Property XL Rating - A reinsurance pricing tool combining experience and exposure rating

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Secura pricing tools

- Tools to price any classical reinsurance treaty type
- Full in-house development
- Developed under the SAS® software suite
- For internal use only
Outline

1. Introduction
2. Traditional methods and their limitations
3. Combining experience and exposure rating
4. Live demonstration
Property per risk XL reinsurance

- Protection for property portfolio of insurer
- Protection against large individual claims (not against events)
- Example of reinsurance program:

<table>
<thead>
<tr>
<th>Layer</th>
<th>Priority</th>
<th>Limit</th>
<th>Reinstatements</th>
</tr>
</thead>
<tbody>
<tr>
<td>$XL_1$</td>
<td>500.000</td>
<td>1.500.000</td>
<td>1@0%, 1@50%, 2@100%</td>
</tr>
<tr>
<td>$XL_2$</td>
<td>1.500.000</td>
<td>10.000.000</td>
<td>2@100%</td>
</tr>
<tr>
<td>$XL_3$</td>
<td>10.000.000</td>
<td>50.000.000</td>
<td>1@100%</td>
</tr>
</tbody>
</table>
Available Information

- Historical premium
- Historical large claims (for example > 300,000)

- Portfolio profiles
  - Current year
  - Past years
**Example of profile**

<table>
<thead>
<tr>
<th>Lower bound</th>
<th>Upper bound</th>
<th>Premium</th>
<th>Number of risks</th>
<th>Total Sum insured</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100.000</td>
<td>4.594.759</td>
<td>67.310</td>
<td>3.579.911.612</td>
</tr>
<tr>
<td>100.000</td>
<td>200.000</td>
<td>24.440.711</td>
<td>136.097</td>
<td>22.653.713.395</td>
</tr>
<tr>
<td>200.000</td>
<td>300.000</td>
<td>26.075.719</td>
<td>91.240</td>
<td>21.759.137.728</td>
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<tr>
<td>300.000</td>
<td>400.000</td>
<td>6.435.353</td>
<td>15.672</td>
<td>5.103.440.419</td>
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<tr>
<td>400.000</td>
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<td>2.057.380</td>
<td>3.808</td>
<td>1.785.717.925</td>
</tr>
<tr>
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<td>600.000</td>
<td>1.180.862</td>
<td>1.778</td>
<td>1.005.702.340</td>
</tr>
<tr>
<td>600.000</td>
<td>700.000</td>
<td>936.016</td>
<td>1.201</td>
<td>799.717.544</td>
</tr>
<tr>
<td>700.000</td>
<td>800.000</td>
<td>775.894</td>
<td>890</td>
<td>662.389.485</td>
</tr>
<tr>
<td>800.000</td>
<td>900.000</td>
<td>651.409</td>
<td>681</td>
<td>590.855.600</td>
</tr>
<tr>
<td>900.000</td>
<td>1.000.000</td>
<td>501.602</td>
<td>530</td>
<td>518.270.225</td>
</tr>
<tr>
<td>1.000.000</td>
<td>1.250.000</td>
<td>1.184.512</td>
<td>1.072</td>
<td>1.099.300.525</td>
</tr>
<tr>
<td></td>
<td>1.500.000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10.000.000</td>
<td>12.500.000</td>
<td>697.406</td>
<td>115</td>
<td>1.325.170.855</td>
</tr>
<tr>
<td>12.500.000</td>
<td>15.000.000</td>
<td>428.088</td>
<td>70</td>
<td>954.756.193</td>
</tr>
<tr>
<td>15.000.000</td>
<td>50.000.000</td>
<td>588.906</td>
<td>56</td>
<td>1.851.132.215</td>
</tr>
</tbody>
</table>
Introduction

