	1.258	1.102	1.056
4		1.890	1.337	1.252	1.092	1.067
5		1.847	1.401	1.205	1.102	1.062

Development Patterns: Quotas and Factors

- If the parameters $\gamma_0, \gamma_1, \ldots, \gamma_n$ form a development pattern for quotas, then the parameters $\varphi_1, \ldots, \varphi_n$ with

$$\varphi_k := \frac{\gamma_k}{\gamma_{k-1}}$$

form a development pattern for factors.

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- If the parameters $\varphi_1, \ldots, \varphi_n$ form a development pattern for factors, then the parameters $\gamma_0, \gamma_1, \ldots, \gamma_n$ with

$$\gamma_k := \prod_{l=k+1}^{n} \frac{1}{\varphi_l}$$

form a development pattern for quotas.

Development Patterns: Estimation of Quotas

For estimation of the parameter γ_k of a development pattern for quotas, the only obvious estimator provided by the run–off triangle is the empirical individual quota

$$\widehat{\gamma}_{0,k} := S_{0,k}/S_{0,n}$$

Accident Year	Development Year 0	1	2	3	4	5
0	0.287	0.533	0.696	0.858	0.958	1.000
1	0.275	0.520	0.686	0.846	0.951	1.000
2	0.270	0.520	0.691	0.850	0.957	1.000
3	0.262	0.506	0.683	0.859	0.947	1.000
4	0.271	0.513	0.686	0.859	0.937	1.000
5	0.274	0.506	0.710	0.855	0.942	1.000

Development Patterns: Estimation of Factors

For estimation of the parameter φ_k of a development pattern for factors, the run–off triangle provides the empirical individual factors

$$\widehat{\varphi}_{i,k} := S_{i,k}/S_{i,k-1}$$

with $i = 0, 1, \ldots, n-k$. Moreover, any weighted mean of these estimators is an estimator as well.

Accident Year	Development Year 0	1	2	3	4	5
0		1.853	1.306	1.233	1.116	1.044
1		1.889	1.319	1.234	1.123	1.052
2		1.923	1.329	1.230	1.126	1.045
3		1.928	1.351	1.258	1.102	1.056
4		1.890	1.337	1.252	1.092	1.067
5		1.847	1.401	1.205	1.102	1.062

Development Patterns: Chain–Ladder Factors

The chain–ladder factors

$$\widehat{\varphi}_k^{\text{CL}} := \frac{\sum_{j=0}^{n-k} S_{j,k}}{\sum_{j=0}^{n-k} S_{j,k-1}} = \sum_{j=0}^{n-k} \frac{S_{j,k-1}}{\sum_{h=0}^{n-k} S_{h,k-1}} \, \widehat{\varphi}_{j,k}$$

are weighted means and may be used to estimate the development factors φ_k.

Accident Year i	Development Year k 0	1	2	3	4	5
0	1001	1855	2423	2988	3335	3483
1	1113	2103	2774	3422	3844	
2	1265	2433	3233	3977		
3	1490	2873	3880			
4	1725	3261				
5	1889					
$\widehat{\varphi}_k^{\text{CL}}$		1.899	1.329	1.232	1.120	1.044

Development Patterns: Chain–Ladder Quotas

The chain–ladder quotas

$$\widehat{\gamma}_k^{\mathrm{CL}} := \prod_{l=k+1}^{n} \frac{1}{\widehat{\varphi}_l^{\mathrm{CL}}}$$

may be used to estimate the development quotas γ_k.

Accident Year i	**Development Year** k 0	1	2	3	4	5
0	1001	1855	2423	2988	3335	3483
1	1113	2103	2774	3422	3844	
2	1265	2433	3233	3977		
3	1490	2873	3880			
4	1725	3261				
5	1889					
$\widehat{\varphi}_k^{\mathrm{CL}}$		1.899	1.329	1.232	1.120	1.044
$\widehat{\gamma}_k^{\mathrm{CL}}$	0.278	0.527	0.701	0.864	0.968	1

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The original Bornhuetter–Ferguson Method (1)

In its original version, the Bornhuetter–Ferguson method aims at predicting calendar year reserves

$$R_i := S_{i,n} - S_{i,n-i}$$

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If $\gamma_0, \gamma_1, \ldots, \gamma_n$ is a development pattern for quotas, then the expected reserves satisfy the model equation

$$E[R_i] = \left(1 - \gamma_{n-i}\right) E[S_{i,n}]$$

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$$E[R_i] = \Big(1 - \gamma_{n-i}\Big)\, E[S_{i,n}]$$

