Data Combination under Basel II and Solvency 2: Operational Risk goes Bayesian

($\Longleftrightarrow$ Give Credit where Credit is due)

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Risk Classes

- **Underwriting Risk**
- **Market Risk**
- **Operational Risk**
- **Credit Risk**
- **Business Risk**

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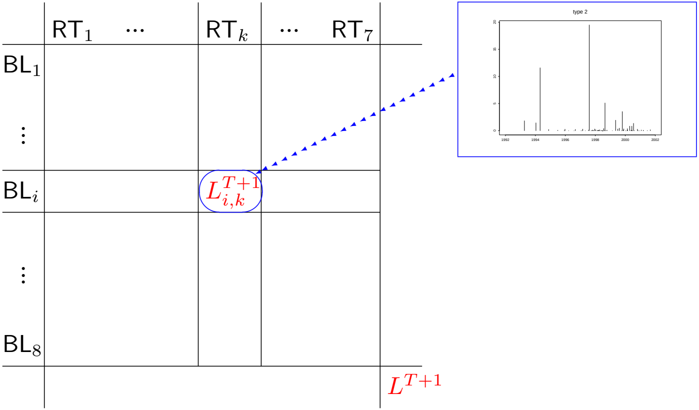
Operational Risk: The risk of loss resulting from inadequate or failed internal processes, people and systems or from external events. Including legal risk, but excluding strategic and reputational risk.

Loss Distribution Approach (LDA)



BL: Business Line
RT: Risk Type

Basel II - Guidelines

- **Risk measure:** VaR
- **Time horizon:** 1 year
- **Level:** 99.9% (1 in 1000 year event!)

- **Otherwise:** Full methodological freedom (within LDA)
- **See:** [Degen, Embrechts, L. (2007)]

Basic Model for Single Risk Cell (i, k)

Compound model for BL i, RT k:

$$L_{i,k} = \sum_{n=1}^{N} X_n$$

N: frequency (e.g., Poisson)
$X_1, X_2, \ldots$ iid: severity (e.g., lognormal, Pareto, g-and-h)

- Assume independence between severity and frequency.
- **Here,** we focus on modeling the severity distribution!
- Frequency distribution is modeled completely analogous.

Data Combination

- **Internal data, external data** and **expert opinion**

- Basel II: "A bank must use scenario analysis of expert opinion in conjunction with external data to evaluate its exposure to high-severity events."
- Practitioners' view: "A big challenge for us is how to mix the internal data with external data; this is something that is still a big problem because I don't think anybody has a solution for that at the moment."

Example: BL 6 (payment and settlement), RT 1 (internal fraud)

- **Internal data:**
  - 17.11.2005: EUR 300'000
  - 02.03.2007: EUR 1'200'000
  - 23.06.2008: EUR 200'000
- **External data:** 300 claims
- **Expert opinion,** scenario analysis

Naive solution 1: put everything in one pot.

Naive solution 2: take convex combination

$$X := \omega_1 X_{\text{int}} + \omega_2 X_{\text{ext}} + (1 - \omega_1 - \omega_2) X_{\text{exp}}$$

But: how to choose ω_1, ω_2? This ad-hoc methods are not robust w.r.t. high quantile estimation (VaR).

Use Bayesian Inference

- Well-understood in an actuarial context!

prior distribution	⟶ B A Y E S ⟶	posterior distribution
↕		↕
market profile Δ	internal data, expert opinion ⟶	company specific model $\Delta \vert \mathbf{X}, \vartheta$
π_Δ	$\longmapsto$	$\widehat{\pi}_{\Delta\vert\mathbf{X},\vartheta} \propto \pi_\Delta \cdot f_{\mathbf{X}\vert\Delta} \cdot f_{\vartheta\vert\Delta}$

The Lognormal(-Normal-Normal) Model

(A realization of) Δ plays the role of the company specific parameter of the loss distribution

- **Market Profile:** $\Delta \sim \mathcal{N}(\mu_{\text{ext}}, \sigma_{\text{ext}})$
- **Expert Opinion:** $\vartheta^{(1)}, \ldots, \vartheta^{(M)} | \Delta \overset{\text{i.i.d.}}{\sim} \mathcal{N}(\Delta, \sigma_{\text{exp}})$
- **Internal Data:** $X_1, \ldots, X_K | \Delta \overset{\text{i.i.d.}}{\sim} \text{LN}(\Delta, \sigma_{\text{int}})$

