Property XL Rating - A reinsurance pricing tool combining experience and exposure rating

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ASTIN, MANCHESTER, JULY 2008

Secura pricing tools

- Tools to price any classical reinsurance treaty type
- Full in-house development
- Developed under the SAS® software suite
- For internal use only

Outline

1. Introduction
2. Traditional methods and their limitations
3. Combining experience and exposure rating
4. Live demonstration

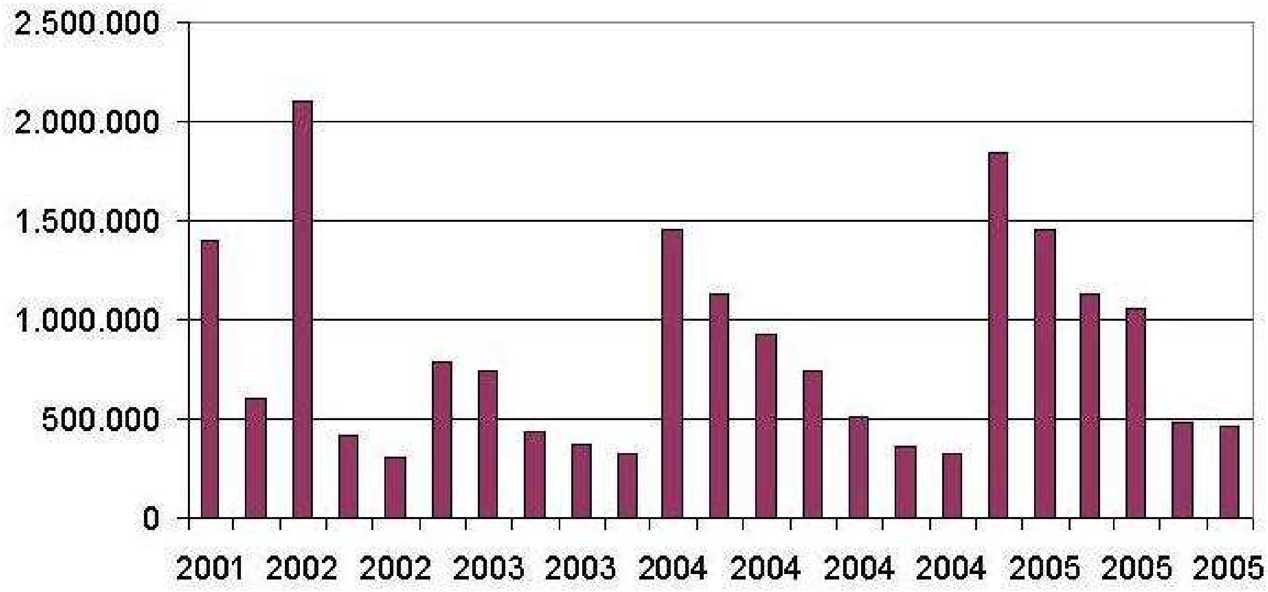
Property per risk XL reinsurance

- Protection for property portfolio of insurer
- Protection against large individual claims (not against events)
- Example of reinsurance program:

Layer	Priority	Limit	Reinstatements
XL_1	500.000	$1.500.000$	1@0%, 1@50%, 2@100%
XL_2	$1.500.000$	$10.000.000$	2@100%
XL_3	$10.000.000$	$50.000.000$	1@100%

Available Information

- Historical premium
- Historical large claims (for example > 300.000)



- Portfolio profiles
  - Current year
  - Past years

Introduction
- Secura pricing tools
- Outline
- Property per risk XL reinsurance
- Available Information