The original Bornhuetter–Ferguson predictors of the reserves R_i are defined as

$$\widehat{R}_i := \Big(1 - \widehat{\gamma}^{\text{CL}}_{n-i}\Big)\, \pi_i\, \widehat{\kappa}_i$$

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$$R_i := S_{i,n} - S_{i,n-i}$$

If $\gamma_0, \gamma_1, \ldots, \gamma_n$ is a development pattern for quotas, then the expected reserves satisfy the model equation

$$E[R_i] = \left(1 - \gamma_{n-i}\right) E[S_{i,n}]$$

The original Bornhuetter–Ferguson predictors of the reserves R_i are defined as

$$\widehat{R}_i := \left(1 - \widehat{\gamma}^{\text{CL}}_{n-i}\right) \pi_i \, \widehat{\kappa}_i$$

where

- $\widehat{\gamma}^{\text{CL}}_{n-i}$ is the current chain–ladder quota,
- π_i is a volume measure, and
- $\widehat{\kappa}_i$ is an estimator of the expected loss ratio $\kappa_i := E[S_{i,n}/\pi_i]$

The original Bornhuetter–Ferguson Method (2)

- Transformation into predictors of the ultimate losses $S_{i,n}$:

$$\widehat{S}_{i,n} := S_{i,n-i} + \left(1 - \widehat{\gamma}^{\text{CL}}_{n-i}\right) \pi_i \, \widehat{\kappa}_i$$

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- Transformation into predictors of other future cumulative losses $S_{i,k}$:

$$\widehat{S}_{i,k} := S_{i,n-i} + \left(\widehat{\gamma}^{\text{CL}}_{k} - \widehat{\gamma}^{\text{CL}}_{n-i}\right) \pi_i \, \widehat{\kappa}_i$$

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Idea:

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- Transformation into predictors of the ultimate losses $S_{i,n}$:

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Idea:

- Replace the chain–ladder quotas by arbitrary estimators of the quotas.

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Idea:

- Replace the chain–ladder quotas by arbitrary estimators of the quotas.
- Replace the estimators $\pi_i \, \widehat{\kappa}_i$ by arbitrary estimators of the expected ultimate losses $E[S_{i,n}]$.

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The extended Bornhuetter–Ferguson Method (1)

The extended Bornhuetter–Ferguson method is based on the assumption, that there exists a development pattern for quotas

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- prior estimators

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  (with $\widehat{\gamma}_n = 1$) of the development quotas $\gamma_0, \gamma_1, \ldots, \gamma_n$ and

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$$\widehat{\alpha}_0, \widehat{\alpha}_1, \ldots, \widehat{\alpha}_n$$

  of the expected ultimate losses

$$\alpha_i := E[S_{i,n}]$$

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These prior estimators can be obtained from

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- a combination of these data.

The extended Bornhuetter–Ferguson Method (3)

The future cumulative losses satisfy the model equation

$$E[S_{i,k}] = E[S_{i,n-i}] + \left(\gamma_k - \gamma_{n-i}\right) E[S_{i,n}]$$

The extended Bornhuetter–Ferguson Method (3)

The future cumulative losses satisfy the model equation

$$E[S_{i,k}] = E[S_{i,n-i}] + \Big(\gamma_k - \gamma_{n-i}\Big) E[S_{i,n}]$$

Accordingly, the extended Bornhuetter–Ferguson predictors of the future cumulative losses are defined as

$$\widehat{S}_{i,k}^{\text{BF}} := S_{i,n-i} + \Big(\widehat{\gamma}_k - \widehat{\gamma}_{n-i}\Big) \widehat{\alpha}_i$$

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Thus:

- The run–off triangle provides information perhaps only via the current losses.

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Thus:

- The run–off triangle provides information perhaps only via the current losses.
- The predictors of the ultimate losses are obtained by linear extrapolation from the current losses.

