An Actuarial Model of Cross Subsidization in Price-Regulated Insurance Markets under Moral Hazard

ASTIN Colloquium
Manchester, UK
July 13-16, 2008

Andreas Milidonis, Ph.D.
Lecturer in Finance
Manchester Business School
University of Manchester

Outline

1. Motivation
2. Literature Review
3. Theoretical Model:
   a. The Compound Aggregate Loss Process
   b. Premium Structure Over Time
      i. Moral Hazard
   c. The Discrete Time Markov Chain
4. Illustrative Examples
5. Conclusion
1. Introduction - Motivation

- Price Regulation $\Leftrightarrow$ Price Ceiling.
- High Risk (HR) Individuals denied coverage.
- HR enter Assigned Risk Pool (ARP).
- ARP divided among insurers in state (California Auto).
- Low Risk (LR) subsidize HR.
- Cross-subsidization triggers transition of LR individuals into ARP due to inflated premiums.
- Process catalyzed by moral hazard and level of regulated premium (price ceiling).
- Vicious Cycle over time due to Price Regulation.
2. Literature

- Theoretical Work
  - Doherty & Garven (JF, 1986).

- Empirical Work
  - Grabowski, Viscusi & Evans (JRI, 1989).
  - Cummins (2002).

2a. Aggregate Loss Model

- Random Aggregate Loss Process:
  \[ S_t = L_{1t} + L_{2t} + \ldots + L_{mt}, \]
  \[ t = 1, 2, \ldots, m \]
  \[ i = 0, 1, 2, \ldots, m \]

- Assumptions:
  - Finite Population
  - No real inflation in claims
  - Discount on Regulated Premium
2a. Aggregate Loss Model

- Compound Density Function:
  \[ F_S(l) = \sum_{n=0}^{m} p_n \Pr(S \leq l | i = n) \]
  \[ = \sum_{n=0}^{m} p_n F_L^{*n}(l) \]

- Frequency Probability Mass Function:
  \[ p_n = \Pr(N = n) \]

- Severity Density Function:
  \[ F_L^{*k}(l) = \int_0^l F_L^{*(k-1)}(l - y) f_L(y) \, dy \]

2b. Premium Structure

- Before Price Regulation:
  All contracts charged their Actuarial Fair Premium (AFP)

- After Price Regulation - Before Surcharge:
  - “V” : AFP < \( \beta \) where \( \beta \) = Price Ceiling
  - “R” : AFP >= \( \beta \)
  - Premium for contract \( i \):
    \[ P_{it} = \min(AFP_{it-1}, \beta) \]
  - Excess loss for contract \( i \):
    \[ XL_{it} = L_{it} - P_{it} \]
  - Total Excess Loss:
    \[ TXL_i = \sum_{i=1}^{m} XL_{it} \]
2b. Premium Structure

Premium Structure in a Price-Regulated Industry

Price Regulation Starts

- After Price Regulation - After Surcharge
  - Surcharge per contract $i$: $\tau_{i3} = TXL_{i2} * w_{i3}$
  - Surcharge weight: $w_{i3} = P_{i2} / \sum_{t=1}^{m} P_{i2}$
  - Premium for $t=2$: $P_{i3} = MIN[(AFP_{i2} + \tau_{i3}), \beta]$
2b. Premium Structure

Moral Hazard Effect

- **BEFORE Regulation**
  - Probability Density Function
  - \( \frac{1}{m} \) at \( \beta \) and \( \gamma \)

- **AFTER Regulation BEFORE Taxes**
  - Probability Density Function
  - \( \frac{1}{m} \) at \( \beta' \) and \( \gamma' \)

2c. The Discrete Time Markov Chain

- **Independent States (V, R)**
- **Rows in Transition Probability Matrix sum to 1**

\[
M[t] = \begin{bmatrix} \pi^t_V, \pi^t_R \end{bmatrix}
\]

\[
M[t = 2]_{1 \times 2} = M[t = 1]_{1 \times 2} \cdot \begin{bmatrix} P^t_{VV} & P^t_{VR} \\ P^t_{RV} & P^t_{RR} \end{bmatrix}_{2 \times 2}
\]

- \( \pi^t_i \) = Population Proportion in state \( i \) at time \( t \)
- \( P^t_{ij} \) = Transition Prob. from state \( i \) to \( j \) at time \( t \)
- **Do you see similarities with Credit Risk Classification?**
2c. The Discrete Time Markov Chain

