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Capital Allocation by Percentile Layer

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Why allocate capital?

- Risk adjusted performance measurement of
  - Operating units
  - Lines of business
  - Policies
  - Underwriters
  - Etc.

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Description of situation and context

- Risk taking financial firm
- Firm has several different business units
- Firm holds capital in order to deal with risk
- Firm incurs cost associated with this capital
- How should firm allocate this cost of capital to its various business units?

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Description of situation and context

- Allocating capital to business units applies to
  - Banks
  - Hedge funds
  - Insurance companies
  - Others

- Our discussion will focus on an insurance company

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Description of situation and context

- Publicly traded insurance company
  - Only covers property business (“short tail”)
- Insurance company must hold capital
  - Required by rating agencies, regulators, investors
- Capital is based upon
  - Value at Risk (VaR) at the 99th percentile
  - Single year time horizon
- Investors require rate of return on capital

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Description of situation and context

- Therefore, the operating environment dictates
  - How much capital to hold
  - How much it costs to hold this capital
- Management can only decide how to allocate this cost
- So we will focus only on <u>allocation</u>
  - Outside of scope:
    - Required capital amount
    - Required rate of return

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A current approach to capital allocation

- Critical feature of any allocation method:
  - Measure risk and capital within a holistic framework
- One notable example: “Co-measures” framework
  - Via Kreps, Mango, others
  - Provides general framework that can be used in several different ways
  - We will focus on one particular approach

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A current approach to capital allocation

- Create simulation model for each “component” to be analyzed
  - Line of business
  - Peril
  - Policy
- Simulate profit and loss results for each component
  - Also tabulate results of total portfolio
- Keep track of components that contribute to each total loss

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A current approach to capital allocation

- One approach to applying this framework:
- Amount of capital = Value at Risk at 99th percentile
- Company holds capital for the 99th percentile loss event
  - Or the “1 in 100 year loss event”
- Allocate cost of capital to simulated “1 in 100 year event”
- Further allocate to perils and sublines that contribute to this loss event

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A current approach to capital allocation

- Assumption:
  - When firm holds VaR(99%) capital, it holds capital for the 99th percentile loss event
    - Or the “1 in 100 year loss scenario”
- Therefore:
  - Allocate cost of capital to lines of business that contribute to the “1 in 100 year loss scenario” or “the tail”.
- Does this make sense?

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Reformulating the meaning of VaR

- What does it mean to hold capital equal to VaR(99%)?
- Current formulation:
  - Holding VaR capital → to hold capital "**for** the 99th percentile loss event"
- Imprecise formulation leads to flawed assumption
  - Holding VaR capital → to hold capital "**only for** the 99th percentile loss event"
    - Therefore, allocate capital only to the 99th percentile loss event

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Reformulating the meaning of VaR

- Problem: is the 99th percentile loss event the <u>only</u> loss that uses capital?
- What about the 98th percentile loss event?
  - 98th percentile loss is still a substantial loss
  - Does it not “deplete” or “consume” capital?
  - Should it not receive an allocation of capital cost?

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Reformulating the meaning of VaR

- What does it mean to hold capital equal to VaR(99%)?
- Proposed new formulation:
  - Holding VaR(99%) capital →
    - To hold capital "<u>even for</u> the 99th percentile loss event"
    - But not "<u>only for</u> the 99th percentile loss event"

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Proposed new approach to capital allocation

- Ramifications of new formulation:
  - Must analyze each <u>percentile layer of capital</u>
  - Hence, “Capital Allocation by Percentile Layer”

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Proposed new approach to capital allocation

- 99th percentile minus 98th percentile
  - Why hold this additional amount of capital?
    - Only used by loss events > 98th percentile loss
- 98th percentile minus 97th percentile
  - Why hold this additional amount of capital?
    - Attributable solely to loss events > 97th percentile
- And so on…

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Proposed new approach to capital allocation