The extended Bornhuetter–Ferguson Method (4)

Accident Year i	**Development Year** k 0	1	2	3	4	5	$\widehat{\alpha}_i$
0						3483	3517
1					3844		3981
2				3977			4598
3			3880				5658
4		3261					6214
5	1889						6325
$\widehat{\gamma}_k$	0.280	0.510	0.700	0.860	0.950	1.000	
$1-\widehat{\gamma}_k$	0.720	0.490	0.300	0.140	0.050	0.000	

The extended Bornhuetter–Ferguson Method (4)

Accident Year i	Development Year k 0	1	2	3	4	5	$\widehat{\alpha}_i$
0						3483	3517
1					3844	4043	3981
2				3977	4391	4621	4598
3			3880	4785	4389	5577	5658
4		3261	4442	5436	5995	6306	6214
5	1889	3344	4546	5558	6127	6443	6325
$\widehat{\gamma}_k$	0.280	0.510	0.700	0.860	0.950	1.000	
$1 - \widehat{\gamma}_k$	0.720	0.490	0.300	0.140	0.050	0.000	

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The Loss–Development Method (1)

The loss–development method is based on the assumption, that there exists a development pattern for quotas and that prior estimators

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The loss–development method does not involve any prior estimators for the expected ultimate losses.

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The loss–development method does not involve any prior estimators for the expected ultimate losses.

The Loss–Development Method (2)

The future cumulative losses satisfy the model equation

$$E[S_{i,k}] = \gamma_k \frac{E[S_{i,n-i}]}{\gamma_{n-i}}$$

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Accordingly, the loss–development predictors of the future cumulative losses are defined as

$$\widehat{S}_{i,k}^{\mathrm{LD}} := \widehat{\gamma}_k \frac{S_{i,n-i}}{\widehat{\gamma}_{n-i}}$$

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Accordingly, the loss–development predictors of the future cumulative losses are defined as

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Thus:

- The run–off triangle provides information perhaps only via the current losses.
- The predictors of the ultimate losses are obtained by scaling the current losses.
- The predictors of other future cumulative losses are obtained by scaling the predictors of the ultimate losses.

The Loss–Development Method (3)

Accident Year i	Development Year k 0	1	2	3	4	5
0						3483
1					3844	
2				3977		
3			3880			
4		3261				
5	1889					
$\widehat{\gamma}_k$	0,280	0,510	0,700	0,860	0,950	1,000

The Loss–Development Method (3)

Accident Year i	Development Year k 0	1	2	3	4	5
0						3483
1					3844	4046
2				3977		4624
3			3880			5543
4		3261				6394
5	1889					6746
$\widehat{\gamma}_k$	0,280	0,510	0,700	0,860	0,950	1,000

The Loss–Development Method (3)

Accident Year i	Development Year k 0	1	2	3	4	5
0						3483
1					3844	4046
2				3977	4393	4624
3			3880	4767	5266	5543
4		3261	4476	5499	6074	6394
5	1889	3440	4722	5802	6409	6746
$\widehat{\gamma}_k$	0,280	0,510	0,700	0,860	0,950	1,000

The Loss–Development Method (4)

Because of the definition

$$\widehat{S}_{i,k}^{\text{LD}} := \widehat{\gamma}_k \, \frac{S_{i,n-i}}{\widehat{\gamma}_{n-i}}$$

the loss–development predictors can be written as

$$\widehat{S}_{i,k}^{\text{LD}} = S_{i,n-i} + \left(\widehat{\gamma}_k - \widehat{\gamma}_{n-i}\right) \frac{S_{i,n-i}}{\widehat{\gamma}_{n-i}}$$

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$$\widehat{S}^{\text{LD}}_{i,k} = S_{i,n-i} + \left(\widehat{\gamma}_k - \widehat{\gamma}_{n-i} \right) \frac{S_{i,n-i}}{\widehat{\gamma}_{n-i}}$$

In this form, the loss–development predictors attain the shape of the extended Bornhuetter–Ferguson predictors with respect to the prior estimators

$$\widehat{\alpha}^{\text{LD}}_i := \frac{S_{i,n-i}}{\widehat{\gamma}_{n-i}}$$

of the expected ultimate losses.

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The Chain–Ladder Method (1)

The chain–ladder method is based on the assumption that there exists a development pattern for factors.

