

Bornhuetter–Ferguson as a General Principle of Loss Reserving

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Abstract

It is the purpose of this paper to demonstrate that a straightforward extension of the Bornhuetter–Ferguson method provides a general principle comprising various methods of loss reserving which are based on run–off triangles. The most prominent methods underlying the Bornhuetter–Ferguson principle are the chain–ladder method and the more general loss–development method, as well as the additive method and the more general Cape–Cod method.

Besides its force of unifying these and other methods of loss reserving, the Bornhuetter–Ferguson principle can be used also to generate new methods of loss reserving, to evaluate the range of predictors depending on different sources of information, and to compare the portfolio under consideration with the market portfolio.

The present paper summarizes results of Schmidt [2006] and of Schmidt and Zocher [2008]; see also Radtke and Schmidt [2004].

Keywords: Additive method, Bornhuetter–Ferguson principle, Bornhuetter–Ferguson method, Cape–Cod method, chain–ladder method, development pattern, loss–development method, multiplicative model.

1 Introduction

The present paper demonstrates that a straightforward extension of the Bornhuetter–Ferguson method provides a general principle comprising various methods of loss reserving which are based on run–off triangles, such as the chain–ladder method and the more general loss–development method, as well as the additive method and the more general Cape–Cod method.

We start with a description of the structure of the data used in the methods of loss reserving considered here (Section 2) and define the multiplicative model and related development patterns (Section 3). We then introduce the Bornhuetter–Ferguson principle as an extension of the famous Bornhuetter–Ferguson method (Section 4) and show that various well–known methods of loss reserving are indeed versions of the Bornhuetter–Ferguson principle. We also present a numerical example (Section 5) which illustrates one of various possibilities of using the Bornhuetter–Ferguson principle in order

- to compare the reserve estimates obtained by different methods,
- to get an idea of the variability of reserve estimates, and
- to compare the portfolio under consideration with the market portfolio.

We conclude the paper with some final remarks (Section 6).

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2 Loss Development Data

We consider a portfolio of risks and we assume that each claim of the portfolio is settled either in the accident year or in the following n development years. The portfolio may be modelled either by incremental losses or by cumulative losses.

- To model a portfolio by incremental losses, we consider a family of random variables $\{Z_{i,k}\}_{i,k \in \{0,1,\dots,n\}}$ and we interpret the random variable $Z_{i,k}$ as the loss of *accident year* i which is settled with a delay of k years and hence in *development year* k and *calendar year* $i+k$. We refer to $Z_{i,k}$ as the *incremental loss* of accident year i and development year k .
- To model a portfolio by cumulative losses, we consider a family of random variables $\{S_{i,k}\}_{i,k \in \{0,1,\dots,n\}}$ and we interpret the random variable $S_{i,k}$ as the loss of accident year i which is settled with a delay of *at most* k years and hence *not later than* in development year k . We refer to $S_{i,k}$ as the *cumulative loss* of accident year i and development year k , to $S_{i,n-i}$ as a *cumulative loss of the present calendar year* n , and to $S_{i,n}$ as an *ultimate cumulative loss*.

The cumulative losses are obtained from the incremental losses by letting

$$S_{i,k} := \sum_{l=0}^k Z_{i,l}$$

and the incremental losses are obtained from the cumulative losses by letting

$$Z_{i,k} := \begin{cases} S_{i,0} & \text{if } k = 0 \\ S_{i,k} - S_{i,k-1} & \text{else} \end{cases}$$

for all $k \in \{0, 1, \dots, n\}$. In the sequel we shall switch between incremental and cumulative losses as necessary.

We assume that the incremental losses $Z_{i,k}$ are *observable* for calendar years $i+k \leq n$ and that they are *non-observable* for calendar years $i+k \geq n+1$. The observable incremental losses are represented by the following *run-off triangle*:

Accident Year	Development Year								
	0	1	...	k	...	$n-i$...	$n-1$	n
0	$Z_{0,0}$	$Z_{0,1}$...	$Z_{0,k}$...	$Z_{0,n-i}$...	$Z_{0,n-1}$	$Z_{0,n}$
1	$Z_{1,0}$	$Z_{1,1}$...	$Z_{1,k}$...	$Z_{1,n-i}$...	$Z_{1,n-1}$	
⋮	⋮	⋮		⋮		⋮			
i	$Z_{i,0}$	$Z_{i,1}$...	$Z_{i,k}$...	$Z_{i,n-i}$			
⋮	⋮	⋮		⋮		⋮			
$n-k$	$Z_{n-k,0}$	$Z_{n-k,1}$...	$Z_{n-k,k}$					
⋮	⋮	⋮							
$n-1$	$Z_{n-1,0}$	$Z_{n-1,1}$							
n	$Z_{n,0}$								