Traditional methods and their limitations

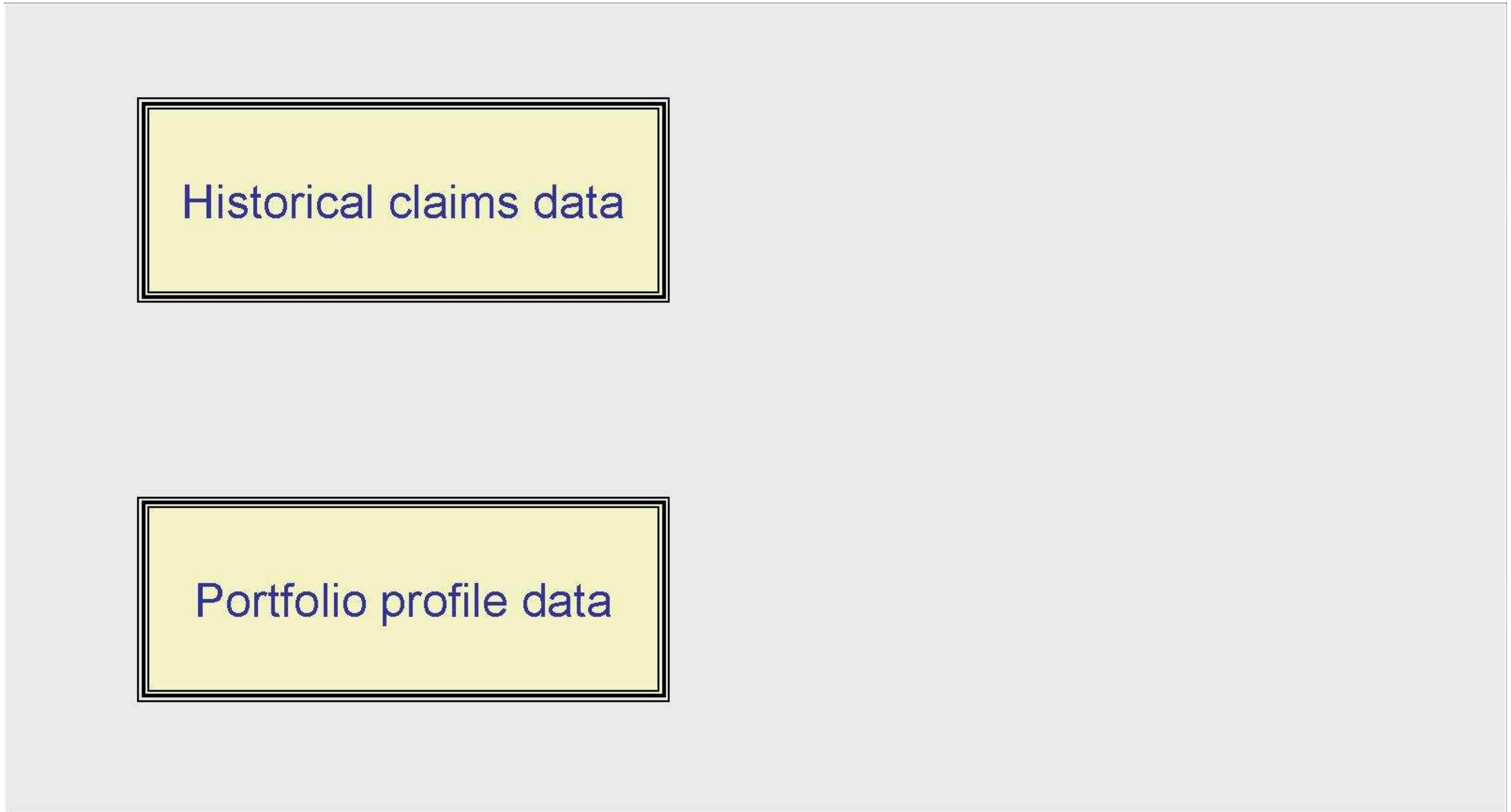
Combined Experience-Exposure Rating

Live demonstration

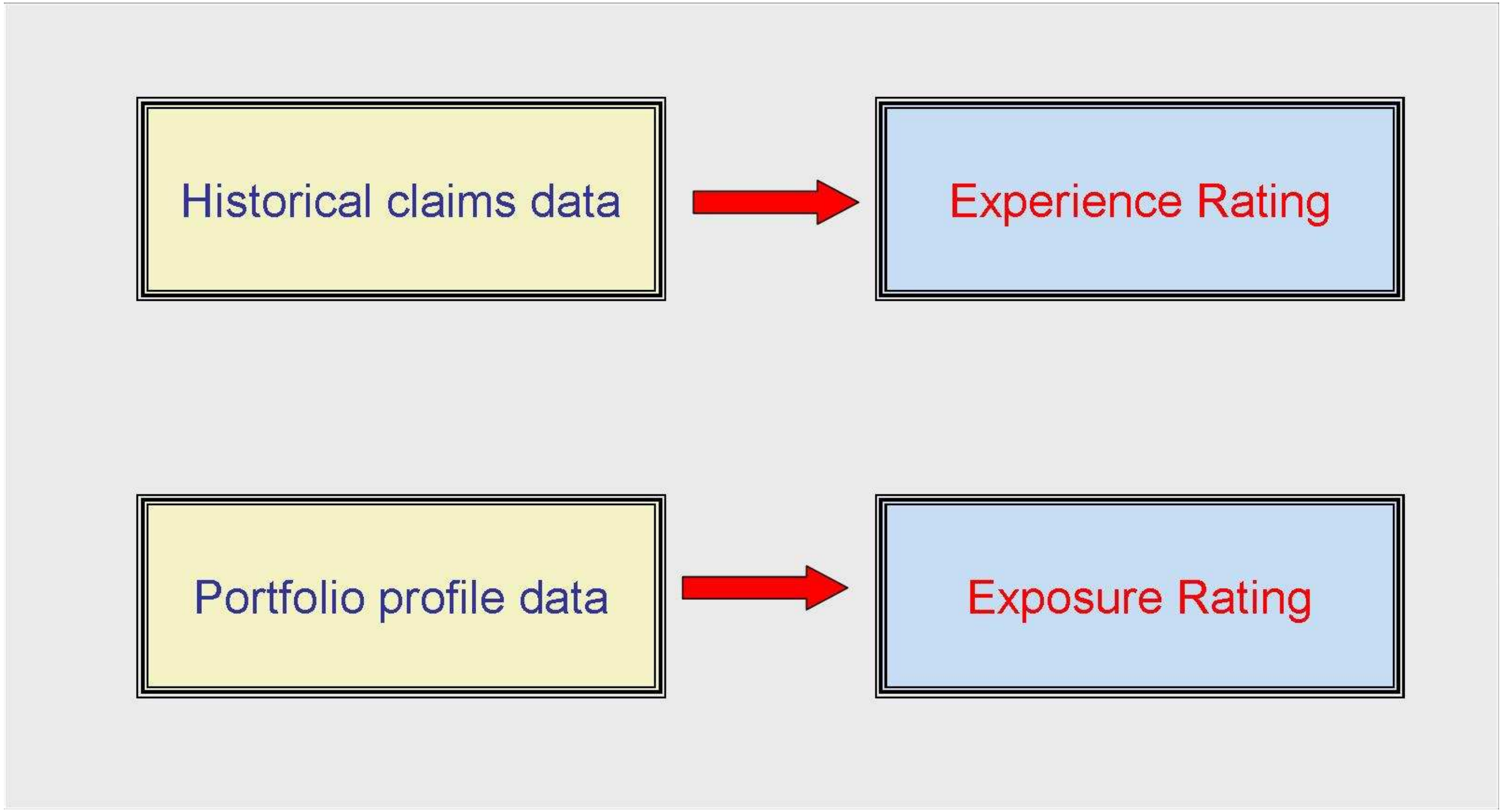
Example of profile

Lower bound	Upper bound	Premium	Number of risks	Total Sum insured
1	100.000	4.594.759	67.310	3.579.911.612
100.000	200.000	24.440.711	136.097	22.653.713.395
200.000	300.000	26.075.719	91.240	21.759.137.728
300.000	400.000	6.435.353	15.672	5.103.440.419
400.000	500.000	2.057.380	3.808	1.785.717.925
500.000	600.000	1.180.862	1.778	1.005.702.340
600.000	700.000	936.016	1.201	799.717.544
700.000	800.000	775.894	890	662.389.485
800.000	900.000	651.409	681	590.855.600
900.000	1.000.000	501.602	530	518.270.225
1.000.000	1.250.000	1.184.512	1.072	1.099.300.525
...	...	...	...	...
10.000.000	12.500.000	697.406	115	1.325.170.855
12.500.000	15.000.000	428.088	70	954.756.193
15.000.000	50.000.000	588.906	56	1.851.132.215