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As estimators of the development factors, the chain–ladder method uses the chain–ladder factors

$$\widehat{\varphi}_k^{\mathrm{CL}} := \frac{\sum_{j=0}^{n-k} S_{j,k}}{\sum_{j=0}^{n-k} S_{j,k-1}} = \sum_{j=0}^{n-k} \frac{S_{j,k-1}}{\sum_{h=0}^{n-k} S_{h,k-1}} \, \widehat{\varphi}_{j,k}$$

The Chain–Ladder Method (2)

The future cumulative losses $S_{i,k}$ satisfy the model equation

$$E[S_{i,k}] = E[S_{i,n-i}] \prod_{l=n-i+1}^{k} \varphi_l$$

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The future cumulative losses $S_{i,k}$ satisfy the model equation

$$E[S_{i,k}] = E[S_{i,n-i}] \prod_{l=n-i+1}^{k} \varphi_l$$

Accordingly, the chain–ladder predictors of the future cumulative losses are defined as

$$\widehat{S}_{i,k}^{\text{CL}} := S_{i,n-i} \prod_{l=n-i+1}^{k} \widehat{\varphi}_l^{\text{CL}}$$

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Thus:

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The future cumulative losses $S_{i,k}$ satisfy the model equation

$$E[S_{i,k}] = E[S_{i,n-i}] \prod_{l=n-i+1}^{k} \varphi_l$$

Accordingly, the chain–ladder predictors of the future cumulative losses are defined as

$$\widehat{S}_{i,k}^{\text{CL}} := S_{i,n-i} \prod_{l=n-i+1}^{k} \widehat{\varphi}_l^{\text{CL}}$$

Thus:

- The chain–ladder method consists in successive scaling of the current loss $S_{i,n-i}$ to the level of the future cumulative loss $S_{i,k}$.

The Chain–Ladder Method (3)

Accident Year i	**Development Year** k 0	1	2	3	4	5
0	1001	1855	2423	2988	3335	3483
1	1113	2103	2774	3422	3844	
2	1265	2433	3233	3977		
3	1490	2873	3880			
4	1725	3261				
5	1889					
$\widehat{\varphi}_k^{CL}$		1,899	1,329	1,232	1,120	1,044

The Chain–Ladder Method (3)

Accident Year i	**Development Year k** 0	1	2	3	4	5
0	1001	1855	2423	2988	3335	3483
1	1113	2103	2774	3422	3844	
2	1265	2433	3233	3977		
3	1490	2873	3880			
4	1725	3261				
5	1889	3587				
$\widehat{\varphi}_k^{\mathrm{CL}}$		1,899	1,329	1,232	1,120	1,044

The Chain–Ladder Method (3)

Accident Year i	**Development Year** k 0	1	2	3	4	5
0	1001	1855	2423	2988	3335	3483
1	1113	2103	2774	3422	3844	
2	1265	2433	3233	3977		
3	1490	2873	3880			
4	1725	3261	4334			
5	1889	3587	4767			
$\widehat{\varphi}_k^{CL}$		1,899	1,329	1,232	1,120	1,044

The Chain–Ladder Method (3)

Accident Year i	**Development Year** k 0	1	2	3	4	5
0	1001	1855	2423	2988	3335	3483
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2	1265	2433	3233	3977		
3	1490	2873	3880	4780		
4	1725	3261	4334	5339		
5	1889	3587	4767	5873		
$\widehat{\varphi}_k^{\mathrm{CL}}$		1,899	1,329	1,232	1,120	1,044

The Chain–Ladder Method (3)

Accident Year i	Development Year k 0	1	2	3	4	5
0	1001	1855	2423	2988	3335	3483
1	1113	2103	2774	3422	3844	
2	1265	2433	3233	3977	4454	
3	1490	2873	3880	4780	5354	
4	1725	3261	4334	5339	5980	
5	1889	3587	4767	5873	6578	
$\widehat{\varphi}_k^{CL}$		1,899	1,329	1,232	1,120	1,044

The Chain–Ladder Method (3)

Accident Year i	**Development Year** k 0	1	2	3	4	5
0	1001	1855	2423	2988	3335	3483
1	1113	2103	2774	3422	3844	4013
2	1265	2433	3233	3977	4454	4650
3	1490	2873	3880	4780	5354	5590
4	1725	3261	4334	5339	5980	6243
5	1889	3587	4767	5873	6578	6867
$\widehat{\varphi}_k^{CL}$		1,899	1,329	1,232	1,120	1,044

The Chain–Ladder Method (4)