Plus: suitable (conditional) independence properties between experts and internal data (given the risk profile Δ)

Aim:
Estimate the company specific model $\Delta | X_1, \ldots, X_K, \vartheta^{(1)}, \ldots, \vartheta^{(M)}$

Give Credit where Credit is due

Theorem [L., Shevchenko, Wüthrich (2007)]:

In the **lognormal model** the following holds for the company specific risk profile $\Delta|\mathbf{X},\boldsymbol{\vartheta}$:

$$\Delta|\mathbf{X},\boldsymbol{\vartheta} \sim \mathcal{N}(\widehat{\mu},\widehat{\sigma}),$$

$$\widehat{\mu} = \mathbb{E}[\Delta|\mathbf{X},\boldsymbol{\vartheta}] = \omega_{\text{ext}}\mu_{\text{ext}} + \omega_{\text{int}}\overline{\log X} + \omega_{\text{exp}}\overline{\vartheta},$$

$$\widehat{\sigma}^2 = \left(1/\sigma^2_{\text{ext}} + K/\sigma^2_{\text{int}} + M/\sigma^2_{\text{exp}}\right)^{-1},$$

where the so-called credibility weights are given by $\omega_{\text{ext}} = \widehat{\sigma}^2/\sigma^2_{\text{ext}}$, $\omega_{\text{int}} = \widehat{\sigma}^2 K/\sigma^2_{\text{int}}$ and $\omega_{\text{exp}} = \widehat{\sigma}^2 M/\sigma^2_{\text{exp}}$.

Give Credit where Credit is due (2)

$$\mathbb{E}[\Delta|\mathbf{X},\boldsymbol{\vartheta}] = \omega_{\text{ext}}\mu_{\text{ext}} + \omega_{\text{int}}\overline{\log X} + \omega_{\text{exp}}\overline{\vartheta}$$

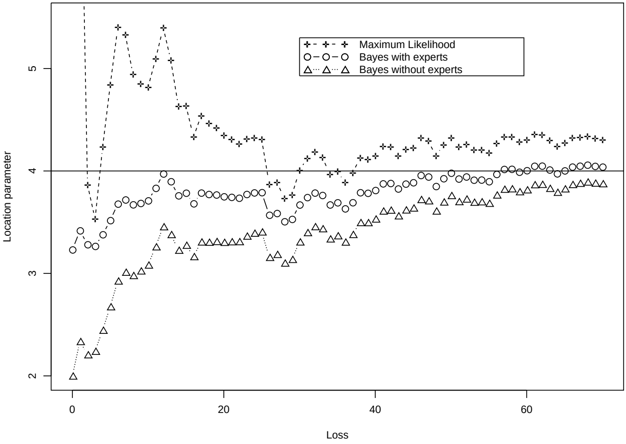
- If information source i is highly inaccurate
  $\Longrightarrow$ the credibility weight $\omega_i \searrow 0$.
  Example (high variance in the market):

$$\sigma_{\text{ext}} \to \infty \Rightarrow \omega_{\text{ext}} \searrow 0$$

- If information source i is very precise
  $\Longrightarrow$ the credibility weight $\omega_i \nearrow 1$.
  Example (many internal losses):

$$K \to \infty \Rightarrow \omega_{\text{int}} \nearrow 1$$

An Example: Estimation of $\Delta | \mathbf{X}, \vartheta$



The Pareto(-Gamma-Gamma) Model

- The same result holds true for other models:
  - **Market Profile:** $\Delta \sim$ Gamma
  - **Expert Opinion:** $\vartheta^{(1)}, \ldots, \vartheta^{(M)} | \Delta \overset{\text{i.i.d.}}{\sim}$ Gamma
  - **Internal Data:** $X_1, \ldots, X_K | \Delta \overset{\text{i.i.d.}}{\sim}$ Pareto(Δ)

Plus: suitable (conditional) independence properties between experts and internal data (given the risk profile Δ)

Theorem [L., Shevchenko, Wüthrich (2007)]:
The posterior distribution is a Generalized Inverse Gaussian (GIG), i.e. $f(x) = cx^{\nu} e^{-\omega x - \phi / x}$, for some parameters ν, ω, ϕ.

Conclusion

- Basel II: You must use external data and scenario analysis of expert opinion.
- Use well-known actuarial theory: Bayesian inference and credibility theory.
- This yields a model with a natural interpretation: give credit where credit is due!
- To do: multivariate models.

References

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