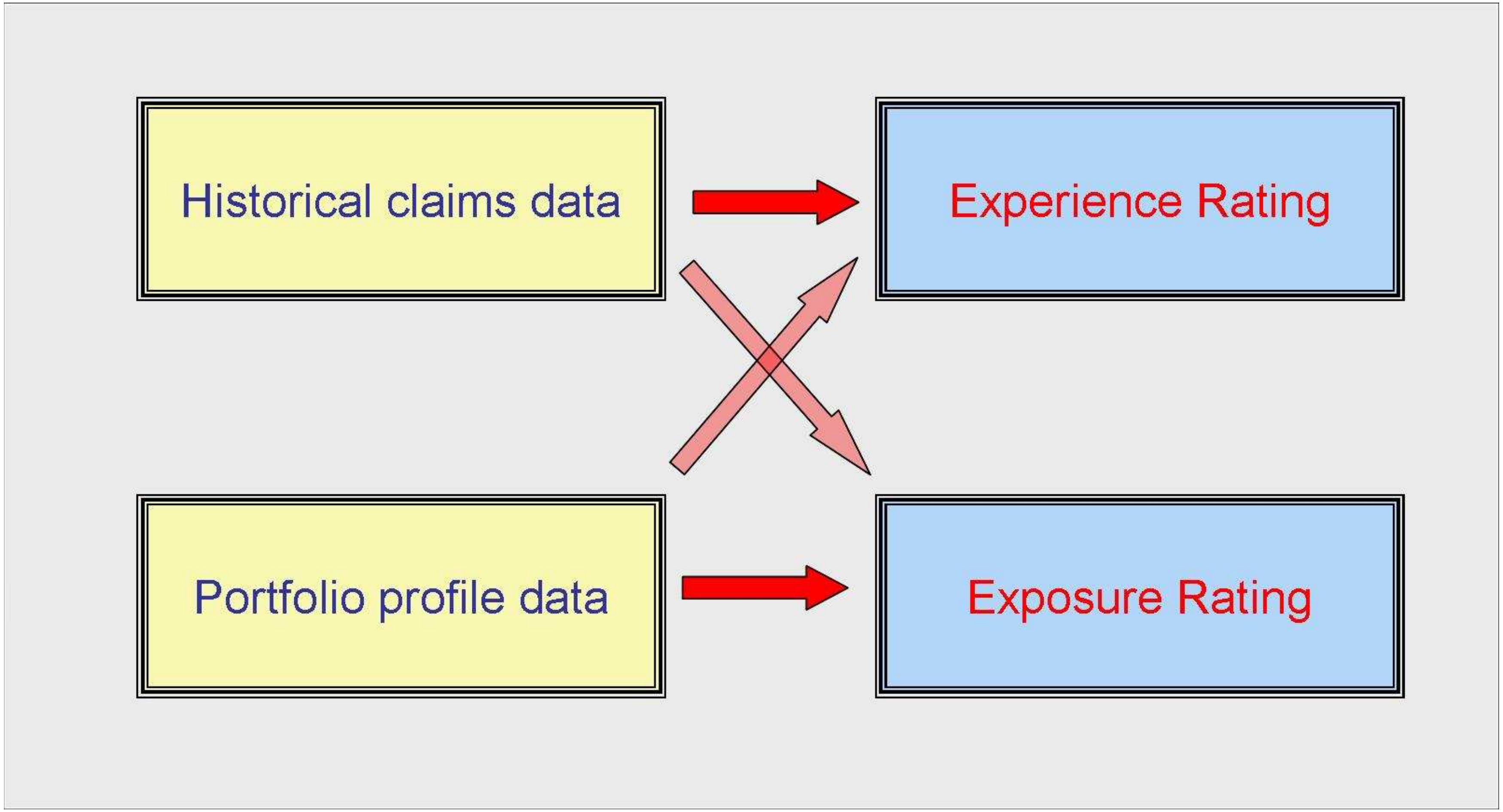
Traditional methods



Traditional methods

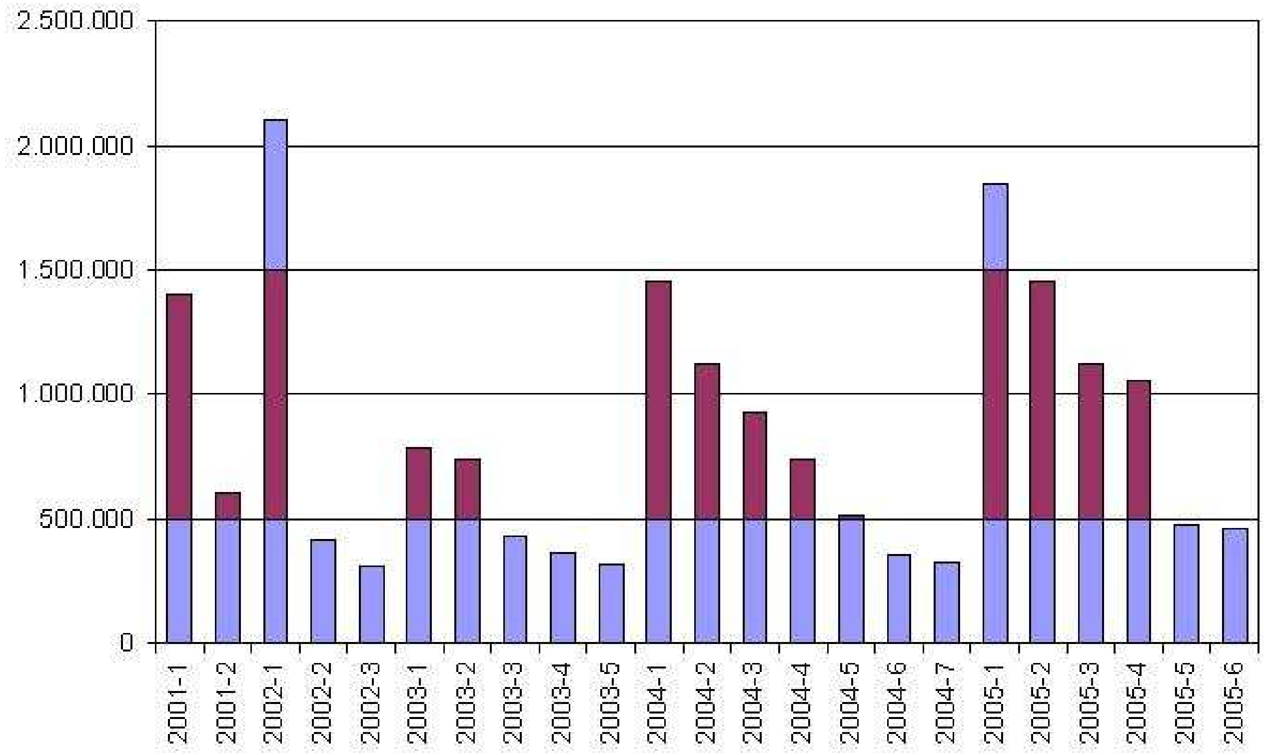


Traditional methods



Traditional Experience Rating

- Burning cost method
- E.g.: Layer 0.5M - 1.5M



- Burning cost = Claims in the layer / Exposed Premium

$$BC = \frac{\sum_{t=1}^{T-1} \sum_{k_t=1}^{n_t} \min(1M; \max(0; C_{t,k_t} - 0.5M))}{\sum_{t=1}^{T-1} P_t}$$

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Estimated Total Cost

- Let P_T = Estimated premium income for year T
- Estimated total cost in the layer L xs D :

$$\begin{aligned} TC &= BC * P_T \\ &= \frac{P_T}{\sum_{t=1}^{T-1} P_t} \sum_{t=1}^{T-1} \sum_{k_t=1}^{n_t} \min(L; \max(0; C_{t,k_t} - D)) \\ &= P_T \frac{\sum_{t=1}^{T-1} N_t}{\sum_{t=1}^{T-1} P_t} \frac{\sum_{t=1}^{T-1} \sum_{k_t=1}^{n_t} \min(L; \max(0; C_{t,k_t} - D))}{\sum_{t=1}^{T-1} N_t} \end{aligned}$$

  The total premium is used as the underlying **measure of exposure**

- Claims and premium are usually adapted to reflect current economic conditions
  - Claims are indexed using e.g. construction price index
  - Premiums are indexed to reflect tariff evolutions

Burning Cost: Limitations

- We do not use all available information :
  - Historic profile information for period $\{1, \ldots, T-1\}$
  - Expected profile for year T
- How to price unused capacity ?
- Portfolio evolutions are not taken into account !

  What to do if the relative portfolio composition has changed ?