- Almost all “percentile layers of capital” will be used by many different loss events.
- How should we allocate each layer to these loss events?
- Allocate using conditional exceedance probability
  - Probability of loss event / (probability of all loss events > lower bound of layer)
  - Allocate only to loss events that can “use” the layer of capital
  - Loss events that are “more likely” receive greater allocation
  - Allocation % on each layer always sums to 100%

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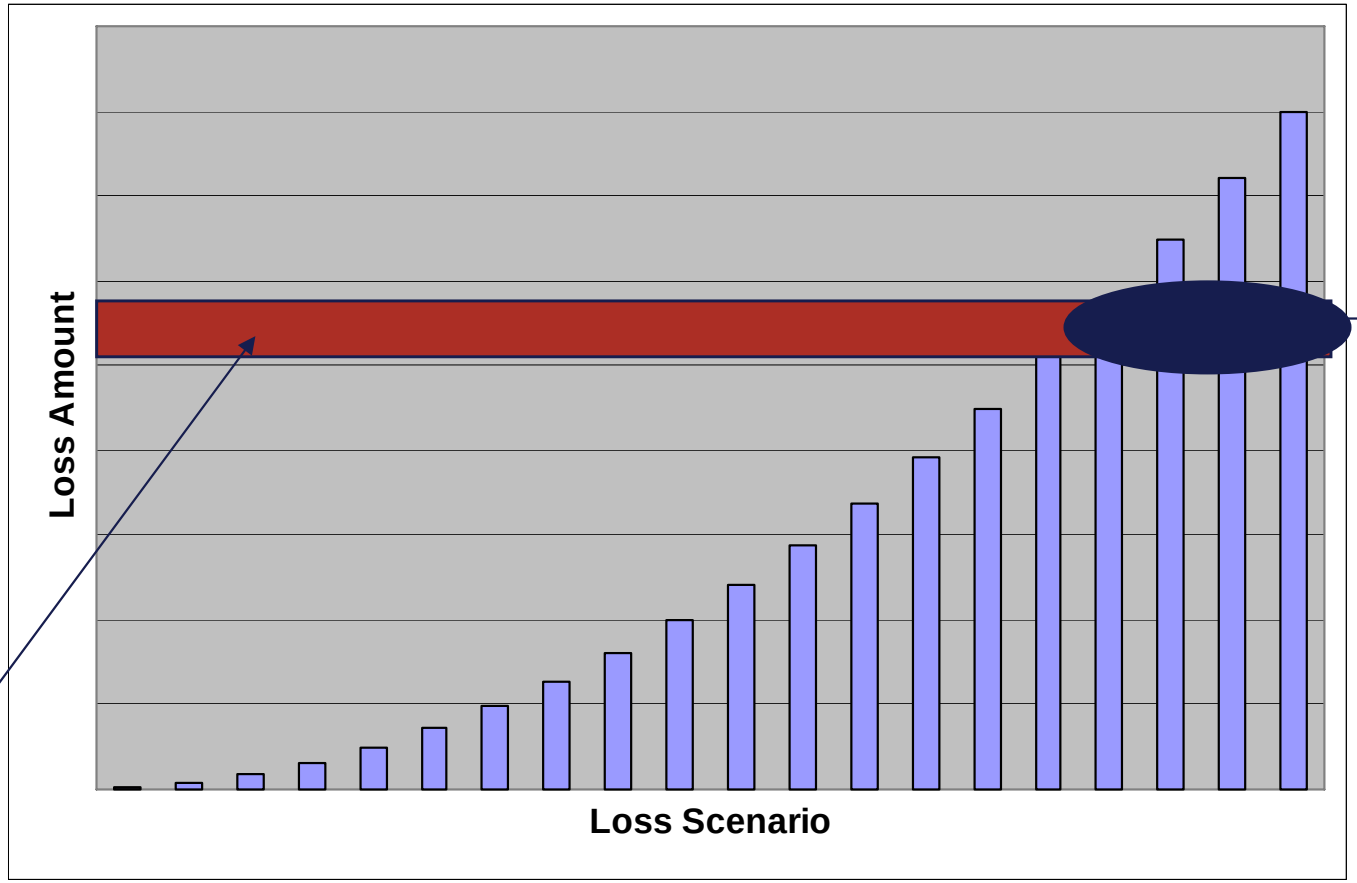
Proposed new approach to capital allocation