Because of the definition

$$\widehat{S}_{i,k}^{\text{CL}} := S_{i,n-i} \prod_{l=n-i+1}^{k} \widehat{\varphi}_l^{\text{CL}}$$

the chain–ladder predictors of the future cumulative losses can be written as

$$\widehat{S}_{i,k}^{\text{CL}} = \widehat{\gamma}_k^{\text{CL}} \frac{S_{i,n-i}}{\widehat{\gamma}_{n-i}^{\text{CL}}}$$

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the chain–ladder predictors of the future cumulative losses can be written as

$$\widehat{S}_{i,k}^{\text{CL}} = \widehat{\gamma}_k^{\text{CL}} \frac{S_{i,n-i}}{\widehat{\gamma}_{n-i}^{\text{CL}}}$$

In this form, the chain–ladder predictors attain the shape of the loss–development predictors with respect to the chain–ladder quotas.

The Chain–Ladder Method (5)

Since

$$\widehat{S}_{i,k}^{\text{CL}} = \widehat{\gamma}_k^{\text{CL}} \, \frac{S_{i,n-i}}{\widehat{\gamma}_{n-i}^{\text{CL}}}$$

the chain–ladder predictors of the future cumulative losses can also be written as

$$\widehat{S}_{i,k}^{\text{CL}} = S_{i,n-i} + \left(\widehat{\gamma}_k^{\text{CL}} - \widehat{\gamma}_{n-i}^{\text{CL}}\right) \frac{S_{i,n-i}}{\widehat{\gamma}_{n-i}^{\text{CL}}}$$

The Chain–Ladder Method (5)

Since

$$\widehat{S}_{i,k}^{\text{CL}} = \widehat{\gamma}_k^{\text{CL}} \frac{S_{i,n-i}}{\widehat{\gamma}_{n-i}^{\text{CL}}}$$

the chain–ladder predictors of the future cumulative losses can also be written as

$$\widehat{S}_{i,k}^{\text{CL}} = S_{i,n-i} + \left(\widehat{\gamma}_k^{\text{CL}} - \widehat{\gamma}_{n-i}^{\text{CL}}\right) \frac{S_{i,n-i}}{\widehat{\gamma}_{n-i}^{\text{CL}}}$$

In this form, the chain–ladder predictors attain the shape of the extended Bornhuetter–Ferguson predictors with respect to the chain–ladder quotas and the prior estimators

$$\widehat{\alpha}_i^{\text{CL}} := \frac{S_{i,n-i}}{\widehat{\gamma}_{n-i}^{\text{CL}}}$$

of the expected ultimate losses.

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The Cape Cod Method (1)

The Cape Cod method is based on the assumption, that there exists a development pattern for quotas and that prior estimators

$$\widehat{\gamma}_0, \widehat{\gamma}_1, \ldots, \widehat{\gamma}_n$$

(with $\widehat{\gamma}_n = 1$) of the development quotas $\gamma_0, \gamma_1, \ldots, \gamma_n$ are available.

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$$\widehat{\gamma}_0, \widehat{\gamma}_1, \dots, \widehat{\gamma}_n$$

(with $\widehat{\gamma}_n = 1$) of the development quotas $\gamma_0, \gamma_1, \dots, \gamma_n$ are available.

It is also based on the assumption that there exist volume measures $\pi_0, \pi_1, \dots, \pi_n$ for the accident years and that the expected ultimate loss ratio

$$\kappa := E\left[\frac{S_{i,n}}{\pi_i}\right]$$

is the same for all accident years.

The Cape Cod Method (2)

The future cumulative losses satisfy the model equation

$$E[S_{i,k}] = E[S_{i,n-i}] + \Big(\gamma_k - \gamma_{n-i}\Big)\pi_i\,\kappa$$

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The future cumulative losses satisfy the model equation

$$E[S_{i,k}] = E[S_{i,n-i}] + \Big(\gamma_k - \gamma_{n-i}\Big)\pi_i\,\kappa$$

Accordingly, the Cape Cod predictors of the future cumulative losses are defined as

$$\widehat{S}_{i,k}^{\text{CC}} := S_{i,n-i} + \Big(\widehat{\gamma}_k - \widehat{\gamma}_{n-i}\Big)\pi_i\,\widehat{\kappa}^{\text{CC}}(\boldsymbol{\pi}, \widehat{\boldsymbol{\gamma}})$$

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Accordingly, the Cape Cod predictors of the future cumulative losses are defined as

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where

$$\widehat{\kappa}^{\text{CC}}(\boldsymbol{\pi},\widehat{\boldsymbol{\gamma}}) := \frac{\sum_{j=0}^{n} S_{j,n-j}}{\sum_{j=0}^{n} \pi_j\,\widehat{\gamma}_{n-j}}$$

is the Cape Cod loss ratio.

