MODELLING AGGREGATE NON-LIFE UNDERWRITING RISK: STANDARD FORMULA VS INTERNAL MODEL

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Abstract
The main target of this paper is to analyse the risk profile of a multi-line non-life insurer. A risk theoretical simulation model is then applied with the aim to predict the risk capital regarding only premium risk. A systematic comparison has been performed between Risk Based Capital obtained by the application of an Internal Model and the corresponding Solvency Capital Requirement as provided by the Solvency II-QIS3 standard formula for different insurers according to dimension and risk distribution. Finally the paper discusses the dependence problem: losses from different line of business are linked by different approaches. At this regard the dependence effect on RBC is examined comparing the QIS3 aggregation formula (using correlation matrix) with Internal Model results. Furthermore different results are obtained applying either elliptical copula functions and approximation formula based on linear correlation

Keywords: Aggregation and dependency in non-life insurance, Premium Risk, Internal Model, Solvency II-QIS3 Standard Formula

1. Introduction

As well known, Solvency II project has articulated a capital requirement framework involving three pillars. The development of the project represents one of the major area of actuarial priorities. CEIOPS work programme and Quantitative Impact Studies lead towards the definition of a final standardized approach for risk capital valuation.
The various sources of risk and methods of determining the factors required for them would be defined with the aim to determine a minimum value of capital strictly correlated to insurers risk profile. For stronger, more technically able companies with effective risk management programs, it may be more appropriate to introduce alternative methods for determining the capital required with respect to specific risk.
Each company should construct its own model: the model must reflect the specific risk profile of a company and must be validated by the supervisor. An appropriate risk management analysis could permit to estimate the actual probability of ruin or appropriate alternative risk measures for a short-term horizon and to analyse the distribution of the return on equity.
In this paper the structure of a classical risk theoretical model is adopted in order to describe the risk profile of a general multi-line insurer. This internal model, well known in actuarial literature[1], follows a risk theoretical approach where premium risk is exclusively dealt with. Model parameters have been calibrated for a specific insurer: it will permit a full comparison between Risk Based Capital obtained by the application of an Internal Model and the corresponding Solvency Capital Requirement (SCR) computed as specified in the QIS3 technical document[2].
Furthermore the paper discusses the dependence problem: losses from different line of business are linked by different approaches. At this regard the dependence effect on Risk Based Capital is examined comparing the QIS3 aggregation formula with Internal Model results obtained applying elliptical copula function and using approximation formula.
For these analyses four different non-life insurance companies are regarded, all of them having different dimension and/or different claim size coefficient of variability (CV).
Internal Model and QIS3 Standard Formula are then applied to these companies with the aim to compare the impact of different parameters. The results emphasize that Internal Model results give a reduced risk capital compared to the QIS3 standard formula for large insurers whereas small insurers should prefer the use of the Standard Formula because, mainly for the lack of a size factor in the QIS3-Standard Formula, a reduced required capital is obtained.

2. QIS3 Standard Formula for Premium Risk

The Solvency II project established by EU since 2001, and still in the due course[3], has structured the insurance solvency supervising according a three-pillars approach as Basle II. The publication of the draft framework directive, confirming this approach, has outlined the basic building blocks for Solvency II.
In Pillar 1, companies have required to hold eligible own funds covering the Solvency Capital Requirement (SCR), which shall be calculated by an appropriate Standard Formula.
This capital requirements, for which it is still under review a *Standard Formula*, should take into account all (measurable) source of risk in order to determine an amount to be sufficient to cover the risk on a time horizon of 1 year for a very high degree of probability (namely 99.5%).
The above mentioned “standard formula” is under review by quantitative impact studies (QIS), carried on the whole EU insurance market on a voluntary basis by solo/group insurers.

[1] For similar applications see, for instance, Daykin et al. (1994) Havning & Savelli (2005) and Ballotta & Savelli (2006)
[2] See QIS3 Technical Specification (2007)
[3] The new Solvency II regime is expected to come in force since 2012

Pillar 2 gives possibilities to consider utilizing the option to base the Solvency Capital Requirement, partially or fully, on the results produced by Internal Model.
Articles 109 to 124 of the Insurance Directive Draft, describe the requirement applying to insurance and reinsurance undertakings using or wishing to use a full or partial Internal Model in the calculations of the SCR.
This model should be approved by the supervisory authorities demonstrating that they meet the use test, statistical quality standards, calibration standards, validation standards, and documentation standards.
Finally, Pillar 3 will introduce some Disclosure Requirement that all companies should give to market and insurers.