  E.g.: the total premium has remained constant but:
  - There are more large risks
  - There are less large risks

Example of portfolio evolution

		Absolute Profile Evolution		
Priority	Year	Premium	Number	Total Sum Insured
0	2001	44.227.501	15.667	39.435.211.295
0	2002	47.311.587	14.195	45.414.605.546
0	2003	80.762.894	13.877	70.158.494.783
0	2004	103.189.000	12.694	87.556.000.000
0	2005	97.146.000	11.833	86.508.000.000
0	2006	95.360.000	11.073	78.294.000.000
500.000	2001	16.119.604	520	24.165.706.663
500.000	2002	19.884.758	632	29.670.906.869
500.000	2003	44.160.705	1.074	50.909.582.253
500.000	2004	62.758.000	1.293	67.798.000.000
500.000	2005	56.662.000	1.395	66.505.000.000
500.000	2006	54.670.000	1.226	58.398.000.000

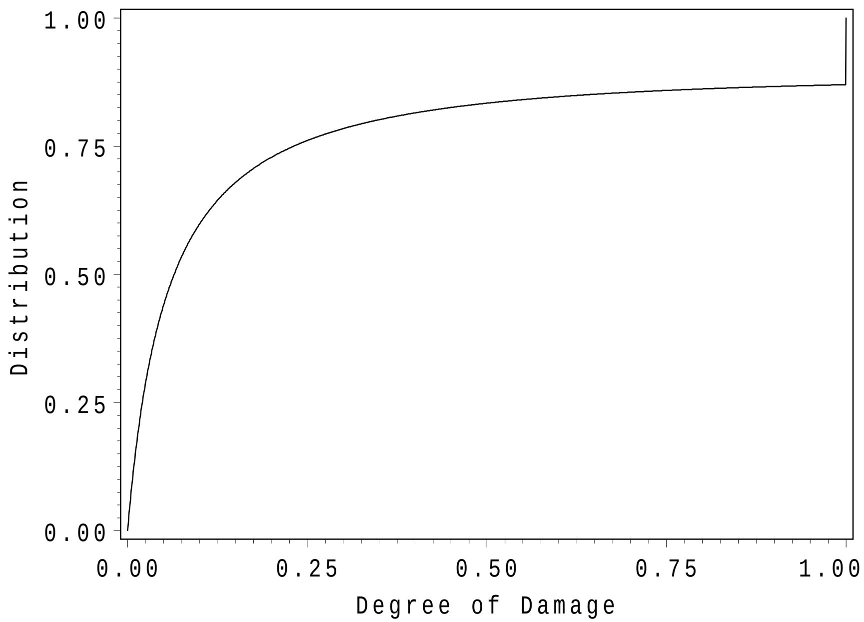
Example of portfolio evolution

		Relative Profile Evolution		
Priority	**Year**	**Premium**	**Number**	**TSI**
0	2001	100	100	100
0	2002	107	91	115
0	2003	183	89	178
0	2004	233	81	222
0	2005	220	76	219
0	2006	216	71	199
500.000	2001	100	100	100
500.000	2002	123	122	123
500.000	2003	274	207	211
500.000	2004	389	249	281
500.000	2005	352	268	275
500.000	2006	339	236	242

- Evolution of the number of risks may be quite different from the evolution of the premium income
- Evolution above the priority may be quite different from the evolution f.g.u.

Traditional Exposure Rating

- Key element : degree of damage
- C = loss for risk with insured value SI, given that there is a loss
- **Degree of damage:** $X = C/SI \in (0,1]$
- Typical distribution of degree of damage:



- Based on the degree of damage dstribution **and** current portfolio profile information, it is easy to price any XL reinsurance layer.
- It is called Exposure Rating.

Limitations of Exposure Rating

- In practice, exposure rating also has important limitations.
- A.o. it does not use all available information:
  - Loss experience
  - Only last profile is used (no historic profile information)
- We will not detail other limitations here.

Combined Experience-Exposure Rating

- Explanation of principles for profile with 1 band and 1 year of claims experience
- Generalization to general profiles is easy
- Aim:
  - Use historical claims information
  - Use information about portfolio evolution

Profile with 1 Band

- Assume the following profile evolution between year 1 and year T:

Year	Nb	SI
1	1.000	200.000
T	1.200	250.000

- Assume construction price index increases by 10% between year 1 and year T
- Observed claims in year 1: $\{C_{1,1}, \ldots, C_{1,n}\}$

$\Rightarrow$ Indexed claims: $\{C^I_{1,1}, \ldots, C^I_{1,n}\}$, where $C^I_{1,k} = 1.1 C_{1,k}$

- Assume we are interested in the claims above 100.000

Profile with 1 Band: Frequency

- Let $\lambda_1^{0,1M}$ = number of indexed claims $> 0,1M$
- Year 1: Let $S_{1,i}$ = random variable describing the indexed loss for risk i. Then the estimated number of indexed losses larger than $0,1M$ is:

$$\begin{aligned} \mathbb{E}[N_1^{0,1M}] &= 1,000\mathbb{P}[S_{1,i} > 0,1M] = 1,000q_1\mathbb{P}\left[X_1 > \frac{0,1M}{1.1 * 0,2M}\right] \\ &= 1,000q_1[1 - F_{X_1}(5/11)], \text{ where} \end{aligned}$$

  - q_1 = from-ground-up probability that the risks produce a loss
  - X_1 = degree of damage of the risks, given that there is a loss
- Year T: Similarly, the estimated number of losses larger than $0,1M$ is:

$$\begin{aligned} \mathbb{E}[N_T^{0,1M}] &= 1,200\mathbb{P}[S_{T,i} > 0,1M] = 1,200q_T\mathbb{P}\left[X_T > \frac{0,1M}{0,25M}\right] \\ &= 1,200q_T[1 - F_{X_T}(2/5)], \end{aligned}$$

$\Rightarrow$ Estimated claims frequency above $0,1M$ in year T: $\hat{\lambda}_T^{0,1M} = \lambda_1^{0,1M} \frac{\mathbb{E}[N_T^{0,1M}]}{\mathbb{E}[N_1^{0,1M}]}$