The Cape Cod Method (3)

Therefore, the Cape Cod predictors of the future cumulative losses have the shape of the extended Bornhuetter–Ferguson predictors with respect to the Cape Cod estimators

$$\widehat{\alpha}_i^{\mathrm{CC}} := \pi_i \, \widehat{\kappa}^{\mathrm{CC}}(\boldsymbol{\pi}, \widehat{\gamma})$$

of the expected ultimate losses.

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The Additive Method (1)

The additive method (or incremental loss ratio method) is based on the assumption, that there exist

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- volume measures $\pi_0, \pi_1, \ldots, \pi_n$ for the accident years, and
- parameters $\zeta_0, \zeta_1, \ldots, \zeta_n$ such that the expected incremental loss ratio

$$\zeta_k := E\left[\frac{Z_{i,k}}{\pi_i}\right]$$

is the same for all accident years, where

$$Z_{i,k} := \begin{cases} S_{i,0} & \text{if } k = 0 \\ S_{i,k} - S_{i,k-1} & \text{else} \end{cases}$$

is the incremental loss of accident year i and development year k.

The Additive Method (2)

The cumulative and incremental losses satisfy the model equation

$$E[S_{i,k}] = E[S_{i,n-i}] + \pi_i \sum_{l=n-i+1}^{k} \zeta_l$$

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is the additive incremental loss ratio of development year k.

The Additive Method (3)

Since

$$\widehat{S}_{i,k}^{\text{AD}} := S_{i,n-i} + \pi_i \sum_{l=n-i+1}^{k} \widehat{\zeta}_l^{\text{AD}}$$

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or as

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In this form, the additive predictors of the future cumulative losses have the shape of the extended Bornhuetter–Ferguson predictors with respect to the additive quotas $\widehat{\gamma}_k^{\text{AD}}(\boldsymbol{\pi})$ and the additive estimators $\widehat{\alpha}_i^{\text{AD}}(\boldsymbol{\pi})$ of the expected ultimate losses.

The Additive Method (4)

Remark:

It can be shown that

$$\widehat{\alpha}_i^{\mathrm{AD}}(\pi) = \widehat{\alpha}_i^{\mathrm{CC}}(\pi, \widehat{\gamma}^{\mathrm{AD}}(\pi))$$

such that the additive method can be viewed as the Cape Cod method with respect to the volume measures π and the additive quotas $\widehat{\gamma}^{\mathrm{AD}}(\pi)$.

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The Bornhuetter–Ferguson Principle (1)

Comparison of certain versions of the extended Bornhuetter–Ferguson method:

Prior Estimators of Expected Ultimate Losses	**Prior Estimators of Cumulative Quotas** $\widehat{\gamma}^{\text{external}}$	$\widehat{\gamma}^{\text{CL}}$	$\widehat{\gamma}^{\text{AD}}(\pi)$
$\widehat{\alpha}^{\text{external}}$	Bornhuetter–Ferguson Method (external)		
$\widehat{\alpha}^{\text{LD}}(\widehat{\gamma})$	Loss–Development Method (external)	Chain–Ladder Method	
$\widehat{\alpha}^{\text{CC}}(\pi, \widehat{\gamma})$	Cape Cod Method (external)		Additive Method

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- the simultaneous application of several versions of the extended Bornhuetter–Ferguson method to a given run–off triangle of cumulative losses.

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- the selection of reliable predictors,
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- the comparison of the given portfolio with a market portfolio, and
- the control of pricing.

In either case, careful actuarial judgement of the quality of the sources of information underlying the different versions of the extended Bornhuetter–Ferguson method is essential.