Layer of capital

Loss events that exceed the lower bound of the layer of capital

Allocate this layer of capital only to losses that “use” the layer of capital

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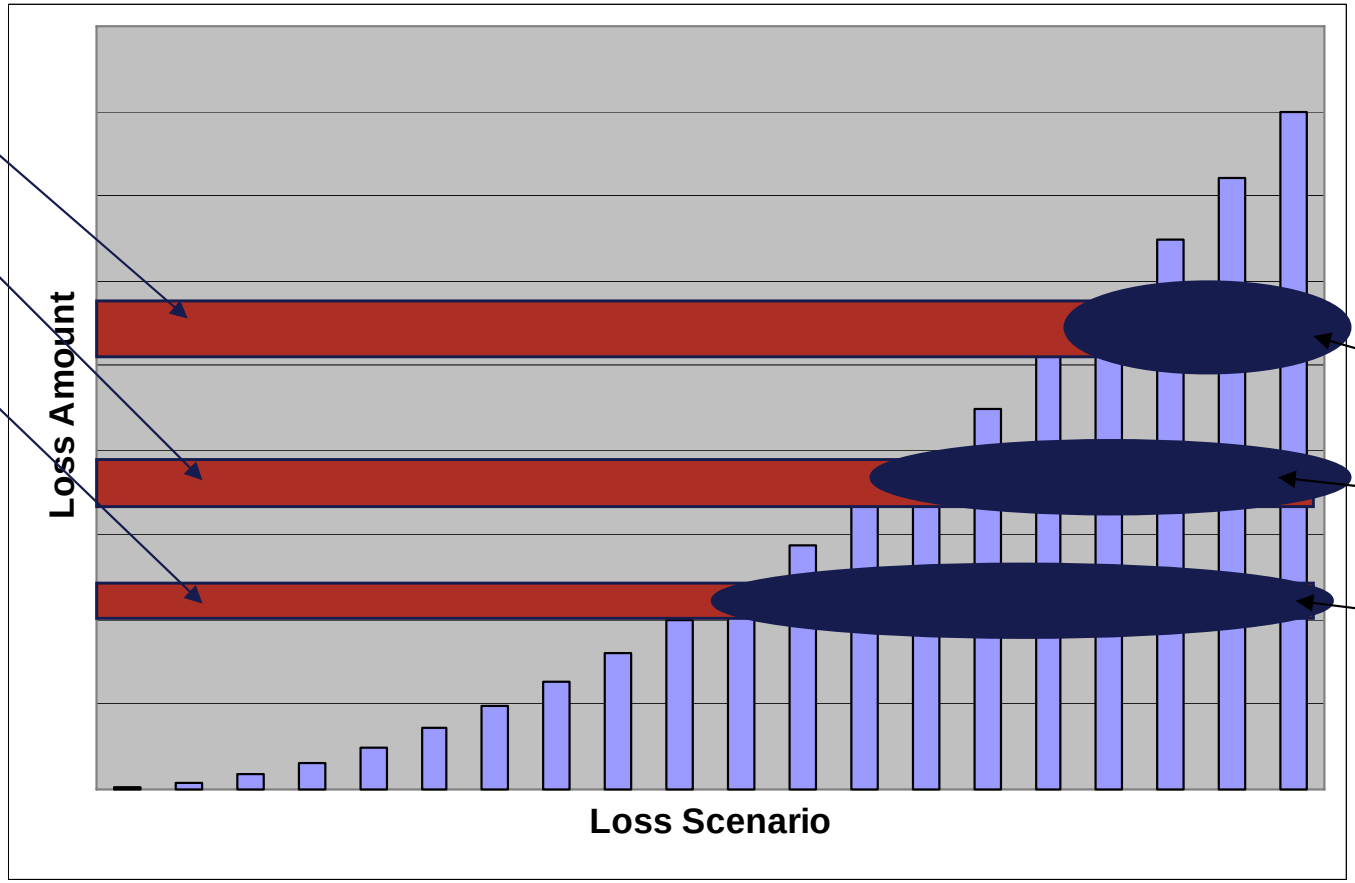
Proposed new approach to capital allocation

