A Loss Reserving Method for Incomplete Claim Data

Or how to close the gap between projections of payments and reported amounts?

René Dahms
Baloise Insurance Switzerland
rene.dahms@baloise.ch

July 2008, International ASTIN Colloquium Manchester

Three motivations

1. The gap between projections of payments and reported amounts

Three motivations

1. The gap between projections of payments and reported amounts
2. Case reserves as exposure measure

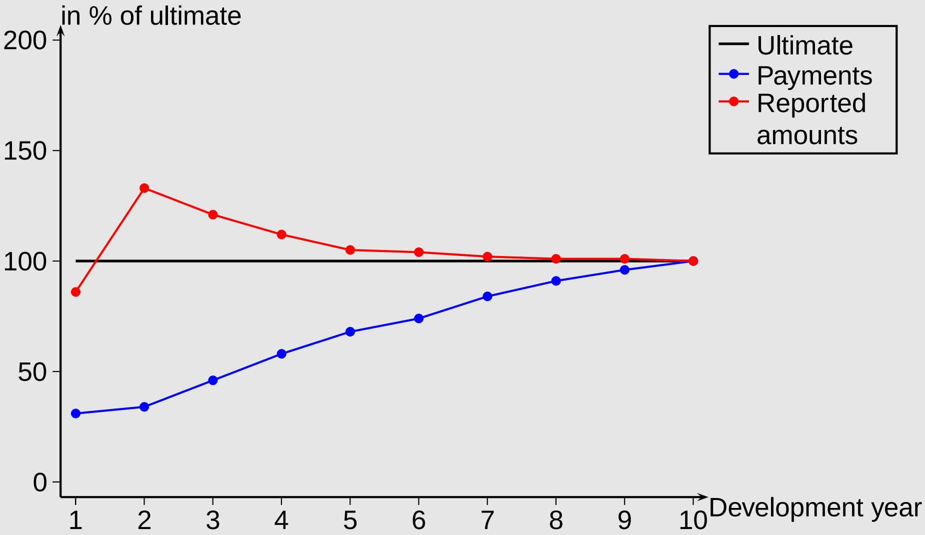
Three motivations

1. The gap between projections of payments and reported amounts
2. Case reserves as exposure measure
3. Reserving based on incomplete data (an example)

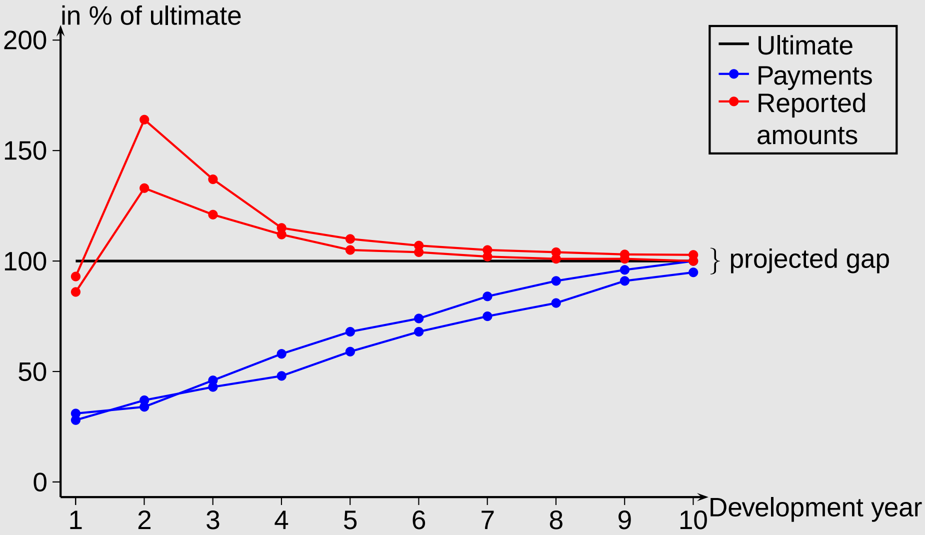
Three motivations

1. The gap between projections of payments and reported amounts
2. Case reserves as exposure measure
3. Reserving based on incomplete data (an example)
4. Summary and outlook