Transition Probability from V to R for contract $i$:

$$P(R_i | V_i) = P(P_{i3} > \beta \delta)$$

$$= P(P_{i2} + w_{i3} \cdot TXL_2 > \beta \delta)$$

$$= 1 - F_{XL_i} \left( \frac{\beta \delta - P_{i2}}{w_{i3}} \right)$$

Random Aggregate Excess Loss Model

---

Transition Probability Matrix

$$P_{i} = \begin{bmatrix} P_{VV}^{i} & P_{VR}^{i} \\ P_{RV}^{i} & P_{RR}^{i} \end{bmatrix}_{2 \times 2}$$

$$P_{VV}^{i} = 1 - P_{VR}^{i}$$

$$P_{VR}^{i} = 1 - F_{XL_i} \sum_{i \in R} \left( \frac{\beta \delta - P_{i1}}{P_{i1}} - 1 \right)$$

$$P_{RV}^{i} = 0$$

$$P_{RR}^{i} = 1$$
3. Illustrative Examples

Price Ceiling (Beta) on Transition Probabilities

<table>
<thead>
<tr>
<th>Beta</th>
<th>Year 2</th>
<th>Year 3</th>
<th>Year 4</th>
<th>Year 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>29.78%</td>
<td>28.49%</td>
<td>28.30%</td>
<td>29.40%</td>
</tr>
<tr>
<td>60</td>
<td>21.07%</td>
<td>16.16%</td>
<td>13.85%</td>
<td>12.24%</td>
</tr>
<tr>
<td>70</td>
<td>13.08%</td>
<td>8.21%</td>
<td>5.60%</td>
<td>3.22%</td>
</tr>
<tr>
<td>80</td>
<td>7.06%</td>
<td>3.19%</td>
<td>1.96%</td>
<td>0.41%</td>
</tr>
</tbody>
</table>

Loss Distribution: Uniform (0, 100)

Price Ceiling (Beta) on "R" population

<table>
<thead>
<tr>
<th>Beta</th>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
<th>Year 4</th>
<th>Year 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>50.3%</td>
<td>65.1%</td>
<td>75.1%</td>
<td>82.1%</td>
<td>87.4%</td>
</tr>
<tr>
<td>60</td>
<td>40.1%</td>
<td>52.7%</td>
<td>60.4%</td>
<td>65.9%</td>
<td>70.0%</td>
</tr>
<tr>
<td>70</td>
<td>30.0%</td>
<td>39.1%</td>
<td>44.1%</td>
<td>47.3%</td>
<td>49.0%</td>
</tr>
<tr>
<td>80</td>
<td>20.0%</td>
<td>25.6%</td>
<td>28.0%</td>
<td>29.4%</td>
<td>29.7%</td>
</tr>
</tbody>
</table>

Loss Distribution: Uniform (0, 100)
3. Illustrative Examples

Moral Hazard (h) on "R" population

<table>
<thead>
<tr>
<th>Population Proportion</th>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
<th>Year 4</th>
<th>Year 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>h=1.05</td>
<td>20.0%</td>
<td>24.4%</td>
<td>26.1%</td>
<td>27.0%</td>
<td>27.0%</td>
</tr>
<tr>
<td>h=1.10</td>
<td>20.0%</td>
<td>25.6%</td>
<td>28.0%</td>
<td>29.4%</td>
<td>29.7%</td>
</tr>
<tr>
<td>h=1.25</td>
<td>20.0%</td>
<td>29.3%</td>
<td>33.2%</td>
<td>36.9%</td>
<td>39.1%</td>
</tr>
<tr>
<td>h=1.50</td>
<td>20.0%</td>
<td>34.6%</td>
<td>41.2%</td>
<td>48.8%</td>
<td>55.0%</td>
</tr>
<tr>
<td>h=2.00</td>
<td>20.0%</td>
<td>43.4%</td>
<td>54.3%</td>
<td>67.9%</td>
<td>78.7%</td>
</tr>
</tbody>
</table>

Loss Distribution: Uniform (0, 100)

3. Illustrative Examples

Discount (delta) on "R" population

<table>
<thead>
<tr>
<th>Population Proportion</th>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
<th>Year 4</th>
<th>Year 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>delta = 97%</td>
<td>20%</td>
<td>28%</td>
<td>33%</td>
<td>38%</td>
<td>43%</td>
</tr>
<tr>
<td>delta = 96%</td>
<td>20%</td>
<td>29%</td>
<td>35%</td>
<td>41%</td>
<td>47%</td>
</tr>
<tr>
<td>delta = 95%</td>
<td>20%</td>
<td>29%</td>
<td>37%</td>
<td>44%</td>
<td>51%</td>
</tr>
</tbody>
</table>
6. Conclusions

• Moral hazard in the Assigned Risk Pool should be taken into account when price ceilings are set.
• Moral Hazard aggravates cross-subsidization.
• In price regulated systems, the shape of the underlying distribution has to be known and monitored over time.

7. Contact Information

• Contact Author:
  Andreas Milidonis, PhD
  Lecturer in Finance
  Accounting & Finance Division
  Manchester Business School

• http://www.personal.mbs.ac.uk/andreas-milidonis/
References