Illustration

- Assume the from ground up probability of producing a loss $q_1 = q_T$
- Assume distribution function of the degree of damage is the same in year 1 and in year T :

Degree of Damage x	$\mathbf{F_X(x)}$
0%	0%
10%	25%
20%	45%
30%	62%
2/5	**76%**
5/11	**79%**
50%	83%
60%	90%
70%	95%
80%	97%
90%	99%
100%	100%

Illustration

- $\mathbb{E}[N_1^{0,1M}] = 1.000 \times q \times [1 - F_X(5/11)]$
  $= 1.000 \times q \times (1 - 0,79)$
- $\mathbb{E}[N_T^{0,1M}] = 1.200 \times q \times [1 - F_X(2/5)]$
  $= 1.200 \times q \times (1 - 0,76)$

$\Rightarrow$ Estimated claims frequency above $0,1M$ in year T:

$$\hat{\lambda}_T^{0,1M} = \lambda_1^{0,1M} \frac{\mathbb{E}[N_T^{0,1M}]}{\mathbb{E}[N_1^{0,1M}]} = 1,37\lambda_1^{0,1M}$$

$\Rightarrow$ We estimate an increase of 37% in the claims frequency above $0,1M$ between year 1 and year T:

- 20% of this increase is due to the change in the number of risks
- Another 17% is due to the fact that the insured value in year T ($0,25M$) is larger than in year 1 ($0,22M$, after indexation)

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Profile with 1 Band: As-if Claims

- Based on the distribution function of the degree of damage, we can calculate the probability distribution of the claim amount, knowing that it is larger than 0,1M.
  - Year 1 : $F_{S_1|S_1 \geqslant 0,1M}(s)$
  - Year T : $F_{S_T|S_T \geqslant 0,1M}(s)$
- We want to create as-if claims above $0,1M$, taking into account:
  - Indexation
  - Evolutions in the portfolio of risks
- Take all claims $C^I_{1,k}$, for $k \in \{1, \ldots, n\}$ for which $C^I_{1,k} > 0,1M$
- Apply the function $F^{-1}_{S_T|S_T \geqslant 0,1M} \circ F_{S_1|S_1 \geqslant 0,1M}$

⇒ We transform an indexed claim

$$C^I_{1,k} > 0,1M$$

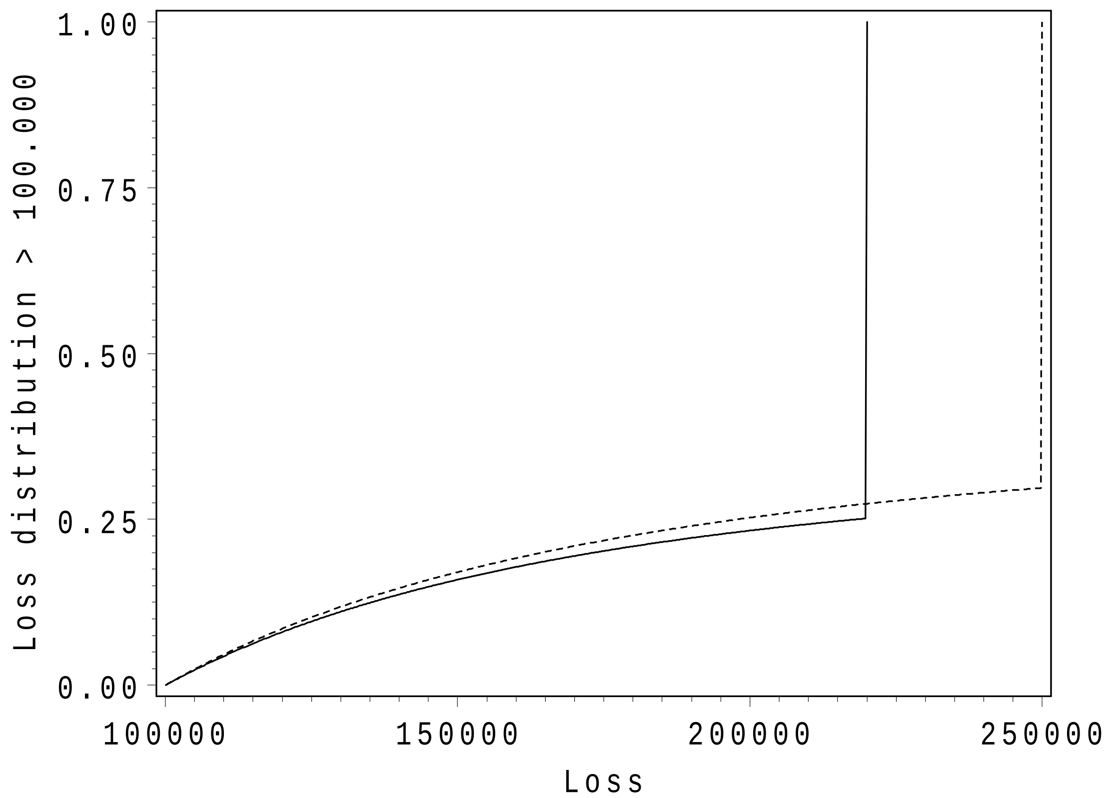
with a probability level

$$F_{S_1|S_1 \geqslant 0,1M}(C^I_{1,k})$$

in year 1 to a claim with the same probability level in year T

Illustration

- Calculate $F_{S_1|S_1 \geqslant 0,1M}$ and $F_{S_T|S_T \geqslant 0,1M}$
- Take $C^I_{1,k} > 0,1M$: e.g. $C^I_{1,k} = 0,2M$
- Apply $F_{S_1|S_1 \geqslant 0,1M}$ to obtain $F_{S_1|S_1 \geqslant 0,1M}(0,2M)$
- Apply $F^{-1}_{S_T|S_T \geqslant 0,1M}$ to obtain $F^{-1}_{S_T|S_T \geqslant 0,1M}(F_{S_1|S_1 \geqslant 0,1M}(0,2M))$