Accordingly, we assume that the cumulative losses $S_{i,k}$ are observable for calendar years $i+k \leq n$ and that they are non-observable for calendar years $i+k \geq n+1$. The observable cumulative losses are represented by the following run-off triangle:

Accident Year	Development Year								
	0	1	...	k	...	$n-i$...	$n-1$	n
0	$S_{0,0}$	$S_{0,1}$...	$S_{0,k}$...	$S_{0,n-i}$...	$S_{0,n-1}$	$S_{0,n}$
1	$S_{1,0}$	$S_{1,1}$...	$S_{1,k}$...	$S_{1,n-i}$...	$S_{1,n-1}$	
⋮	⋮	⋮		⋮		⋮			
i	$S_{i,0}$	$S_{i,1}$...	$S_{i,k}$...	$S_{i,n-i}$			
⋮	⋮	⋮		⋮		⋮			
$n-k$	$S_{n-k,0}$	$S_{n-k,1}$...	$S_{n-k,k}$					
⋮	⋮	⋮							
$n-1$	$S_{n-1,0}$	$S_{n-1,1}$							
n	$S_{n,0}$								

The problem is to *predict* reserves like

- the *accident year reserves*

$$R_i := \sum_{l=n-i+1}^n Z_{i,l}$$

with $i \in \{1, \dots, n\}$,

- the *calendar year reserves*

$$R^{(n+p)} := \sum_{i=p}^n Z_{i,n-i+p}$$

- with $p \in \{1, \dots, n\}$, or
- the *total reserve*

$$R := \sum_{j=1}^n \sum_{l=n-j+1}^n Z_{j,l}$$

This can be achieved by predicting the non-observable incremental losses $Z_{i,k}$ or by predicting the non-observable cumulative losses $S_{i,k}$, with $i + k \geq n + 1$ in both cases.

3 Multiplicative Model and Development Patterns

The use of run-off triangles in loss reserving is usually justified by the assumption that, except for random effects, the columns of the array of incremental losses are proportional to each other; this is equivalent to the assumption that, except for random effects, the rows of the array of incremental losses are proportional to each other.

In mathematical terms, these assumptions can be formalized by the *multiplicative model* which consists in the assumption that the expected incremental losses satisfy

$$E[Z_{i,k}] = \alpha_i \vartheta_k \quad (1)$$

for all $i, k \in \{0, 1, \dots, n\}$ and some parameters $\alpha_0, \alpha_1, \dots, \alpha_n$ and $\vartheta_0, \vartheta_1, \dots, \vartheta_n$; since this representation of the expected incremental losses is not unique, it is usually assumed that $\sum_{l=0}^n \vartheta_l = 1$. This normalizing assumption yields

$$\alpha_i = E[S_{i,n}]$$

and hence

$$\vartheta_k = E[Z_{i,k}] / E[S_{i,n}]$$

such that α_i is the *expected ultimate loss* of accident year and ϑ_k is something like the *proportion* of losses settled in development year k , which is common to all accident years. The parameters $\vartheta_0, \vartheta_1, \dots, \vartheta_n$ are referred to as a *development pattern for incremental quotas*.

The multiplicative model can also be expressed in terms of cumulative instead of incremental losses, since it is equivalent with the assumption that the expected cumulative losses satisfy

$$E[S_{i,k}] = \alpha_i \gamma_k \quad (2)$$

for all $i, k \in \{0, 1, \dots, n\}$ and some parameters $\alpha_0, \alpha_1, \dots, \alpha_n$ and $\gamma_0, \gamma_1, \dots, \gamma_n$ with $\gamma_n = 1$. Here again we have $\alpha_i = E[S_{i,n}]$, and the parameters $\gamma_0, \gamma_1, \dots, \gamma_n$ are referred to as a *development pattern for (cumulative) quotas*. Obviously, the development patterns for incremental and cumulative quotas are related to each other by the identities

$$\gamma_k = \sum_{l=0}^k \vartheta_l$$

and

$$\vartheta_k = \begin{cases} \gamma_0 & \text{if } k = 0 \\ \gamma_k - \gamma_{k-1} & \text{else} \end{cases}$$

for all $k \in \{0, 1, \dots, n\}$.

