Bootstrap Estimation of the Predictive Distributions of Reserves Using Paid and Incurred Claims

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Outline

- Introduction
- Munich chain ladder (MCL)
- Bootstrap Method
- Bootstrap the MCL Method
- A Numerical Example
- Conclusions

Introduction

- Reserving Exercise
  - Mostly used Data - Paid and Incurred Claims, etc
  - Mostly Used Method – Chain ladder method, etc
- Issues with existing Approach
  - Inconsistent ultimate loss estimates when applying CL, separately.
- Solution?
  - Munich chain ladder method
- Uncertainty of the MCL Reserve?
  - A Data-based simulation approach: Bootstrapping

Munich Chain Ladder Method - Summary

- To model both paid and incurred claims information, dependently. And it was introduced by Quarg and Mack (2004).
- Based on Mack's distribution free model
- It models the natural dependency between paid and incurred by using correlations within the GLM framework.
- It produces more consistent ultimate loss estimates from paid and incurred claims.

Munich Chain Ladder Method - Algorithm

- Apply the Mack's distribution free model to paid and incurred claims, separately. This results the same paid and incurred development factors as the chain ladder method.
- Obtain the ratios of paid divided by incurred and the reverse
- Calculate the residuals of the four sets of ratios so that the accident/development year effects are removed.
- Calculate the correlations between paid dev factors and the (I/P) ratios, and the correlations between incurred dev factors and the (P/I) ratios.
- Adjust the original paid and incurred dev factors using the correlations obtained
- Apply the adjusted paid and incurred claims for the future projections.

AGCS CRO Actuarial/ AGCS Bootstrap Estimation of the Predictive Distributions of Reserves Using Paid and Incurred Claims (6/20)

Munich Chain Ladder Method - Data

Paid Claims

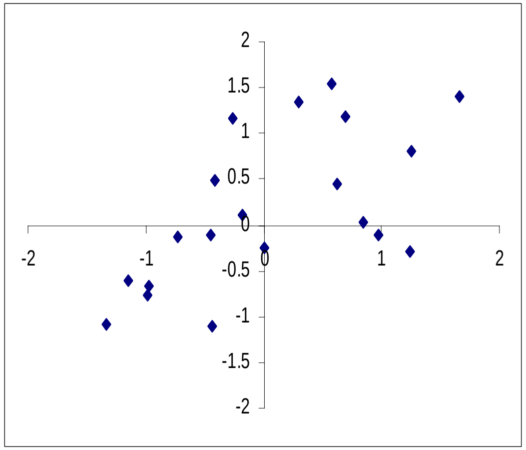
$$\begin{array}{cccc} P_{11} & P_{12} & \cdots & P_{1n} \\ P_{21} & \cdots & P_{2,n-1} & \\ \vdots & \iddots & & \\ P_{n1} & & & \end{array}$$

Incurred Claims

$$\begin{array}{cccc} I_{11} & I_{12} & \cdots & I_{1n} \\ I_{21} & \cdots & I_{2,n-1} & \\ \vdots & \iddots & & \\ I_{n1} & & & \end{array}$$

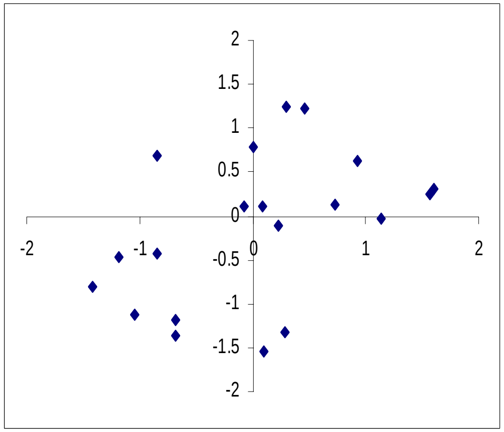
Munich Chain Ladder Method – Residuals Plots