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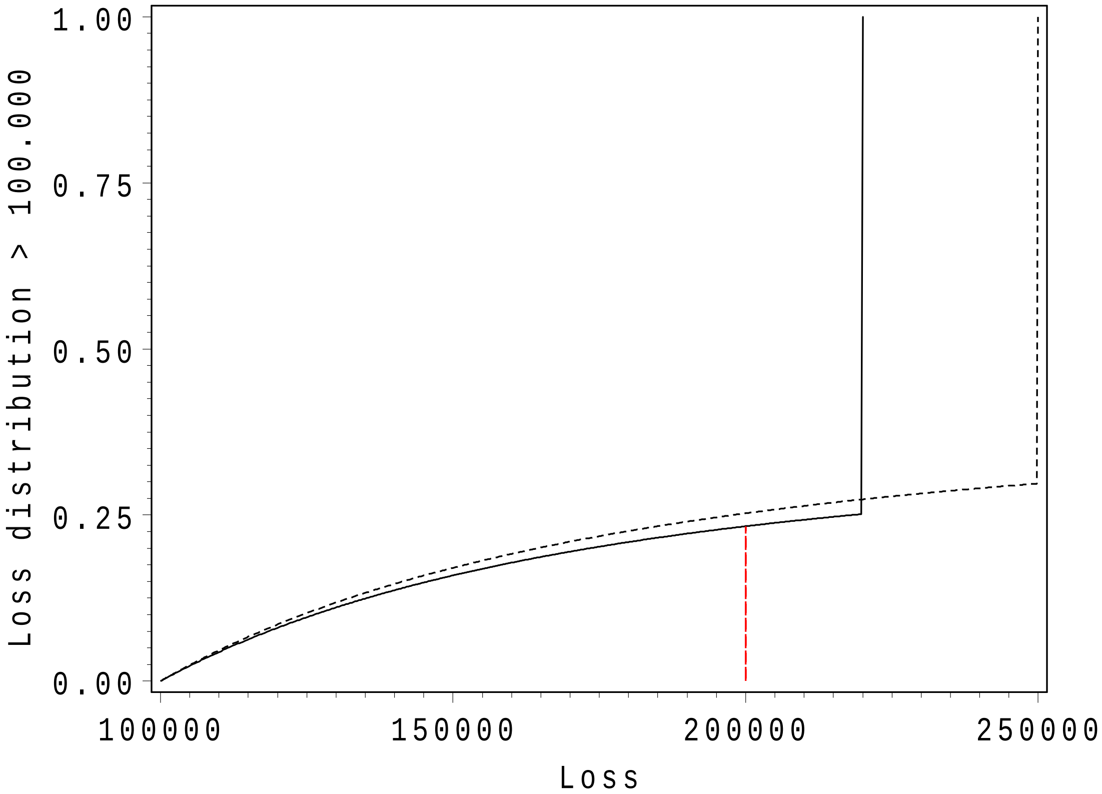

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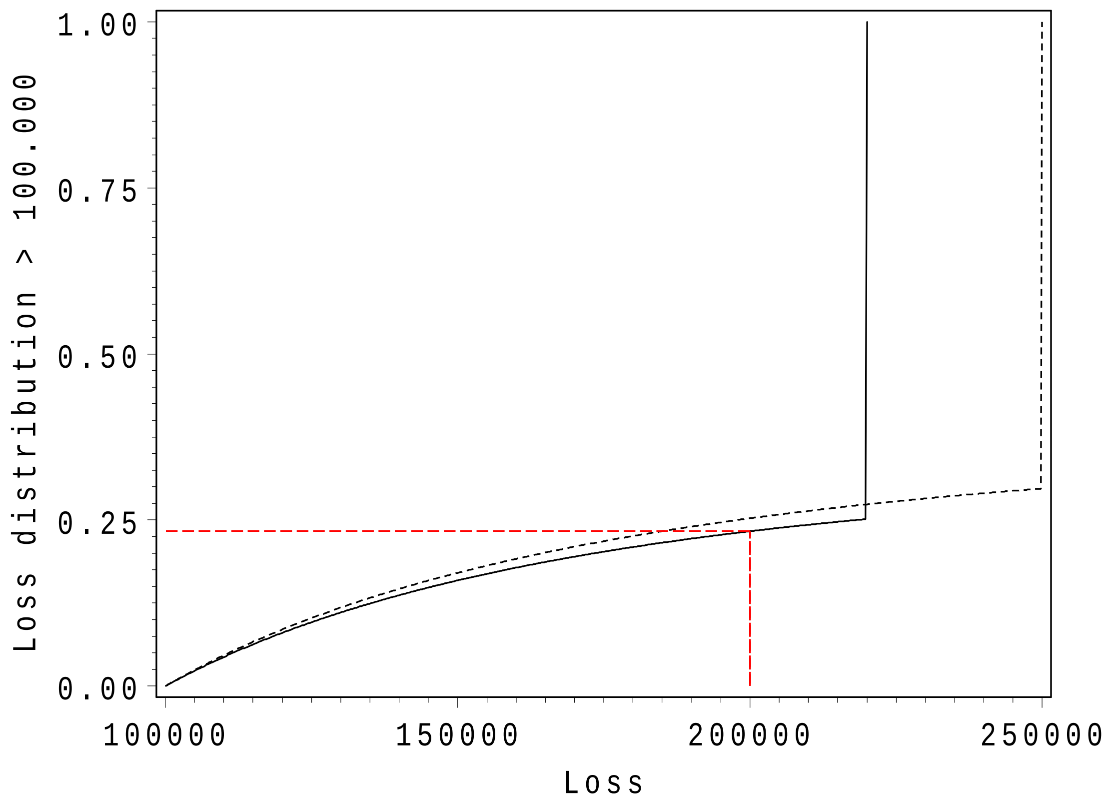