Finally, the multiplicative model is also equivalent with the assumption that the expected cumulative losses satisfy

$$E[S_{i,k}] = E[S_{i,k-1}] \varphi_k$$

for all $k \in \{1, \dots, n\}$ and $i \in \{0, 1, \dots, n\}$ and some parameters $\varphi_1, \dots, \varphi_n$. The parameters $\varphi_1, \dots, \varphi_n$ are referred to as a *development pattern for factors*. The development patterns for quotas and factors are related by the identity

$$\gamma_k = \gamma_{k-1} \varphi_k$$

which because of $\gamma_n = 1$ (and with $\prod_{l=n+1}^n (1/\varphi_l) := 1$) yields

$$\gamma_k = \prod_{l=k+1}^n \frac{1}{\varphi_l}$$

for all $k \in \{0, 1, \dots, n\}$.

The assumption of the multiplicative model and hence the equivalent assumption of an underlying development pattern for incremental quotas, (cumulative) quotas or factors can be viewed as a primitive stochastic model which is essential for all methods of loss reserving based of run-off triangles.

We assume henceforth that the assumptions of the multiplicative model are fulfilled.

4 The Bornhuetter–Ferguson Principle

The incremental form (1) of the multiplicative model suggests to predict a future or nonobservable incremental loss $Z_{i,k}$ (with $i + k \geq n + 1$) by a predictor of the form

$$\widehat{Z}_{i,k} := \widehat{\alpha}_i \widehat{\vartheta}_k \quad (3)$$

where $\widehat{\alpha}_i$ is a prior estimator of the expected ultimate loss $\alpha_i = E[S_{i,n}]$ and $\widehat{\vartheta}_k$ is a prior estimator of the parameter $\vartheta_k = E[Z_{i,k}]/E[S_{i,n}]$ of the development pattern of incremental quotas.

Similarly, the cumulative form (2) of the multiplicative model suggests to predict a future or nonobservable cumulative loss $S_{i,k}$ (with $i + k \geq n + 1$) by a predictor of the form

$$\widehat{S}_{i,k} := \widehat{\alpha}_i \widehat{\gamma}_k \quad (4)$$

where $\widehat{\alpha}_i$ is a prior estimator of the expected ultimate loss $\alpha_i = E[S_{i,n}]$ and $\widehat{\gamma}_k$ is a prior estimator of the parameter $\gamma_k = E[S_{i,k}]/E[S_{i,n}]$ of the development pattern of cumulative quotas.

In either case, the term *prior estimator* is not being used in a Bayesian sense but rather to indicate that these estimators present a prior step to the solution of the prediction problem. As will be pointed out later, these prior estimators may be obtained from the run-off triangle and/or from other sources of information like premiums or market statistics.

Since the accident year reserves can be written as $R_i = S_{i,n} - S_{i,n-i}$, they satisfy $E[R_i] = E[S_{i,n}] - E[S_{i,n-i}] = \alpha_i (1 - \gamma_{n-i})$ which suggests to use predictors of the form

$$\widehat{R}_i := \widehat{\alpha}_i (1 - \widehat{\gamma}_{n-i})$$

These predictors are abstract versions of the predictors proposed by Bornhuetter and Ferguson [1972] who considered the special case with

$$\begin{aligned} \widehat{\alpha}_i &:= \pi_i \widehat{\kappa}_i \\ \widehat{\gamma}_{n-i} &:= \widehat{\gamma}_{n-i}^{\text{CL}} \end{aligned}$$

where π_i is the premium income of accident year i , $\widehat{\kappa}_i$ is an estimator of the expected loss ratio $\kappa_i := E[S_{i,n}/\pi_i]$ of accident year i , and the estimator $\widehat{\gamma}_{n-i}^{\text{CL}}$ is obtained from the chain-ladder method and will be defined below.