Paid dev. factors vs. I/P



$$\rho = 0.64$$

Incurred dev. factors vs. P/I



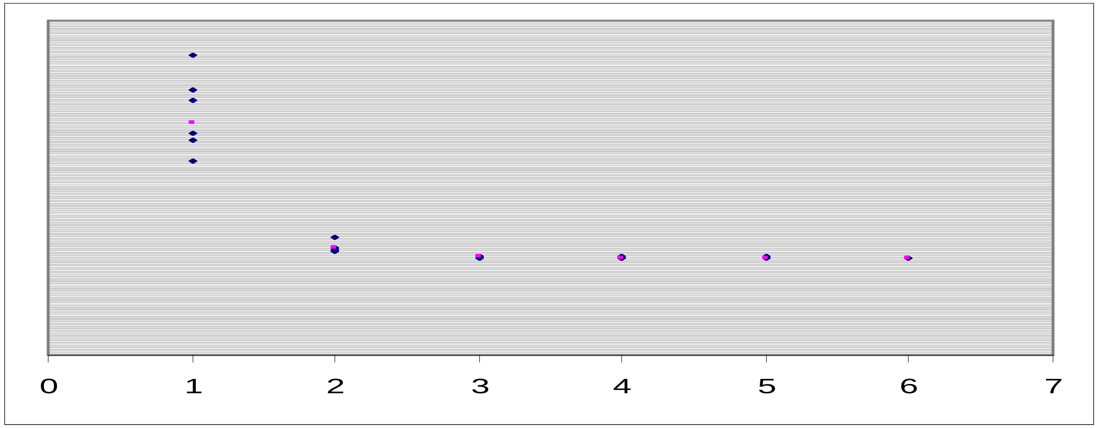
$$\rho = 0.44$$

Munich Chain Ladder Method

By using the correlation, the MCL model adjusts each CL development factor to individual ‘link ratio’ according to different accident year.

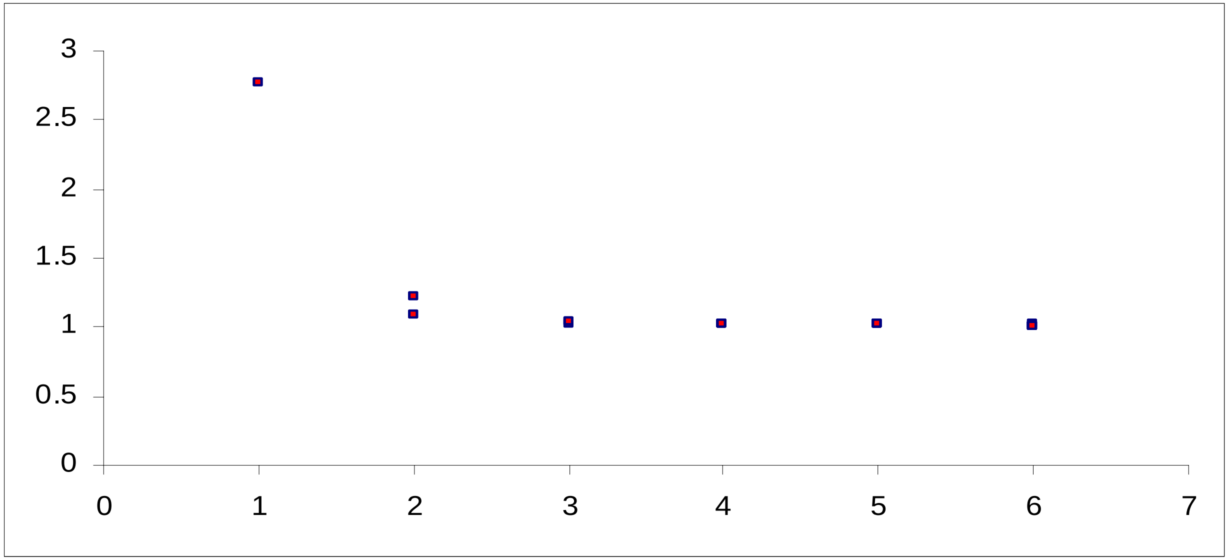
In other words, for one dev year, the CL dev factor are ‘expended’ into a few ‘link ratios’ according to the accident year effect.