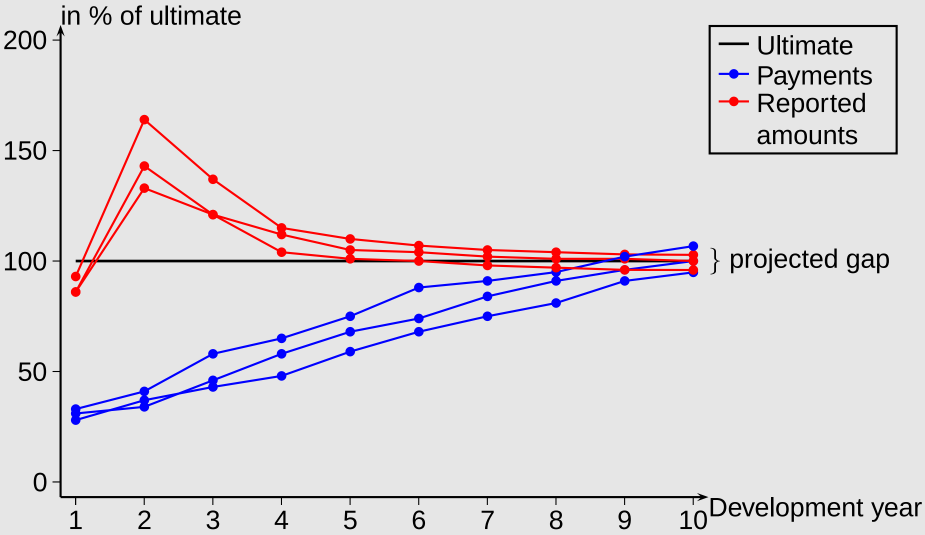
The typical outcome of projections



The typical outcome of projections



The typical outcome of projections



How to close the gap?

- Munich-Chain-Ladder tries to deal with the systematic gap of Chain-Ladder-projections, but still leads to different estimates for payments and reported amounts.

How to close the gap?

- Munich-Chain-Ladder tries to deal with the systematic gap of Chain-Ladder-projections, but still leads to different estimates for payments and reported amounts.
- Weighting of ultimates like in Bornhuetter-Ferguson or Cape Cod (or using credibility theory). We often use

$$\frac{L_P U_P + L_A U_A}{L_P + L_A},$$

where U_P and U_A are the projected ultimates based on payments and reported amounts, respectively, and L_P and L_A are the corresponding lag factors or their reciprocal values, depending on which are smaller.

How to close the gap?

- Munich-Chain-Ladder tries to deal with the systematic gap of Chain-Ladder-projections, but still leads to different estimates for payments and reported amounts.
- Weighting of ultimates like in Bornhuetter-Ferguson or Cape Cod (or using credibility theory). We often use

$$\frac{L_P U_P + L_A U_A}{L_P + L_A},$$

  where U_P and U_A are the projected ultimates based on payments and reported amounts, respectively, and L_P and L_A are the corresponding lag factors or their reciprocal values, depending on which are smaller.
- Making assumptions on the joint distribution of payments and reported amounts.

How to close the gap?

- Munich-Chain-Ladder tries to deal with the systematic gap of Chain-Ladder-projections, but still leads to different estimates for payments and reported amounts.
- Weighting of ultimates like in Bornhuetter-Ferguson or Cape Cod (or using credibility theory). We often use

$$\frac{L_P U_P + L_A U_A}{L_P + L_A},$$

where U_P and U_A are the projected ultimates based on payments and reported amounts, respectively, and L_P and L_A are the corresponding lag factors or their reciprocal values, depending on which are smaller.

- Making assumptions on the joint distribution of payments and reported amounts.

Is there a distribution free stochastic model which combines the information of payments and reported amounts?

What is the correct exposure for the payments of the next development year?

At first some definitions for accident year i and development year j:

- $C_{i,j}$ - cumulative payments
- $S_{i,j}$ - incremental payments ($C_{i,j} = \sum_{k=1}^{j} S_{i,k}$)

Baloise Insurance

What is the correct exposure for the payments of the next development year?