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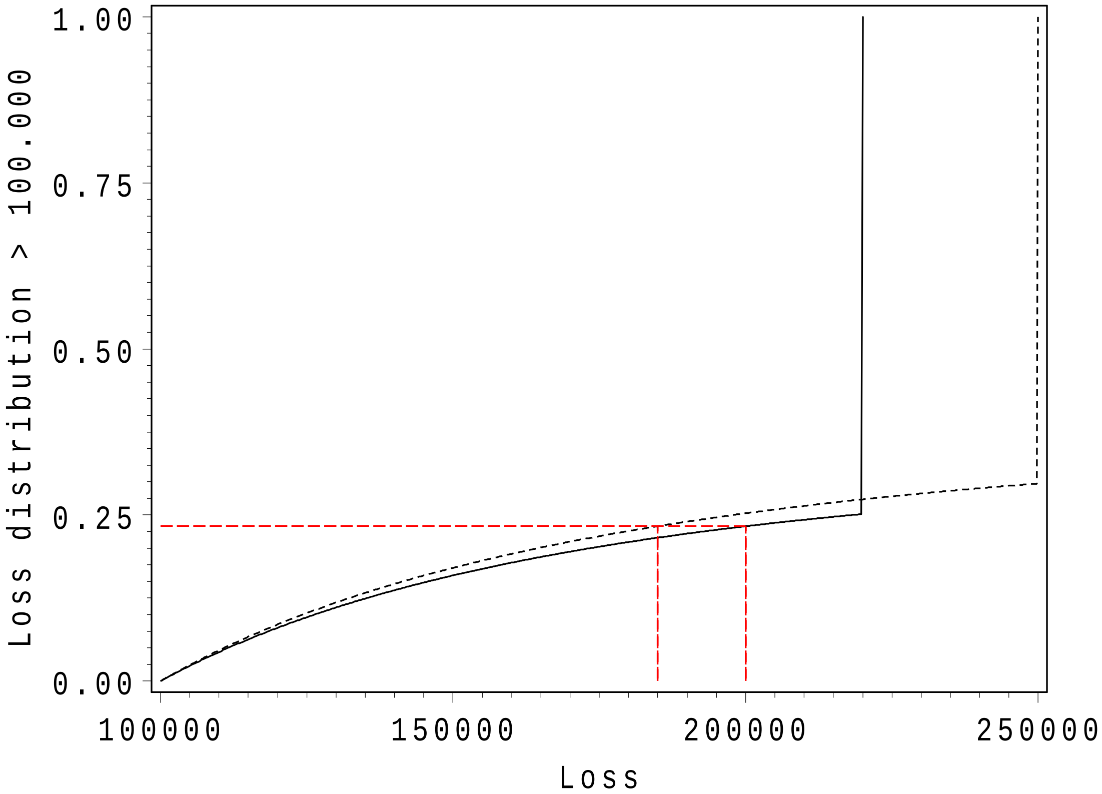

Working and Non-Working Layers

- For working layers, we use combined experience-exposure rating. This means we use:
  - Historically observed indexed claims
  - Indexed profile
  - Detailed portfolio evolutions (taken into account with exposure curves) to estimate:
    - Claims frequency
    - As-if claims
- For non-working layers, we use exposure rating, calibrated on the experience of a working layer

Live demonstration

Appendix : formulas as-if claims

- For year 1, we define for $s \geqslant 0,1M$:

$$\begin{aligned}
F_{S_1|S_1\geqslant 0,1M(s)} &= \mathbb{P}[S_1 \leqslant s | S_1 > 0,1M] \\
&= \frac{\mathbb{P}[S_1 \leqslant s \cap S_1 > 0,1M]}{\mathbb{P}[S_1 > 0,1M]} \\
&= \frac{\mathbb{P}[0,22M.X_1 \leqslant s \cap 0,22M.X_1 > 0,1M]}{q_1(1 - F_{X_1}(5/11))} \\
&= \frac{F_{X_1}(s/0,22M) - F_{X_1}(5/11)}{1 - F_{X_1}(5/11)}
\end{aligned}$$

- Similarly, for year T, we define for $s \geqslant 0,1M$:

$$\begin{aligned}
F_{S_T|S_T\geqslant 0,1M}(s) &= \mathbb{P}[S_T \leqslant s | S_T > 0,1M] \\
&= \frac{F_{X_T}(s/0,25M) - F_{X_T}(2/5)}{1 - F_{X_T}(2/5)}
\end{aligned}$$