Plot of the link ratios

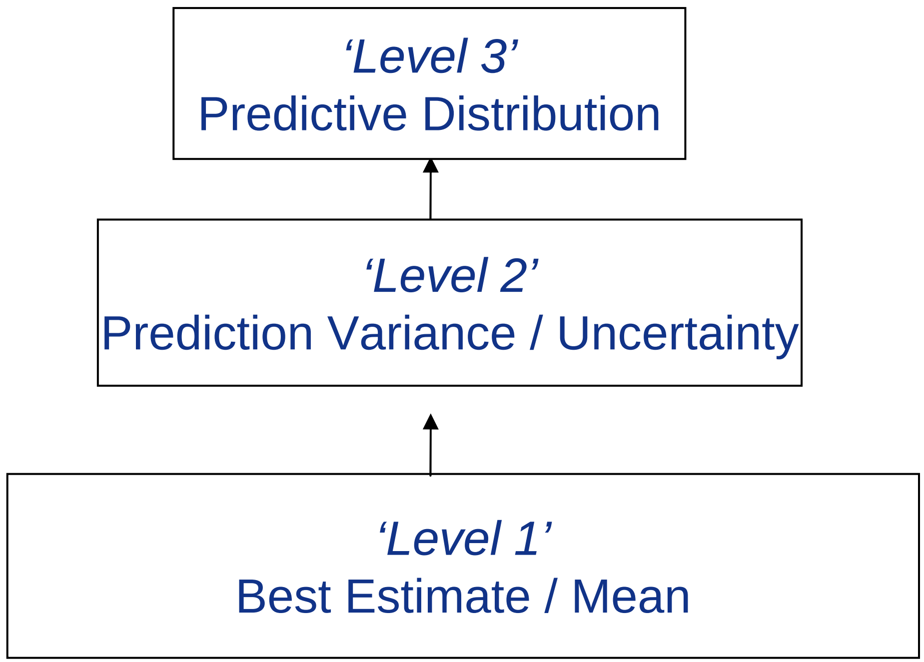


Munich Chain Ladder Method

Plot of the 'link ratios' adjusted by the MCL method



Stochastic Reserving



Bootstrapping - Overview

- The Bootstrap is a data-based simulation method for statistical inference
- It can be applied to obtain an approximation to the prediction error or the predictive distributions of the parameter in interest, e.g. reserve estimate, when including process error.
- Bootstrapping has become very popular in stochastic claims reserving because of the simplicity and flexibility.
- It is important to realise that bootstrapping is not a “model”, and therefore it is important to ensure that the underlying reserving models are correctly calibrated to the observed data.