Layers of capital



Loss events that exceed the lower bound of each layer

Perform allocation for all layers of capital (up to required VaR capital)

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Proposed new approach to capital allocation

Simplified numerical example: simulation model

- LOB A: “Fire”
  - 25% chance of loss
    - If there is loss, severity is exponential, mean = 4m
- LOB B: “Wind”
  - 5% chance of loss
    - If there is loss, severity is exponential, mean = 20m
- LOB C: “EQ”
  - 1% chance of loss
    - If there is loss, severity is exponential, mean = 100m

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Proposed new approach to capital allocation

Simplified numerical example: allocation results

Allocation Method	LOB A (Fire)	LOB B (Wind)	LOB C (EQ)
coTVaR(99%)	0%	24%	76%
Capital allocation by percentile layer	17%	53%	30%

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Proposed new approach to capital allocation

- We have analyzed discrete layers of capital
- What if we set
  - The number of loss events infinitely large
  - The number of layers of capital infinitely large
  - The “width” of each layer of capital infinitesimally small
- Now we have a continuous case

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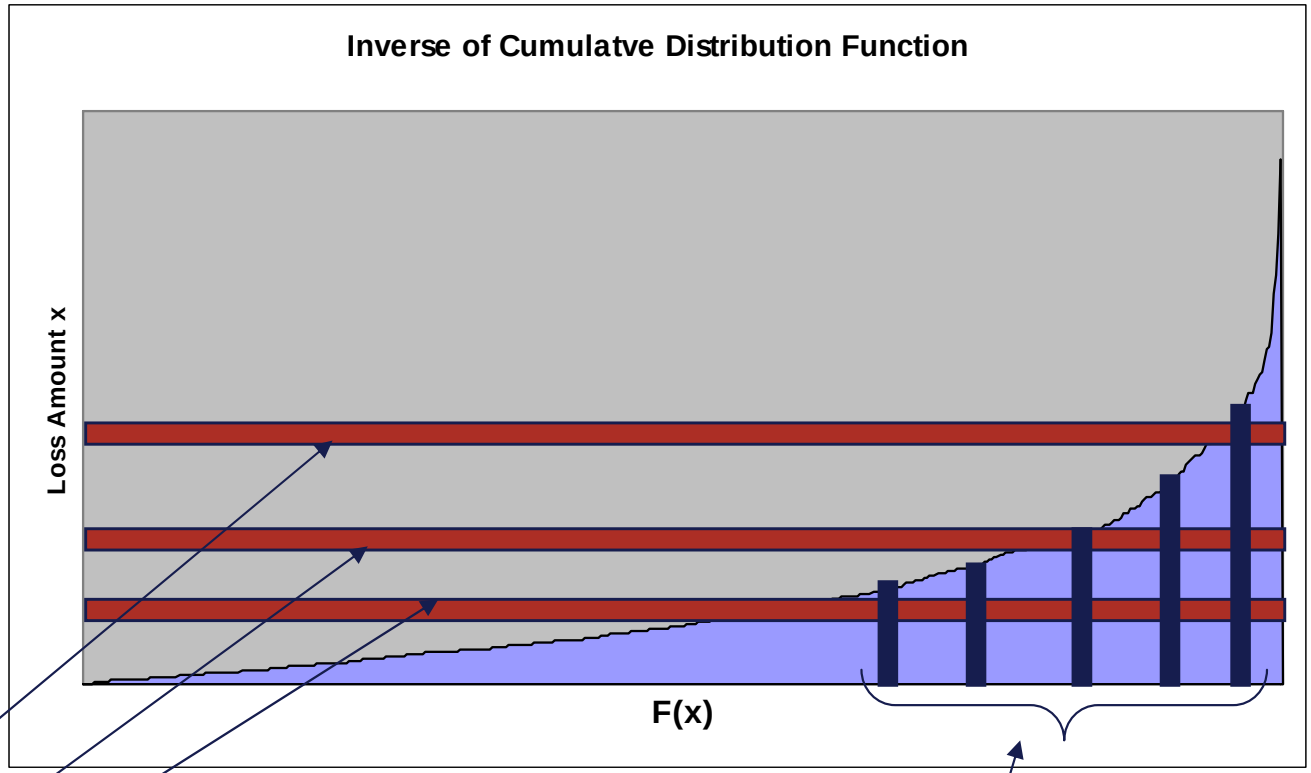
Proposed new approach to capital allocation