Traditional methods and their limitations

- Traditional Experience Rating
- Traditional Exposure Rating

Combined Experience-Exposure Rating

Live demonstration

**Traditional methods**

- **Historical claims data**
- **Portfolio profile data**
Traditional methods

- Traditional Experience Rating
- Traditional Exposure Rating

Combined
Experience-Exposure Rating

Live demonstration
Traditional methods

- Traditional Experience Rating
- Traditional Exposure Rating

Combined Experience-Exposure Rating

Live demonstration
Traditional Experience Rating

- Burning cost method
- E.g.: Layer 0.5M - 1.5M

\[
BC = \frac{\sum_{t=1}^{T-1} \sum_{k_t=1}^{n_t} \min(1M; \max(0; C_{t,k_t} - 0.5M))}{\sum_{t=1}^{T-1} P_t}
\]
Estimated Total Cost

- Let \( P_T \) = Estimated premium income for year \( T \)
- Estimated total cost in the layer \( L \) xs \( D \):

\[
TC = BC \times P_T
\]

\[
= \frac{P_T}{\sum_{t=1}^{T-1} P_t} \sum_{t=1}^{T-1} \sum_{k_t=1}^{n_t} \min(L; \max(0; C_{t,k_t} - D))
\]

\[
= \frac{P_T}{\sum_{t=1}^{T-1} P_t} \frac{\sum_{t=1}^{T-1} N_t}{\sum_{t=1}^{T-1} P_t} \frac{\sum_{t=1}^{T-1} \sum_{k_t=1}^{n_t} \min(L; \max(0; C_{t,k_t} - D))}{\sum_{t=1}^{T-1} N_t}
\]

The total premium is used as the underlying **measure of exposure**
- Claims and premium are usually adapted to reflect current economic conditions
  - Claims are indexed using e.g. construction price index
  - Premiums are indexed to reflect tariff evolutions
Burning Cost: Limitations

- We do not use all available information:
  - Historic profile information for period \(1, \ldots, T-1\)
  - Expected profile for year \(T\)

- How to price unused capacity?

- Portfolio evolutions are not taken into account!
  What to do if the relative portfolio composition has changed?
  E.g.: the total premium has remained constant but:
  - There are more large risks
  - There are less large risks
### Example of portfolio evolution

#### Absolute Profile Evolution

<table>
<thead>
<tr>
<th>Priority</th>
<th>Year</th>
<th>Premium</th>
<th>Number</th>
<th>Total Sum Insured</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2001</td>
<td>44.227.501</td>
<td>15.667</td>
<td>39.435.211.295</td>
</tr>
<tr>
<td>0</td>
<td>2002</td>
<td>47.311.587</td>
<td>14.195</td>
<td>45.414.605.546</td>
</tr>
<tr>
<td>0</td>
<td>2003</td>
<td>80.762.894</td>
<td>13.877</td>
<td>70.158.494.783</td>
</tr>
<tr>
<td>0</td>
<td>2004</td>
<td>103.189.000</td>
<td>12.694</td>
<td>87.556.000.000</td>
</tr>
<tr>
<td>0</td>
<td>2005</td>
<td>97.146.000</td>
<td>11.833</td>
<td>86.508.000.000</td>
</tr>
<tr>
<td>0</td>
<td>2006</td>
<td>95.360.000</td>
<td>11.073</td>
<td>78.294.000.000</td>
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<tr>
<td>500.000</td>
<td>2001</td>
<td>16.119.604</td>
<td>520</td>
<td>24.165.706.663</td>
</tr>
<tr>
<td>500.000</td>
<td>2002</td>
<td>19.884.758</td>
<td>632</td>
<td>29.670.906.869</td>
</tr>
<tr>
<td>500.000</td>
<td>2003</td>
<td>44.160.705</td>
<td>1.074</td>
<td>50.909.582.253</td>
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<tr>
<td>500.000</td>
<td>2004</td>
<td>62.758.000</td>
<td>1.293</td>
<td>67.798.000.000</td>
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<tr>
<td>500.000</td>
<td>2005</td>
<td>56.662.000</td>
<td>1.395</td>
<td>66.505.000.000</td>
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<tr>
<td>500.000</td>
<td>2006</td>
<td>54.670.000</td>
<td>1.226</td>
<td>58.398.000.000</td>
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</table>
Example of portfolio evolution