At first some definitions for accident year i and development year j:

- $C_{i,j}$ - cumulative payments
- $S_{i,j}$ - incremental payments ($C_{i,j} = \sum_{k=1}^{j} S_{i,k}$)
- $D_{i,j}$ - reported amounts
- $T_{i,j}$ - changes of reported amounts ($D_{i,j} = \sum_{k=1}^{j} T_{i,k}$)

Baloise Insurance

What is the correct exposure for the payments of the next development year?

At first some definitions for accident year i and development year j:

- $C_{i,j}$ - cumulative payments
- $S_{i,j}$ - incremental payments ($C_{i,j} = \sum_{k=1}^{j} S_{i,k}$)
- $D_{i,j}$ - reported amounts
- $T_{i,j}$ - changes of reported amounts ($D_{i,j} = \sum_{k=1}^{j} T_{i,k}$)
- $R_{i,j}$ - case reserves ($R_{i,j} = D_{i,j} - C_{i,j}$)

What is the correct exposure for the payments of the next development year?

At first some definitions for accident year i and development year j:

- $C_{i,j}$ - cumulative payments
- $S_{i,j}$ - incremental payments ($C_{i,j} = \sum_{k=1}^{j} S_{i,k}$)
- $D_{i,j}$ - reported amounts
- $T_{i,j}$ - changes of reported amounts ($D_{i,j} = \sum_{k=1}^{j} T_{i,k}$)
- $R_{i,j}$ - case reserves ($R_{i,j} = D_{i,j} - C_{i,j}$)
- Δ_k^C, Δ_k^D and Δ_k are the σ-algebras containing all the information of the payment triangle, the reported amount triangle and both triangles, respectively, up to development period k.

What is the correct exposure for the payments of the next development year?

- Mack's model of the Chain-Ladder-Method assumes that

$$E[S_{i,k+1}|\Delta_k^C] = (f_k - 1)\, C_{i,k}.$$

Therefore, if you believe in Chain-Ladder you have to believe in the cumulative payments as exposure measure for the payments of the next development year.

What is the correct exposure for the payments of the next development year?

- Mack's model of the Chain-Ladder-Method assumes that

$$E[S_{i,k+1}|\Delta_k^C] = (f_k - 1)\, C_{i,k}.$$

  Therefore, if you believe in Chain-Ladder you have to believe in the cumulative payments as exposure measure for the payments of the next development year.

- Additive method (Complementary Loss Ratio Method) assume that

$$E[S_{i,k+1}|\Delta_k^C] = q_k\, P_i,$$

  where P_i is an external given risk measure for accident year i, for instance the risk premium. Therefore, if you believe in this method you have to believe that the risk measure P_i is the correct one.

Assumptions for the presented method

We assume that

- $$E[S_{i,k+1}|\Delta_k] = \alpha_k R_{i,k} \quad \text{and} \quad E[T_{i,k+1}|\Delta_k] = \beta_k R_{i,k},$$
  for some constants α_k and β_k.

Assumptions for the presented method

We assume that

- $$E[S_{i,k+1}|\Delta_k] = \alpha_k R_{i,k} \quad \text{and} \quad E[T_{i,k+1}|\Delta_k] = \beta_k R_{i,k},$$
  for some constants α_k and β_k.
  This means, that we take the opening case reserves $R_{i,k}$ as exposure measure for the development during the year.

Assumptions for the presented method

We assume that

- $$E[S_{i,k+1}|\Delta_k] = \alpha_k R_{i,k} \quad \text{and} \quad E[T_{i,k+1}|\Delta_k] = \beta_k R_{i,k},$$
  for some constants α_k and β_k.
  This means, that we take the opening case reserves $R_{i,k}$ as exposure measure for the development during the year.
  It follows that the case reserves meet a Chain-Ladder like assumption:
  $$E[R_{i,k+1}|\Delta_k] = f_k R_{i,k},$$
  where $f_k = 1 - \alpha_k + \beta_k$.

Assumptions for the presented method

We assume that

- $$E[S_{i,k+1}|\Delta_k] = \alpha_k R_{i,k} \qquad \text{and} \qquad E[T_{i,k+1}|\Delta_k] = \beta_k R_{i,k},$$
  for some constants α_k and β_k.
  This means, that we take the opening case reserves $R_{i,k}$ as exposure measure for the development during the year.
  It follows that the case reserves meet a Chain-Ladder like assumption:
  $$E[R_{i,k+1}|\Delta_k] = f_k R_{i,k},$$
  where $f_k = 1 - \alpha_k + \beta_k$.
- All accident years are independent.