The idea underlying the construction of the abstract Bornhuetter–Ferguson predictors of accident year reserves can also be used for the construction of predictors of future cumulative losses: Since $S_{i,k} = S_{i,n-i} + (S_{i,k} - S_{i,n-i})$ and $E[S_{i,k} - S_{i,n-i}] = \alpha_i (\gamma_k - \gamma_{n-i})$, we define the (abstract) *Bornhuetter–Ferguson predictors* of future cumulative losses by letting

$$\widehat{S}_{i,k}^{\text{BF}} := S_{i,n-i} + \widehat{\alpha}_i (\widehat{\gamma}_k - \widehat{\gamma}_{n-i}) \quad (5)$$

(with $i + k \geq n + 1$). In a sense, these predictors are better than those given under (4) since they take into account the known current cumulative loss $S_{i,n-i}$. Also, since

$$\begin{aligned} \widehat{S}_{i,n}^{\text{BF}} &= S_{i,n-i} + \widehat{\alpha}_i (1 - \widehat{\gamma}_{n-i}) \\ &= S_{i,n-i} + \widehat{R}_i \end{aligned}$$

the Bornhuetter–Ferguson predictor of an ultimate loss is precisely the sum of the current cumulative loss and the Bornhuetter–Ferguson predictor of the accident year reserve.

In Schmidt [2006] and in Schmidt and Zocher [2008] it has been shown that, for many methods of loss reserving, the resulting predictors of future cumulative losses can be cast into the form (5), with appropriate choices of the prior estimators $\hat{\alpha}_i$ and $\hat{\gamma}_k$. A method for which this is possible will be said to be a *version* of the Bornhuetter–Ferguson principle.

Loss–Development Method. The predictors of the loss–development method are defined as

$$\hat{S}_{i,k}^{\text{LD}} := \hat{\gamma}_k \frac{S_{i,n-i}}{\hat{\gamma}_{n-i}}$$

with arbitrary prior estimators of the development pattern for quotas. Since

$$\hat{S}_{i,k}^{\text{LD}} = S_{i,n-i} + \frac{S_{i,n-i}}{\hat{\gamma}_{n-i}} (\hat{\gamma}_k - \hat{\gamma}_{n-i})$$

we see that the loss–development method is a version of the Bornhuetter–Ferguson principle with $\hat{\alpha}_i := S_{i,n-i}/\hat{\gamma}_{n-i}$.

Chain–Ladder Method. The predictors of the chain–ladder method are defined as

$$\hat{S}_{i,k}^{\text{CL}} := S_{i,n-i} \prod_{l=n-i+1}^k \hat{\varphi}_l^{\text{CL}}$$

where

$$\hat{\varphi}_k^{\text{CL}} := \frac{\sum_{j=0}^{n-k} S_{j,k}}{\sum_{j=0}^{n-k} S_{j,k-1}}$$

is the *chain–ladder factor* of development year k . The conversion of a development pattern for factors into a development pattern for quotas suggests to convert the chain–ladder factors into the *chain–ladder quotas*

$$\hat{\gamma}_k^{\text{CL}} := \prod_{l=k+1}^n \frac{1}{\hat{\varphi}_l^{\text{CL}}}$$

Then the chain–ladder predictors can be written as

$$\hat{S}_{i,k}^{\text{CL}} = \hat{\gamma}_k^{\text{CL}} \frac{S_{i,n-i}}{\hat{\gamma}_{n-i}^{\text{CL}}}$$

which means that the chain–ladder method is a special case of the loss–development method with $\widehat{\gamma}_k := \widehat{\gamma}_k^{\text{CL}}$. Furthermore, as noticed before, the previous identity can be written as

$$\widehat{S}_{i,k}^{\text{CL}} = S_{i,n-i} + \frac{S_{i,n-i}}{\widehat{\gamma}_{n-i}^{\text{CL}}} (\widehat{\gamma}_k^{\text{CL}} - \widehat{\gamma}_{n-i}^{\text{CL}})$$

which means that the chain–ladder method is a version of the Bornhuetter–Ferguson principle with $\widehat{\gamma}_k := \widehat{\gamma}_k^{\text{CL}}$ and $\widehat{\alpha}_i := S_{i,n-i}/\widehat{\gamma}_{n-i}^{\text{CL}}$.

Cape–Cod Method. The predictors of the Cape–Cod method are defined as

$$\widehat{S}_{i,k}^{\text{CC}} := S_{i,n-i} + \pi_i \widehat{\kappa}^{\text{CC}} (\widehat{\gamma}_k - \widehat{\gamma}_{n-i})$$

where π_i is the premium of accident year i and

$$\widehat{\kappa}^{\text{CC}} := \frac{\sum_{j=0}^n S_{j,n-j}}{\sum_{j=0}^n \widehat{\gamma}_{n-j} \pi_j}$$

is the *Cape–Cod loss ratio* which is an estimator of the expected loss ratio $\kappa = E[S_{i,n}/\pi_i]$ (supposed to be common to all accident years). Of course, the Cape–Cod method is a version of the Bornhuetter–Ferguson principle with $\widehat{\alpha}_i := \pi_i \widehat{\kappa}^{\text{CC}}$.