<table>
<thead>
<tr>
<th>Priority</th>
<th>Year</th>
<th>Premium</th>
<th>Number</th>
<th>TSI</th>
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</thead>
<tbody>
<tr>
<td>0</td>
<td>2001</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>0</td>
<td>2002</td>
<td>107</td>
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<td>115</td>
</tr>
<tr>
<td>0</td>
<td>2003</td>
<td>183</td>
<td>89</td>
<td>178</td>
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<tr>
<td>0</td>
<td>2004</td>
<td>233</td>
<td>81</td>
<td>222</td>
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<tr>
<td>0</td>
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<td>220</td>
<td>76</td>
<td>219</td>
</tr>
<tr>
<td>0</td>
<td>2006</td>
<td>216</td>
<td>71</td>
<td>199</td>
</tr>
<tr>
<td>500.000</td>
<td>2001</td>
<td>100</td>
<td>100</td>
<td>100</td>
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<tr>
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</tr>
<tr>
<td>500.000</td>
<td>2003</td>
<td>274</td>
<td>207</td>
<td>211</td>
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<tr>
<td>500.000</td>
<td>2004</td>
<td>389</td>
<td>249</td>
<td>281</td>
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<tr>
<td>500.000</td>
<td>2005</td>
<td>352</td>
<td>268</td>
<td>275</td>
</tr>
<tr>
<td>500.000</td>
<td>2006</td>
<td>339</td>
<td>236</td>
<td>242</td>
</tr>
</tbody>
</table>

- Evolution of the number of risks may be quite different from the evolution of the premium income.
- Evolution above the priority may be quite different from the evolution f.g.u.
Traditional Exposure Rating

- Key element: degree of damage
- $C$ = loss for risk with insured value $SI$, given that there is a loss
- **Degree of damage:** $X = C/SI \in (0, 1]$
- Typical distribution of degree of damage:

Based on the degree of damage distribution and current portfolio profile information, it is easy to price any XL reinsurance layer.
- It is called **Exposure Rating**.
Limitations of Exposure Rating

- In practice, exposure rating also has important limitations.
- A.o. it does not use all available information:
  - Loss experience
  - Only last profile is used (no historic profile information)
- We will not detail other limitations here.
Combined Experience-Exposure Rating

- Explanation of principles for profile with 1 band and 1 year of claims experience
- Generalization to general profiles is easy
- Aim:
  - Use historical claims information
  - Use information about portfolio evolution
Profile with 1 Band

- Assume the following profile evolution between year 1 and year $T$:

<table>
<thead>
<tr>
<th>Year</th>
<th>Nb</th>
<th>SI</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.000</td>
<td>200.000</td>
</tr>
<tr>
<td>$T$</td>
<td>1.200</td>
<td>250.000</td>
</tr>
</tbody>
</table>

- Assume construction price index increases by 10% between year 1 and year $T$

- Observed claims in year 1: $\{C_{1,1}, \ldots, C_{1,n}\}$

  $\Rightarrow$ Indexed claims: $\{C^I_{1,1}, \ldots, C^I_{1,n}\}$, where $C^I_{1,k} = 1.1C_{1,k}$

- Assume we are interested in the claims above 100,000
Profile with 1 Band: Frequency

- Let $\lambda_{1}^{0,1M}$ = number of indexed claims $> 0,1M$
- Year 1: Let $S_{1,i} = \text{random variable describing the indexed loss for risk } i$.
  Then the estimated number of indexed losses larger than $0,1M$ is:

$\mathbb{E}[N_{1}^{0,1M}] = 1,000\mathbb{P}[S_{1,i} > 0,1M] = 1,000q_{1}\mathbb{P}[X_{1} > \frac{0,1M}{1,1*0,2M}]$

$= 1,000q_{1}[1 - F_{X_{1}}(5/11)]$, where

- $q_{1} = \text{from-ground-up probability that the risks produce a loss}$
- $X_{1} = \text{degree of damage of the risks, given that there is a loss}$

- Year $T$: Similarly, the estimated number of losses larger than $0,1M$ is:

$\mathbb{E}[N_{T}^{0,1M}] = 1,200\mathbb{P}[S_{T,i} > 0,1M] = 1,200q_{T}\mathbb{P}[X_{T} > \frac{0,1M}{0,25M}]$

$= 1,200q_{T}[1 - F_{X_{T}}(2/5)]$,

$\Rightarrow$ Estimated claims frequency above $0,1M$ in year $T$: $\hat{\lambda}_{T}^{0,1M} = \frac{\lambda_{1}^{0,1M} \mathbb{E}[N_{T}^{0,1M}]}{\mathbb{E}[N_{1}^{0,1M}]}$
Illustration

- Assume the from ground up probability of producing a loss $q_1 = q_T$
- Assume distribution function of the degree of damage is the same in year 1 and in year $T$:

<table>
<thead>
<tr>
<th>Degree of Damage $x$</th>
<th>$F_x(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>10%</td>
<td>25%</td>
</tr>
<tr>
<td>20%</td>
<td>45%</td>
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<tr>
<td>30%</td>
<td>62%</td>
</tr>
<tr>
<td>2/5</td>
<td>76%</td>
</tr>
<tr>
<td>5/11</td>
<td>79%</td>
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<tr>
<td>50%</td>
<td>83%</td>
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<td>60%</td>
<td>90%</td>
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<tr>
<td>70%</td>
<td>95%</td>
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<td>80%</td>
<td>97%</td>
</tr>
<tr>
<td>90%</td>
<td>99%</td>
</tr>
<tr>
<td>100%</td>
<td>100%</td>
</tr>
</tbody>
</table>
Illustration

- \( \mathbb{E}[N_{1,1M}^0] = 1.000 \times q \times [1 - F_X(5/11)] \)
  \[ = 1.000 \times q \times (1 - 0.79) \]
- \( \mathbb{E}[N_{T,1M}^0] = 1.200 \times q \times [1 - F_X(2/5)] \)
  \[ = 1.200 \times q \times (1 - 0.76) \]

⇒ Estimated claims frequency above 0, 1M in year \( T \):

\[
\hat{\lambda}_{T,1M}^0 = \lambda_{1,1M}^0 \frac{\mathbb{E}[N_{T,1M}^0]}{\mathbb{E}[N_{1,1M}^0]}
\]

\[ = 1.37 \lambda_{1,1M}^0 \]

⇒ We estimate an increase of 37% in the claims frequency above 0, 1M between year 1 and year \( T \):
  - 20% of this increase is due to the change in the number of risks
  - Another 17% is due to the fact that the insured value in year \( T \) (0, 25M) is larger than in year 1 (0, 22M, after indexation)
Profile with 1 Band: As-if Claims

- Based on the distribution function of the degree of damage, we can calculate the probability distribution of the claim amount, knowing that it is larger than 0.1M.
  - Year 1: \( F_{S_1 | S_1 \geq 0.1M}(s) \)
  - Year \( T \): \( F_{S_T | S_T \geq 0.1M}(s) \)

- We want to create as-if claims above 0.1M, taking into account:
  - Indexation
  - Evolutions in the portfolio of risks

- Take all claims \( C_{1,k}^I \), for \( k \in \{1, \ldots, n\} \) for which \( C_{1,k}^I > 0.1M \)

- Apply the function \( F_{S_T | S_T \geq 0.1M}^{-1} \circ F_{S_1 | S_1 \geq 0.1M} \)

\[ \Rightarrow \text{We transform an indexed claim} \]

\[ C_{1,k}^I > 0.1M \]

with a probability level

\[ F_{S_1 | S_1 \geq 0.1M}(C_{1,k}^I) \]

in year 1 to a claim with the same probability level in year \( T \)
Illustration