We focus here on the QIS3 Standard Formula[4] for the valuation of a particular subcategory of the underwriting risk non – life: the premium risk[5].
QIS3 has introduced a unique module for the joined valuation of both risks related to future claims arising during and after the period until the time horizon for the solvency assessment (Premium Risk) and the risk relate to a non-sufficient amount of the technical provisions (Reserve Risk).
The Premium Risk could be estimated according two different approaches, the first based on a *market-wide approach* and the second one taking into account the specific technical data of the company by the loss ratios[6] (*undertaking-specific approach*) :

(1) $$NL_{pre}^{mark} = \rho(\sigma_M) \cdot P \qquad\qquad NL_{pre}^{utk} = \rho(\sigma_U) \cdot P$$

being P the next year gross premium volume (net of reinsurance) and the function ρ(X)[7] as estimate of the 99,5% VaR of a probability distribution with standard deviation X :

(2) $$\rho(x) = \frac{\exp\left(N_{0,995}\sqrt{\log(x^2+1)}\right)}{\sqrt{(x^2+1)}} - 1$$

being $N_{0.995}$ the 99,5% quantile of the standard normal distribution.
In both approaches the overall volatility (σ) is obtained as follows

(3) $$\sigma = \sqrt{\frac{1}{V^2}\sum_{r,c} CorrLob^{rxc} \cdot V_r \cdot V_c \cdot \alpha_r \cdot \alpha_c}$$

where CorrLob is the correlation matrix between different lines of business (LoB), for single premium and reserve risk (given as an input by CEIOPS),V representing the volume measure and α the single-LoB volatility.
This formula provides a correlation between volatility for premium and reserve risk of different LoBs using the correlation coefficient of premium reduced of 50%.
The differences between market-wide and undertaking-specific approach are noticeable in the single-LoB volatility valuation. Market Approach is based on a market-wide estimate of the standard-deviation for premium risk, determined on a specific volatility factor given as input by CEIOPS. For instance, as to the main non-life lines, these factors are given in the next Figure 1:

[4] For the complete formula see QIS3 Technical Specifications
[5] QIS3 has introduced for the category underwriting risk non-life only two sub-modules: Premium & Reserve risk and Cat risk.
[6] QIS3 has based undertaking-specific approach on Loss ratios. CEIOPS says, in technical specifications, that Loss ratios (rather than combined ratios, as in QIS2) are used since these provide a more objective basis for the measurement of volatility, and since this lessens the burden on undertakings with respect to data collection.
[7] Roughly, ρ(σ) ≈ 3 • σ

Figure 1: Volatility factor (QIS3 – Premium Risk)

Premium Risk (QIS3 Standard Formula)	Accident	Mot. Damages	Property	MTPL	GTPL
Volatility Factor	5%	10%	10%	10%	10%

The undertaking-specific estimate of the standard deviation for premium risk is determined on the basis of the volatility of historic loss ratios. In this second approach the volatility for premium risk in the individual LoB is derived as a credibility mix of the undertaking-specific estimate and of the market-wide estimate as follows:

(4) $$\sigma_{U,lob} = \sqrt{c_{lob}\sigma^2_{cr,lob} + (1-c_{lob})\sigma^2_{M,lob}} \ .$$

The credibility factor is greater than zero only when company has available at least seven loss ratios. However it always depends on number of loss ratios:

(5) $$\begin{cases} c_{lob} = \dfrac{n_{lob}}{n_{lob}+4} & if \ \ n_{lob} \geq 7 \\ c_{lob} = 0 & otherwise \end{cases}$$

The credibility formula shows that the volatility will never be determined only using the standard deviation of loss ratio[8].

3. The Collective Risk Model: structure and parameters

The framework of the model provides a risk theoretical approach where only the premium risk is mainly dealt with and whereas the run-off risk arising from loss reserves is not considered as well as the reinsurance covers.

A Collective Risk Simulation Model[9] is here applied with the aim to quantify the capital required for premium risk for a multi-line insurer.

Following the collective approach, the aggregate claims amount $\tilde{X}_t$ is given by a compound process, where:

- the number of claims distribution is the Poisson law, and, having assumed a dynamic portfolio[10], the Poisson parameter will be increasing (or decreasing) recursively year by year by the real growth rate *g*. A structure variable $\tilde{q}$ will be also introduced to represent short-term fluctuations, where $\tilde{q}$ is supposed Gamma distributed with equal parameters. It means that the number of claims $\tilde{k}_t$ are Negative Binomial distributed with mean $n_t = n_0 \cdot (1+g)^t$
- the claim size amounts, denoted by $\tilde{Z}_{i,t}$, are assumed to be i.i.d. random variables with a LogNormal distribution (with mean m_0) and to be scaled by only the claim inflation rate *i* in each year.