Bootstrapping – Plug-in Principle

The Plug-in Principle for the Real World and the Bootstrap World

Real World

$$F \rightarrow \vec{x} = \left(x_1, x_2, \ldots, x_n\right)$$

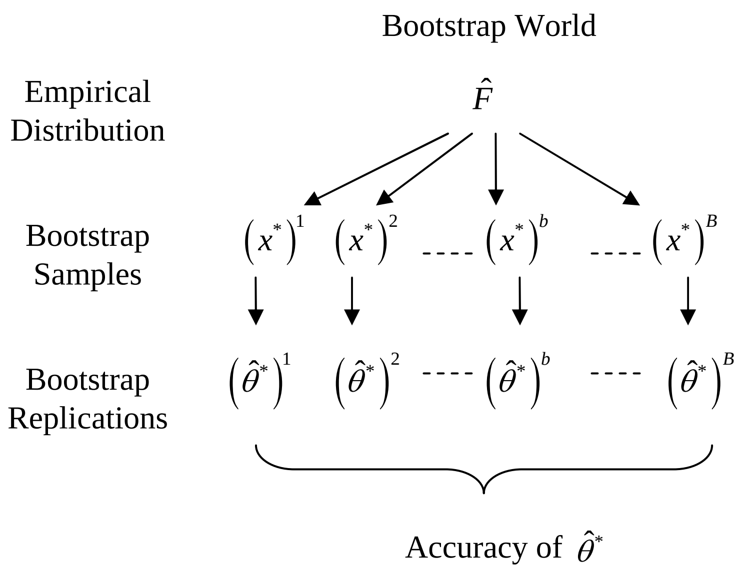
Plug-in Principle

Bootstrap World

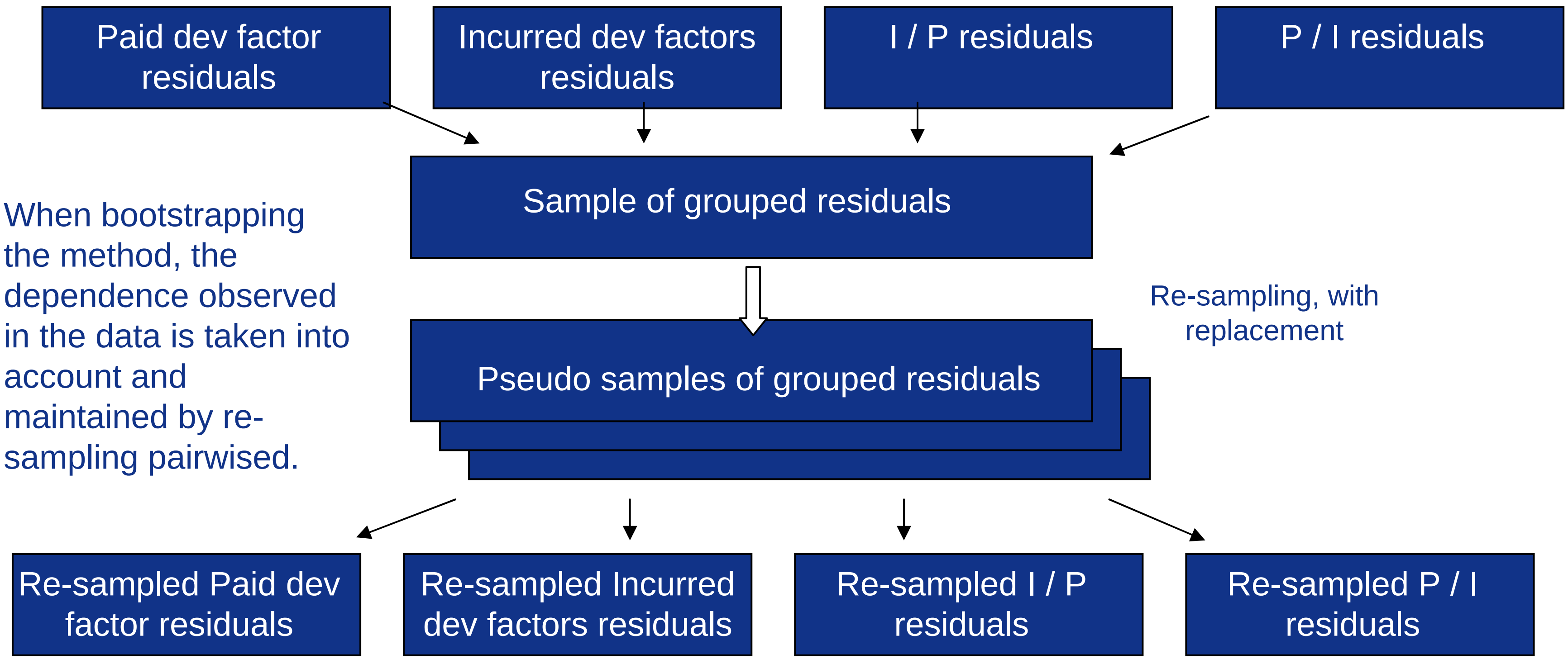
$$\hat{F} \rightarrow \vec{x}^* = \left(x_1^*, x_2^*, \ldots, x_n^*\right)$$

Bootstrapping - Algorithm

Bootstrap Algorithm of Estimating the Accuracy of a Statistic



Bootstrap MCL Method



When bootstrapping the method, the dependence observed in the data is taken into account and maintained by re-sampling pairwised.

Numerical Example

Table 1. Paid Claim Data from Quarg and Mack (2004)

576	1,804	1,970	2,024	2,074	2,102	2,131
866	1,948	2,162	2,232	2,284	2,348	
1,412	3,758	4,252	4,416	4,494		
2,286	5,292	5,724	5,850			
1,868	3,778	4,648				
1,442	4,010					
2,044						

Table 2. Incurred Claim Data from Quarg and Mack (2004)

978	2,104	2,134	2,144	2,174	2,182	2,174
1,844	2,552	2,466	2,480	2,508	2,454	
2,904	4,354	4,698	4,600	4,644		
3,502	5,958	6,070	6,142			
2,812	4,882	4,852				
2,642	4,406					
5,022						