- Calculate $F_{S_1|S_1 \geq 0.1M}$ and $F_{S_T|S_T \geq 0.1M}$
- Take $C_{1,k}^I > 0.1M$: e.g. $C_{1,k}^I = 0.2M$
- Apply $F_{S_1|S_1 \geq 0.1M}$ to obtain $F_{S_1|S_1 \geq 0.1M}(0, 2M)$
- Apply $F_{S_T|S_T \geq 0.1M}^{-1}$ to obtain $F_{S_T|S_T \geq 0.1M}^{-1}(F_{S_1|S_1 \geq 0.1M}(0, 2M))$
Illustration

- Calculate $F_{S_1|S_1 \geq 0.1M}$ and $F_{S_T|S_T \geq 0.1M}$
- Take $C_{1,k}^I > 0.1M$: e.g. $C_{1,k}^I = 0.2M$
- Apply $F_{S_1|S_1 \geq 0.1M}$ to obtain $F_{S_1|S_1 \geq 0.1M}(0, 2M)$
- Apply $F_{S_T|S_T \geq 0.1M}^{-1}$ to obtain $F_{S_T|S_T \geq 0.1M}^{-1}(F_{S_1|i|S_1,i \geq 0.1M}(0, 2M))$
Illustration

- Calculate $F_{S1 | S1 \geq 0,1M}$ and $F_{ST | ST \geq 0,1M}$
- Take $C_{1,k}^I > 0,1M$: e.g. $C_{1,k}^I = 0,2M$
- Apply $F_{S1 | S1 \geq 0,1M}$ to obtain $F_{S1 | S1 \geq 0,1M}(0, 2M)$
- Apply $F_{ST}^{-1} | ST \geq 0,1M$ to obtain $F_{ST}^{-1} | ST \geq 0,1M(F_{S1 | S1 \geq 0,1M}(0, 2M))$
Illustration

- Calculate \( F_{S_1 | S_1 \geq 0,1M} \) and \( F_{S_T | S_T \geq 0,1M} \)
- Take \( C_{1,k}^I > 0,1M \): e.g. \( C_{1,k}^I = 0,2M \)
- Apply \( F_{S_1 | S_1 \geq 0,1M} \) to obtain \( F_{S_1 | S_1 \geq 0,1M}(0,2M) \)
- Apply \( F_{S_T | S_T \geq 0,1M}^{-1} \) to obtain \( F_{S_T | S_T \geq 0,1M}(F_{S_1 | S_1 \geq 0,1M}(0,2M)) \)
Working and Non-Working Layers

- For **working layers**, we use combined experience-exposure rating. This means we use:
  - Historically observed indexed claims
  - Indexed profile
  - Detailed portfolio evolutions (taken into account with exposure curves) to estimate:
    - Claims frequency
    - As-if claims

- For **non-working layers**, we use exposure rating, calibrated on the experience of a working layer
Live demonstration
Appendix: formulas as-if claims

- For year 1, we define for $s \geq 0, 1M$:

$$F_{S_1|S_1\geq0,1M}(s) = \frac{\mathbb{P}[S_1 \leq s|S_1 > 0, 1M]}{\mathbb{P}[S_1 > 0, 1M]} = \frac{\mathbb{P}[S_1 \leq s \cap S_1 > 0, 1M]}{\mathbb{P}[S_1 > 0, 1M]}$$

$$= \frac{\mathbb{P}[0, 22M.X_1 \leq s \cap 0, 22M.X_1 > 0, 1M]}{q_1(1 - F_{X_1}(5/11))}$$

$$= \frac{F_{X_1}(s/0, 22M) - F_{X_1}(5/11)}{1 - F_{X_1}(5/11)}$$

- Similarly, for year $T$, we define for $s \geq 0, 1M$:

$$F_{S_T|S_T\geq0,1M}(s) = \frac{\mathbb{P}[S_T \leq s|S_T > 0, 1M]}{\mathbb{P}[S_T > 0, 1M]} = \frac{\mathbb{P}[0, 25M.X_1 \leq s \cap 0, 25M.X_1 > 0, 1M]}{q_1(1 - F_{X_T}(2/5))}$$

$$= \frac{F_{X_T}(s/0, 25M) - F_{X_T}(2/5)}{1 - F_{X_T}(2/5)}$$