Additive Method. The discussion of the additive method (which is also known as the *incremental loss ratio method*) is slightly more involved:

The additive method is based on the assumption that the expected incremental losses satisfy

$$E[Z_{i,k}] = \pi_i \zeta_k$$

for all $i, k \in \{0, 1, \dots, n\}$, where π_i is the known premium of accident year i and the parameter ζ_k is the unknown *expected incremental loss ratio* of development year k . Letting

$$\begin{aligned} \alpha_i &:= \pi_i \sum_{l=0}^n \zeta_l \\ \gamma_k &:= \frac{\sum_{l=0}^k \zeta_l}{\sum_{l=0}^n \zeta_l} \end{aligned}$$

we obtain

$$E[S_{i,k}] = \alpha_i \gamma_k$$

as well as $\alpha_i = E[S_{i,n}]$ and $\gamma_k = E[S_{i,k}]/E[S_{i,n}]$ for all $i, k \in \{0, 1, \dots, n\}$, such that the assumptions of the multiplicative model are indeed fulfilled.

The predictors of the additive method are defined as

$$\widehat{S}_{i,k}^{\text{AD}} := S_{i,n-i} + \pi_i \sum_{l=n-i+1}^k \widehat{\zeta}_l^{\text{AD}}$$

where

$$\widehat{\zeta}_k^{\text{AD}} := \frac{\sum_{j=0}^{n-k} Z_{j,k}}{\sum_{j=0}^{n-k} \pi_j}$$

is the *additive incremental loss ratio* of development year k . Letting

$$\begin{aligned} \widehat{\alpha}_i^{\text{AD}} &:= \pi_i \sum_{l=0}^n \widehat{\zeta}_l^{\text{AD}} \\ \widehat{\gamma}_k^{\text{AD}} &:= \frac{\sum_{l=0}^k \widehat{\zeta}_l^{\text{AD}}}{\sum_{l=0}^n \widehat{\zeta}_l^{\text{AD}}} \end{aligned}$$

we obtain

$$\widehat{S}_{i,k}^{\text{AD}} := S_{i,n-i} + \widehat{\alpha}_i^{\text{AD}} (\widehat{\gamma}_k^{\text{AD}} - \widehat{\gamma}_{n-i}^{\text{AD}})$$

which means that the additive method is a version of the Bornhuetter–Ferguson principle with $\widehat{\gamma}_k := \widehat{\gamma}_k^{\text{AD}}$ and $\widehat{\alpha}_i := \widehat{\alpha}_i^{\text{AD}}$.

It can also be shown that the additive method is a special case of the Cape–Cod method with $\widehat{\gamma}_k := \widehat{\gamma}_k^{\text{AD}}$; see Schmidt and Zocher [2008].

The following table compares the methods of loss reserving considered before with regard to the choices of the prior estimators $\widehat{\alpha}_i$ of the expected ultimate losses and the prior estimators $\widehat{\gamma}_k$ of the cumulative quotas:

Prior Estimators of Expected Ultimate Losses	Prior Estimators of Cumulative Quotas		
	Arbitrary	$\widehat{\gamma}_k^{\text{CL}}$	$\widehat{\gamma}_k^{\text{AD}}$
Arbitrary	Bornhuetter–Ferguson Principle		
$\widehat{\alpha}_i^{\text{LD}}(\widehat{\gamma})$	Loss–Development Method	Chain–Ladder Method	
$\widehat{\alpha}_i^{\text{CC}}(\widehat{\gamma})$	Cape–Cod Method	Additive Method	

Here and in the sequel we use the abbreviations

$$\begin{aligned} \widehat{\alpha}_i^{\text{LD}}(\widehat{\gamma}) &:= \frac{S_{i,n-i}}{\widehat{\gamma}_{n-i}} \\ \widehat{\alpha}_i^{\text{CC}}(\widehat{\gamma}) &:= \pi_i \frac{\sum_{j=0}^n S_{j,n-j}}{\sum_{j=0}^n \widehat{\gamma}_{n-j} \pi_j} \end{aligned}$$

Of course, the four other possible combinations of prior estimators, which apparently have not been given a name in the literature, could be used as well, and even other choices of the prior estimators of the expected ultimate losses and of the cumulative quotas could be considered; see Schmidt and Zocher [2008].