Assumptions for the presented method

We assume that

- $$E[S_{i,k+1}|\Delta_k] = \alpha_k R_{i,k} \quad \text{and} \quad E[T_{i,k+1}|\Delta_k] = \beta_k R_{i,k},$$
  for some constants α_k and β_k.
  This means, that we take the opening case reserves $R_{i,k}$ as exposure measure for the development during the year.
  It follows that the case reserves meet a Chain-Ladder like assumption:
  $$E[R_{i,k+1}|\Delta_k] = f_k R_{i,k},$$
  where $f_k = 1 - \alpha_k + \beta_k$.
- All accident years are independent.
- $$Cov\left[\left.\binom{S_{i,k+1}}{T_{i,k+1}}, \binom{S_{i,k+1}}{T_{i,k+1}}\right|\Delta_k\right] = R_{i,k} \cdot \Sigma_k^2$$
  for some positive definite, symmetric matrices Σ_k.

Derived estimators

- One estimator for the reserves of accident year i:

$$\widehat{Reserve}_i = R_{i,n+1-i} \sum_{k=n+1-i}^{n-1} \widehat{\alpha}_k \prod_{l=n+1-i}^{k-1} \widehat{f}_l$$

Baloise Insurance

Derived estimators

- One estimator for the reserves of accident year i:

$$\widehat{Reserve}_i = R_{i,n+1-i} \sum_{k=n+1-i}^{n-1} \widehat{\alpha}_k \prod_{l=n+1-i}^{k-1} \widehat{f}_l$$

$$= R_{i,n+1-i} \left(1 + \sum_{k=n+1-i}^{n-1} \widehat{\beta}_k \prod_{l=n+1-i}^{k-1} \widehat{f}_l \right),$$

Baloise Insurance

Derived estimators

- One estimator for the reserves of accident year i:

$$\widehat{Reserve}_i = R_{i,n+1-i} \sum_{k=n+1-i}^{n-1} \widehat{\alpha}_k \prod_{l=n+1-i}^{k-1} \widehat{f}_l$$
$$= R_{i,n+1-i} \left(1 + \sum_{k=n+1-i}^{n-1} \widehat{\beta}_k \prod_{l=n+1-i}^{k-1} \widehat{f}_l \right),$$

with estimators

$$\widehat{\alpha}_k = \frac{\sum_{i=1}^{n-k} S_{i,k+1}}{\sum_{i=1}^{n-k} R_{i,k}}, \quad \widehat{\beta}_k = \frac{\sum_{i=1}^{n-k} T_{i,k+1}}{\sum_{i=1}^{n-k} R_{i,k}} \quad \text{and} \quad \widehat{f}_k = 1 - \widehat{\alpha}_k + \widehat{\beta}_k.$$

Derived estimators

- One estimator for the reserves of accident year i:

$$\widehat{Reserve}_i = R_{i,n+1-i} \sum_{k=n+1-i}^{n-1} \widehat{\alpha}_k \prod_{l=n+1-i}^{k-1} \widehat{f}_l$$
$$= R_{i,n+1-i} \left(1 + \sum_{k=n+1-i}^{n-1} \widehat{\beta}_k \prod_{l=n+1-i}^{k-1} \widehat{f}_l\right),$$

with estimators

$$\widehat{\alpha}_k = \frac{\sum_{i=1}^{n-k} S_{i,k+1}}{\sum_{i=1}^{n-k} R_{i,k}}, \quad \widehat{\beta}_k = \frac{\sum_{i=1}^{n-k} T_{i,k+1}}{\sum_{i=1}^{n-k} R_{i,k}} \quad \text{and} \quad \widehat{f}_k = 1 - \widehat{\alpha}_k + \widehat{\beta}_k.$$

- Two estimators for the (conditional) mean squared error of the estimated reserves. One depends more on the payments and the other more on the reported amounts.