Results

Table 3. Bootstrap Reserves and MCL Reserves

	Bootstrap		MCL		CL	
	Paid	Incurred	Paid	Incurred	Paid	Incurred
i=1	0	43	0	43	0	43
i=2	35	95	35	96	32	97
i=3	106	128	103	135	159	86
i=4	275	317	269	326	333	275
i=5	294	287	289	302	407	192
i=6	672	649	646	655	924	464
i=7	5512	5655	5505	5606	4095	6388
Overall Total	6893	7175	6846	7163	5950	7544

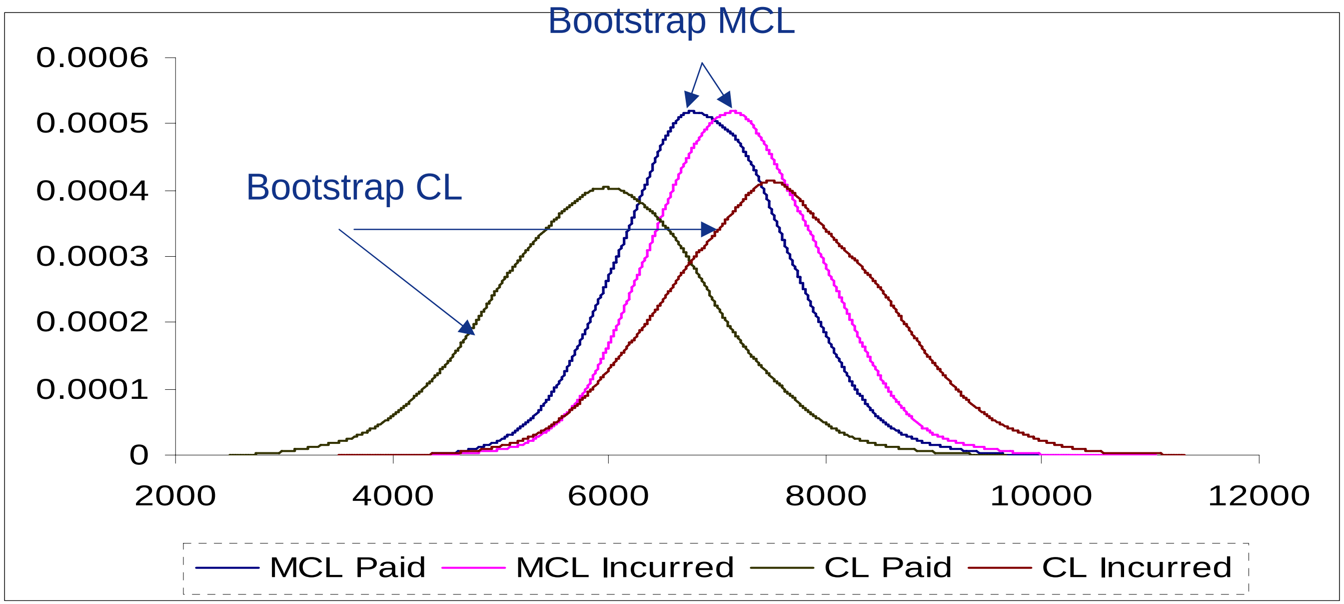
Results

Table 4. Bootstrap Prediction Errors of MCL and CL Method

	MCL		CL	
	Paid	Incurred	Paid	Incurred
i=1	0	0	0	0
i=2	5	5	15	9
i=3	44	67	52	82
i=4	57	84	69	103
i=5	69	99	72	118
i=6	207	204	286	217
i=7	723	695	891	865
Overall Total	755	762	980	986

Results

Predictive Distributions – A comparison between CL and MCL



Conclusions

- Bootstrapping is well-suited for these purposes the ideal candidate from a the practical point of view, since it avoids the complicated theoretical calculations and is easily to be implemented by in a simple spreadsheet.
- However, the MCL model relies on the strong dependency between data sets and does not always produce better results than the straightforward chain ladder model.
- As a consequence, it is important for the data to be carefully checked to test whether the dependency assumptions of the MCL model are valid for each data set before it is applied.

Conclusions

The End

Thank You for Your Attention!