Layers of capital

Loss scenarios

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Proposed new approach to capital allocation

- Let x = loss amount
- Let y = capital
- Let layer of capital span (y, y+dy)
- dy = width of layer of capital

- Allocate to each loss event using conditional probability
  - Probability of loss event: $f(x)$
  - Probability of all losses that use the layer of capital: $1 - F(y)$
  - Conditional probability of loss event: $f(x)/(1-F(y))$

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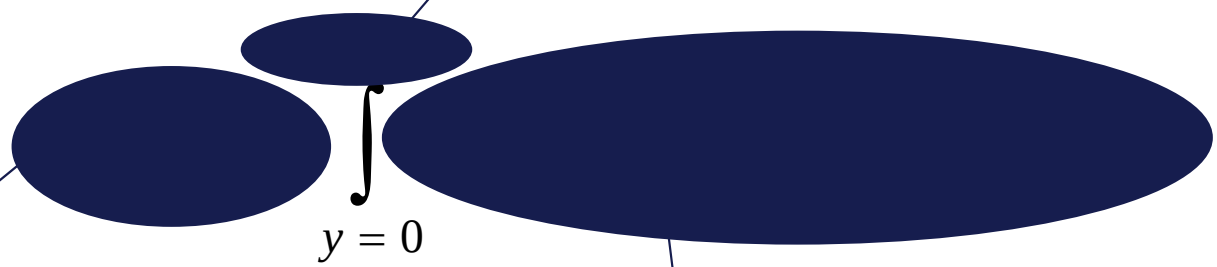
Proposed new approach to capital allocation

- A loss event's allocated capital thus depends upon:

Severity of the loss event (i.e., greater severity → loss event receives allocation across more layers of capital)



Probability (or "frequency") of the loss event

Loss event's inability to share capital burden with other loss events (on each layer of capital). Or, measure of loss event's "dissimilarity" to other loss events

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Summary

- New formulation of the meaning of holding VaR capital
  - Hold capital to pay “even for” 99th percentile loss
  - But not “only for” this loss
- Similar logic extends to TVaR (tail value at risk)

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Summary

- Ramifications for allocating capital
  - Must allocate capital by layer or “percentile layer”
  - Allocate each layer of capital using conditional probability
    - Allocate only to loss events that exceed lower bound of the layer
  - Continuous results show new forms for
    - Risk measure
    - Disutility
    - Risk load
    - Transformed probability

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Summary

- Results of new proposal for capital allocation
  - Allocate capital to all loss events, not just in the tail
  - Smaller loss events below the tail percentile receive <u>some</u> allocation
    - Versus zero via some current methods
  - Largest loss events still receive large allocation
    - But less than “tail based” allocation methods
  - Can alter the profitability of various lines of business

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Comments?

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