Data Combination under Basel II and Solvency 2: Operational Risk goes Bayesian

(⇐⇒ Give Credit where Credit is due)

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Risk Classes

- Underwriting Risk
- Market Risk
- Operational Risk
- Credit Risk
- Business Risk
Risk Classes

- Underwriting Risk
- Market Risk
- Operational Risk
- Credit Risk
- Business Risk

**Operational Risk**: The risk of loss resulting from inadequate or failed internal processes, people and systems or from external events. Including legal risk, but excluding strategic and reputational risk.
# Loss Distribution Approach (LDA)

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BL: Business Line  
RT: Risk Type

\[ L^{T+1} \]
Basel II - Guidelines

- **Risk measure:** VaR
- **Time horizon:** 1 year
- **Level:** 99.9% (1 in 1000 year event!)

► **Otherwise:** Full methodological freedom (within LDA)
► **See:** [Degen, Embrechts, L. (2007)]
Basic Model for Single Risk Cell \((i, k)\)

Compound model for BL \(i\), RT \(k\):

\[
L_{i,k} = \sum_{n=1}^{N} X_n
\]

\(N\): frequency (e.g., Poisson)
\(X_1, X_2, \ldots\) iid: severity (e.g., lognormal, Pareto, g-and-h)

- Assume independence between severity and frequency.
- **Here**, we focus on modeling the **severity** distribution!
- **Frequency** distribution is modeled completely analogous.
Data Combination

• Internal data, external data and expert opinion

▶ Basel II: “A bank must use scenario analysis of expert opinion in conjunction with external data to evaluate its exposure to high-severity events.”

▶ Practitioners’ view: “A big challenge for us is how to mix the internal data with external data; this is something that is still a big problem because I don’t think anybody has a solution for that at the moment.”
Example: BL 6 (payment and settlement), RT 1 (internal fraud)

- **Internal data:**
  - 17.11.2005: EUR 300’000
  - 02.03.2007: EUR 1’200’000
  - 23.06.2008: EUR 200’000

- **External data:** 300 claims

- **Expert opinion, scenario analysis**

**Naive solution 1:** put everything in one pot.

**Naive solution 2:** take convex combination

\[ X := \omega_1 X_{\text{int}} + \omega_2 X_{\text{ext}} + (1 - \omega_1 - \omega_2) X_{\text{exp}} \]

**But:** how to choose \( \omega_1, \omega_2 \)? This ad-hoc methods are not robust w.r.t. high quantile estimation (VaR).
Use Bayesian Inference

- Well-understood in an actuarial context!

prior distribution $\rightarrow$ Bayesian $\rightarrow$ posterior distribution

market profile $\Delta$ $\rightarrow$ internal data, expert opinion $\rightarrow$ company specific model $\Delta | X, \vartheta$

$\pi_\Delta \mapsto \hat{\pi}_\Delta | X, \vartheta \propto \pi_\Delta \cdot f_X | \Delta \cdot f_\vartheta | \Delta$
The Lognormal(-Normal-Normal) Model

(A realization of) \( \Delta \) plays the role of the company specific parameter of the loss distribution

- **Market Profile:** \( \Delta \sim \mathcal{N}(\mu_{\text{ext}}, \sigma_{\text{ext}}) \)
- **Expert Opinion:** \( \vartheta^{(1)}, \ldots, \vartheta^{(M)}|\Delta \overset{\text{i.i.d.}}{\sim} \mathcal{N}(\Delta, \sigma_{\text{exp}}) \)
- **Internal Data:** \( X_1, \ldots, X_K|\Delta \overset{\text{i.i.d.}}{\sim} \mathcal{LN}(\Delta, \sigma_{\text{int}}) \)

**Plus:** suitable (conditional) independence properties between experts and internal data (given the risk profile \( \Delta \))

**Aim:**
Estimate the company specific model \( \Delta|X_1, \ldots, X_K, \vartheta^{(1)}, \ldots, \vartheta^{(M)} \)
Give Credit where Credit is due

**Theorem** [L., Shevchenko, Wüthrich (2007)]:

In the **lognormal model** the following holds for the company specific risk profile $\Delta | X, \vartheta$:

$$
\Delta | X, \vartheta \sim \mathcal{N}(\hat{\mu}, \hat{\sigma}),
$$

$$
\hat{\mu} = \mathbb{E}[\Delta | X, \vartheta] = \omega_{\text{ext}} \mu_{\text{ext}} + \omega_{\text{int}} \log X + \omega_{\text{exp}} \vartheta,
$$

$$
\hat{\sigma}^2 = \left(1/\sigma_{\text{ext}}^2 + K/\sigma_{\text{int}}^2 + M/\sigma_{\text{exp}}^2 \right)^{-1}
$$

where the so-called credibility weights are given by

$$
\omega_{\text{ext}} = \hat{\sigma}^2/\sigma_{\text{ext}}^2, \quad \omega_{\text{int}} = \hat{\sigma}^2 K/\sigma_{\text{int}}^2, \quad \text{and} \quad \omega_{\text{exp}} = \hat{\sigma}^2 M/\sigma_{\text{exp}}^2.
$$
Give Credit where Credit is due (2)

\[ E[\Delta|X,\vartheta] = \omega_{\text{ext}} \mu_{\text{ext}} + \omega_{\text{int}} \log X + \omega_{\exp} \vartheta \]

- If information source \( i \) is highly inaccurate
  \[ \implies \text{the credibility weight } \omega_i \downarrow 0. \]
  
  **Example** (high variance in the market):
  \[ \sigma_{\text{ext}} \rightarrow \infty \implies \omega_{\text{ext}} \downarrow 0 \]

- If information source \( i \) is very precise
  \[ \implies \text{the credibility weight } \omega_i \uparrow 1. \]
  
  **Example** (many internal losses):
  \[ K \rightarrow \infty \implies \omega_{\text{int}} \uparrow 1 \]
An Example: Estimation of $\Delta|X, \vartheta$
The Pareto(-Gamma-Gamma) Model

- The same result holds true for other models:
  - **Market Profile**: $\Delta \sim \text{Gamma}$
  - **Expert Opinion**: $\vartheta^{(1)}, \ldots, \vartheta^{(M)}|\Delta \overset{\text{i.i.d.}}{\sim} \text{Gamma}$
  - **Internal Data**: $X_1, \ldots, X_K|\Delta \overset{\text{i.i.d.}}{\sim} \text{Pareto}(\Delta)$

**Plus**: suitable (conditional) independence properties between experts and internal data (given the risk profile $\Delta$)

**Theorem** [L., Shevchenko, Wüthrich (2007)]:
The posterior distribution is a Generalized Inverse Gaussian (GIG), i.e. $f(x) = cx^{\nu}e^{-\omega x - \phi/x}$, for some parameters $\nu, \omega, \phi$. 
Conclusion

▶ Basel II: You must use external data and scenario analysis of expert opinion.

▶ Use well-known actuarial theory: Bayesian inference and credibility theory.

▶ This yields a model with a natural interpretation: give credit where credit is due!

▶ To do: multivariate models.
References