In conclusion, the Bornhuetter–Ferguson principle unifies known methods and it also creates new methods of loss reserving.

5 Numerical Example

To illustrate the possible use of the Bornhuetter–Ferguson principle, let us now consider a numerical example. Of course, any observations and comments we shall make refer only to the numerical example under consideration, and different data would lead to different observations and conclusions. It is also evident that in actuarial practice a much more refined analysis would be required which, however, could still be performed in the same spirit.

We hope to show that the Bornhuetter–Ferguson principle can be used to select a version which is appropriate for *a given* run–off triangle. By contrast, since the selection process is driven by the data and actuarial judgment, it should be clear that the Bornhuetter–Ferguson principle cannot be used to identify a single version which would be superior *for every* run–off triangle.

Table 2 contains for $n = 5$

- the realization of a run–off triangle of cumulative losses $S_{i,k}$,
 - premiums π_i of the accident years,
 - prior estimators $\hat{\alpha}_i^{\text{external}}$ the expected ultimate losses, and
 - prior estimators $\hat{\gamma}_k^{\text{external}}$ of the cumulative quotas,
- where all $\hat{\alpha}_i^{\text{external}}$ and $\hat{\gamma}_k^{\text{external}}$ are based on external information:

Accident Year i	Development Year k						π_i	$\hat{\alpha}_i^{\text{external}}$
	0	1	2	3	4	5		
0	1001	1855	2423	2988	3335	3483	4000	3520
1	1113	2103	2774	3422	3844		4500	3980
2	1265	2433	3233	3977			5300	4620
3	1490	2873	3880				6000	5660
4	1725	4261					6900	6210
5	1889						8200	6330
$\hat{\gamma}_k^{\text{external}}$	0.2800	0.5300	0.7100	0.8600	0.9500	1.0000		

Table 2

The comparison of the cumulative losses of development years 0 and 1 indicates that the realization of $S_{4,1}$ could be an outlier, maybe due to a single large claim.

Table 3 displays the prior estimates of the cumulative quotas which are used in the different versions of the Bornhuetter–Ferguson principle:

Prior Quotas	Development Year k					
	0	1	2	3	4	5
$\hat{\gamma}_k^{\text{external}}$	0.2800	0.5300	0.7100	0.8600	0.9500	1.0000
$\hat{\gamma}_k^{\text{CL}}$	0.2546	0.5222	0.6939	0.8549	0.9575	1.0000
$\hat{\gamma}_k^{\text{AD}}$	0.2626	0.5430	0.7091	0.8623	0.9600	1.0000

Table 3

Table 4 displays the prior estimates of the expected ultimate losses which are used in the different versions of the Bornhuetter–Ferguson principle;

	Prior Expected Ultimate Losses	Prior Quotas	Accident Year i					
			0	1	2	3	4	5
M11	$\hat{\alpha}_i^{\text{external}}$	$\hat{\gamma}^{\text{external}}$	3520	3980	4620	5660	6210	6330
M12	$\hat{\alpha}_i^{\text{external}}$	$\hat{\gamma}^{\text{CL}}$	3520	3980	4620	5660	6210	6330
M13	$\hat{\alpha}_i^{\text{external}}$	$\hat{\gamma}^{\text{AD}}$	3520	3980	4620	5660	6210	6330
M21	$\hat{\alpha}_i^{\text{LD}}(\hat{\gamma}^{\text{external}})$	$\hat{\gamma}^{\text{external}}$	3483	4046	4624	5465	8040	6746
M22	$\hat{\alpha}_i^{\text{LD}}(\hat{\gamma}^{\text{CL}})$	$\hat{\gamma}^{\text{CL}}$	3483	4015	4652	5592	8160	7420
M23	$\hat{\alpha}_i^{\text{LD}}(\hat{\gamma}^{\text{AD}})$	$\hat{\gamma}^{\text{AD}}$	3483	4004	4612	5472	7848	7195
M31	$\hat{\alpha}_i^{\text{CC}}(\hat{\gamma}^{\text{external}})$	$\hat{\gamma}^{\text{external}}$	3703	4166	4906	5554	6387	7591
M32	$\hat{\alpha}_i^{\text{CC}}(\hat{\gamma}^{\text{CL}})$	$\hat{\gamma}^{\text{CL}}$	3760	4230	4982	5641	6487	7709
M33	$\hat{\alpha}_i^{\text{CC}}(\hat{\gamma}^{\text{AD}})$	$\hat{\gamma}^{\text{AD}}$	3703	4166	4907	5555	6388	7591

Table 4

Due to the outlier in accident year 4 and development year 1, the prior estimates of the expected ultimate losses of accident year 4 obtained by the loss development method are extremely high.