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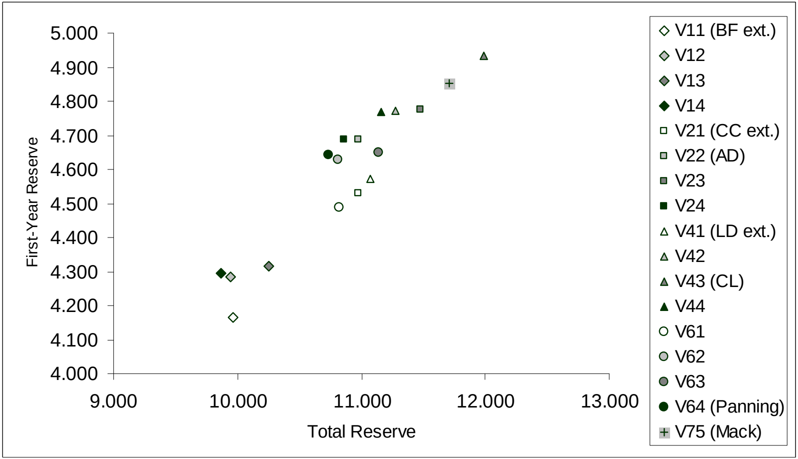
An Example

Modified example:

Acc. Year i	**Development Year** k 0	1	2	3	4	5	π_i	$\widehat{\alpha}_i^{\text{ext}}$
0	1001	1855	2423	2988	3335	3483	4000	3520
1	1113	2103	2774	3422	3844		4500	3980
2	1265	2433	3233	3977			5300	4620
3	1490	2873	3880				6000	5660
4	1725	4261					6900	6210
5	1889						8200	6330
$\widehat{\gamma}_k^{\text{ext}}$	0.2800	0.5300	0.7100	0.8600	0.9500	1.0000		

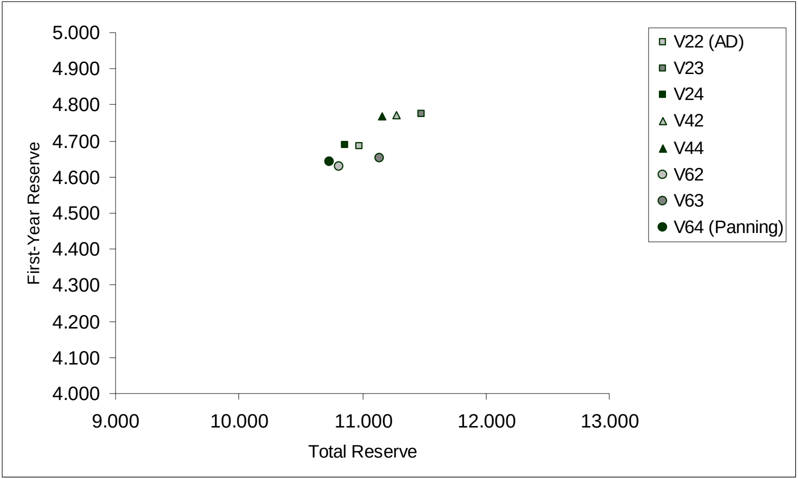
An Example

Predictors of various versions:



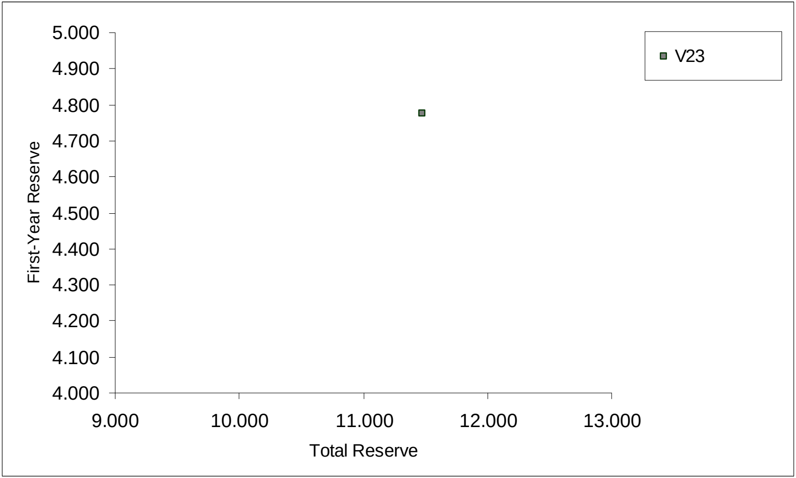
An Example

Reliable predictors:



An Example

Selected predictor:



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