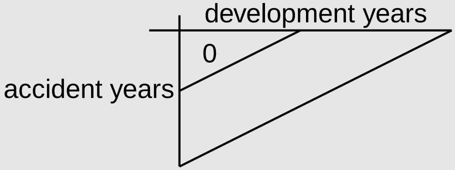
Separating accident damage from bodily injury claims

- The bodily injury flag has been introduced some time ago, but has been applied to new, still open or reopened claims, only.

Baloise Insurance

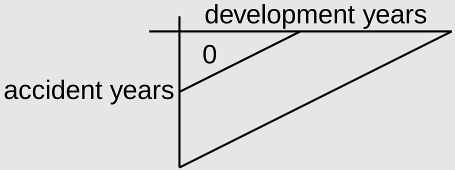
Separating accident damage from bodily injury claims

- The bodily injury flag has been introduced some time ago, but has been applied to new, still open or reopened claims, only. This means, the incremental triangles are of the form



Separating accident damage from bodily injury claims

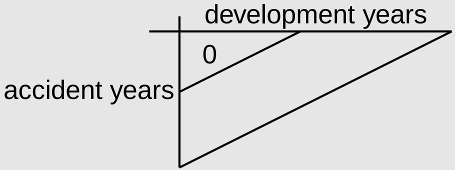
- The bodily injury flag has been introduced some time ago, but has been applied to new, still open or reopened claims, only. This means, the incremental triangles are of the form



- The same kind of data you may get if your company acquires another company and only migrates all open (and reopened) claims.

Separating accident damage from bodily injury claims

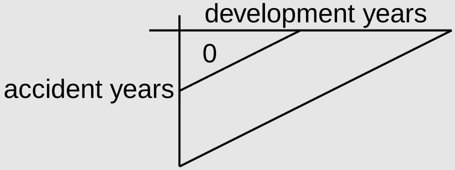
- The bodily injury flag has been introduced some time ago, but has been applied to new, still open or reopened claims, only. This means, the incremental triangles are of the form



- The same kind of data you may get if your company acquires another company and only migrates all open (and reopened) claims.
- Most of the standard reserving methods will not work on such data.

Separating accident damage from bodily injury claims

- The bodily injury flag has been introduced some time ago, but has been applied to new, still open or reopened claims, only. This means, the incremental triangles are of the form



- The same kind of data you may get if your company acquires another company and only migrates all open (and reopened) claims.
- Most of the standard reserving methods will not work on such data.
- The presented method will work if case reserves are available.

Benefits of the presented method

- Combines information of payments and reported amounts in a natural way to estimate the reserves.

Benefits of the presented method

- Combines information of payments and reported amounts in a natural way to estimate the reserves.
- Works for some kind of incomplete data.

Benefits of the presented method

- Combines information of payments and reported amounts in a natural way to estimate the reserves.
- Works for some kind of incomplete data.
- Easy to compute.

Drawbacks of the presented method

- Does not work if the case reserves are too small or even non positive. For instance, in cases with a huge impact by late reported or reopened claims.

Drawbacks of the presented method

- Does not work if the case reserves are too small or even non positive. For instance, in cases with a huge impact by late reported or reopened claims.
- No statement about the quality of the estimator of the reserves. We only know that it is conditionally mean preserving. For the estimator of the mean squared error we do not even know that.

Baloise Insurance

Drawbacks of the presented method

- Does not work if the case reserves are too small or even non positive. For instance, in cases with a huge impact by late reported or reopened claims.
- No statement about the quality of the estimator of the reserves. We only know that it is conditionally mean preserving. For the estimator of the mean squared error we do not even know that.
- More difficult to handle than two separate projections.

Outlook

- There is a paper in preparation (M. Merz, M. Wüthrich and R. D.) that gives an estimator for the one year solvency risk.

Outlook

- There is a paper in preparation (M. Merz, M. Wüthrich and R. D.) that gives an estimator for the one year solvency risk.
- A model based on assumptions on the joint distribution of payments and reported amounts which leads to the same estimators as in the presented model would be convenient. Such a model may give us a better understanding of the method itself and may lead to further stochastic statements about the distribution of the estimated reserves.

Questions?

Questions?

Enjoy your meal.