Table 5 displays the first–year reserves and the total reserves which are obtained from the data by applying different versions of the Bornhuetter–Ferguson principle:

	Prior Expected Ultimate Losses	Prior Quotas	First–Year Reserve	Total Reserve
M11	$\hat{\alpha}^{\text{external}}$	$\hat{\gamma}^{\text{external}}$	4164	9964
M12	$\hat{\alpha}^{\text{external}}$	$\hat{\gamma}^{\text{CL}}$	4315	10258
M22	$\hat{\alpha}^{\text{external}}$	$\hat{\gamma}^{\text{AD}}$	4284	9948
M21	$\hat{\alpha}^{\text{LD}}(\hat{\gamma}^{\text{external}})$	$\hat{\gamma}^{\text{external}}$	4572	11071
M22	$\hat{\alpha}^{\text{LD}}(\hat{\gamma}^{\text{CL}})$	$\hat{\gamma}^{\text{CL}}$	4935	11987
M23	$\hat{\alpha}^{\text{LD}}(\hat{\gamma}^{\text{AD}})$	$\hat{\gamma}^{\text{AD}}$	4770	11279
M31	$\hat{\alpha}^{\text{CC}}(\hat{\gamma}^{\text{external}})$	$\hat{\gamma}^{\text{external}}$	4530	10973
M32	$\hat{\alpha}^{\text{CC}}(\hat{\gamma}^{\text{CL}})$	$\hat{\gamma}^{\text{CL}}$	4776	11475
M33	$\hat{\alpha}^{\text{CC}}(\hat{\gamma}^{\text{AD}})$	$\hat{\gamma}^{\text{AD}}$	4687	10976
Minimum			4164	9948
Maximum			4935	11987

Table 5

The pairs of reserves presented in Table 5 are plotted in Figure 1:

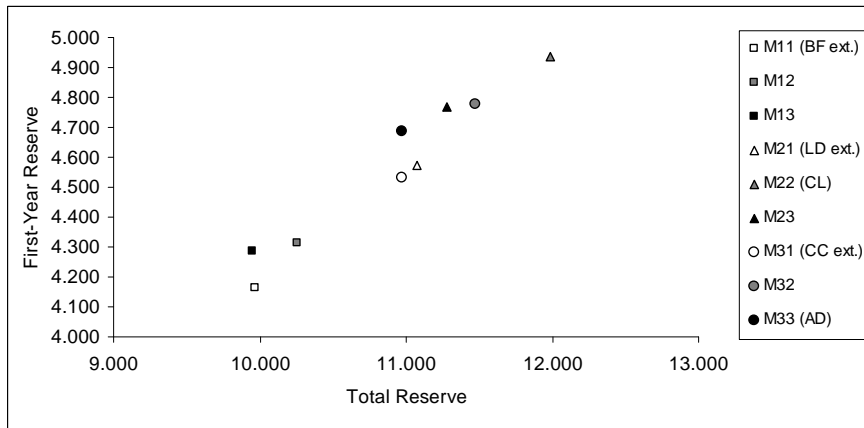


Figure 1

Figure 1 shows that there is a strong positive correlation between the first-year reserves and the total reserves. Moreover, we make the following observations:

- Both reserves are low for the versions M11, M12, M13 using the external prior estimators of the expected ultimate losses.
- Both reserves are relatively low for the versions M11, M21, M31 using the external prior estimators of the cumulative quotas.
- Both reserves are relatively high for the versions M12, M22, M32 using the chain-ladder quotas; this is due to the outlier in accident year 4 and development year 1.
- Both reserves are high for the version M22 (chain-ladder method).

Moreover, there is a high volatility between the pairs of reserves produced by the different versions of the Bornhuetter–Ferguson principle.

When combined with actuarial judgement, the previous observations may be used to select predictors providing reliable reserves:

- If the data of the run-off triangle are judged to be highly reliable, then the low predictors which are based of the external prior estimators of the expected ultimate losses and/or the quotas could be discarded.
- The remaining predictors provide ranges for the first-year reserve and for the total reserve which are not too large.

The remaining pairs of reserves could be judged as being reliable and are plotted in Figure 2:

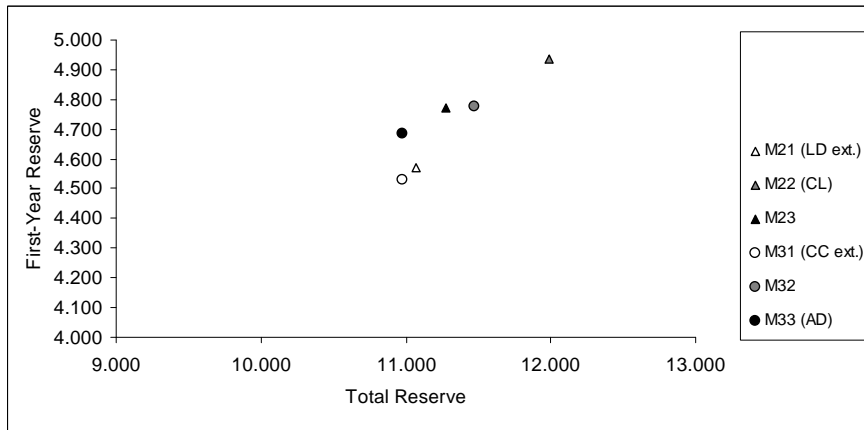


Figure 2

Figure 2 shows that the reliable pairs of reserves yield a rather small range for the first-year reserves and a slightly larger range for the total reserves.

Once the reliable reserves are determined, the final problem is to select predictors which can be regarded as best predictors of the ultimate losses. For example, if particularly prudent reserves are required, then one might select the predictors of version M32 (Cape-Cod method with chain-ladder quotas) which are plotted in Figure 3:

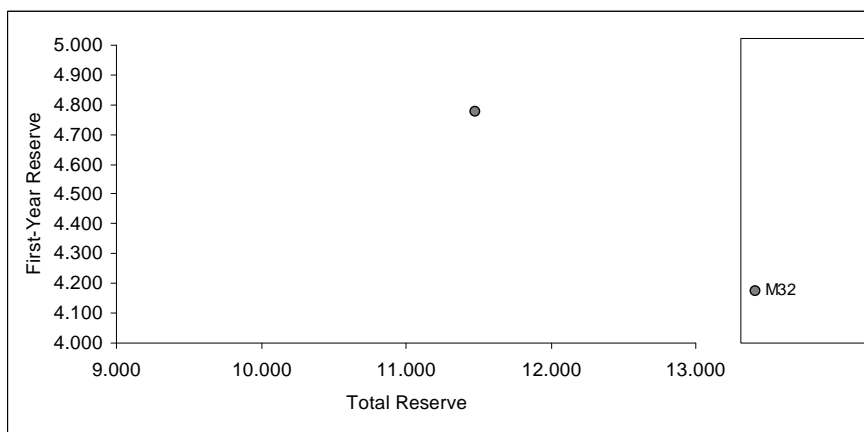


Figure 3

Of course, actuarial judgement could also lead to the selection of another version of the Bornhuetter–Ferguson principle among those which produce reliable reserves.

6 Remarks

The selection of a particular version of the Bornhuetter–Ferguson principle provides predictors which can be regarded as best predictors of the ultimate losses. However,

the rules of accounting tend to require not only best predictors but also ranges which reflect the uncertainty of the best predictors.

The Bornhuetter–Ferguson principle provides an approximate answer to this requirement: Since the different versions of the Bornhuetter–Ferguson principle generate a variety of reserves, they can be used to determine reliable ranges for the ultimate losses. These ranges are, of course, non–probabilistic ones; instead, they reflect the uncertainty caused by the different sources of information used in the different versions of the Bornhuetter–Ferguson principle.

Beyond the selection of best predictors and ranges, the Bornhuetter–Ferguson principle may also be used to analyze the run–off triangle and hence the portfolio under consideration. We only mention two rather obvious aspects of such an analysis:

- In the case where the predictors based on external prior estimators obtained from a market portfolio differ strongly from the other predictors, the structure of the portfolio under consideration is likely to differ from the market portfolio.
- In the case where the predictors based on premiums differ strongly from the other predictors, there might be something wrong with rate–making.

The plot presented in Figure 1 is just one among various possibilities of analyzing the effects of the different versions of the Bornhuetter–Ferguson principle. Other two–dimensional plots could be designed for representing certain pairs of predictors produced by the different versions of the Bornhuetter–Ferguson principle and could be used for selecting best predictors and ranges or for analyzing the run–off triangle.

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