[8] If insurer has all 15 loss ratios requested by CEIOPS, c_{lob} will be almost 79%.

[9] For stochastic Risk Reserve see Daykin, Pentikainen, Pesonen (1994). For exact moments of aggregate claims see Havning & Savelli (2005)

[10] For each single line of business both the nominal gross premium volume $B_{t,lob}$ and the risk premium $P_{t,lob}$ increase yearly by either the claim inflation rate(i) and the real growth rate (g)

For these analyses four different non-life insurance companies are regarded (their figures are summed up in Figure 2), all of them having different dimension and/or different claim size coefficient of variability (CV). Furthermore all insurers underwrite business in the same 5 lines of business (Accident, Motor Damages, Property, Motor Third-Party Liability and General Third Party Liability) with the same weight on the gross written premiums volume (rather similar to the actual proportion in the Italian insurance market):

- LoB 1: Accident: 10%
- LoB 2: Motor Damages : 10%
- LoB 3: Property: 15%.
- LoB4 : MTPL: 55%
- LOB5: GTPL: 10%

Consequently, the examined companies have the following initial total gross premium volume (without regarding the increase by approximately 5% in the forthcoming year, relevant for our risk capital evaluation):

- Company **OMEGA**: 1.000 millions of Euros
- Company **TAU**: 500 millions of Euros
- Company **TAU HIGH**: 500 millions of Euros
- Company **EPSILON**: 100 millions of Euros.

As we can see from Figure 2 companies TAU and TAU HIGH have the same volume of premiums (50% of Company OMEGA) but they differ for the claim size CV c_Z (standard deviation/mean), higher than 50% for the Company TAU HIGH. Finally, Company EPSILON has the same parameters of insurers OMEGA and TAU but it has a largely minor dimension (1/10 of OMEGA):

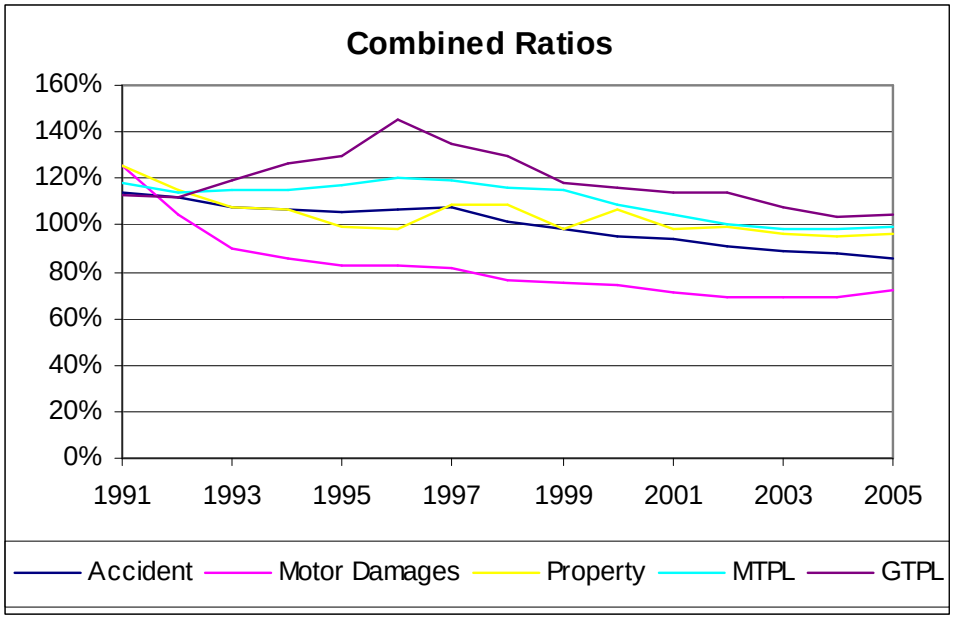
Figure 2: Parameters for premium and claims

	LoBs	n_0	σ(q)	g	m_0	c_z	i	λ	exp
OMEGA	LoB1	**17.374**	14,0%	1,9%	3.200	3	3%	22,40%	31,95%
	LoB2	**18.515**	28,9%	1,9%	2.500	2	3%	64,25%	23,98%
	LoB3	**16.580**	11,2%	1,9%	6.000	8	3%	6,28%	29,51%
	LoB4	**111.316**	8,7%	1,9%	4.000	4	3%	1,88%	17,52%
	LoB5	**7.721**	13,9%	1,9%	10.000	12	3%	-7,03%	28,22%
TAU	LoB1	**8.687**	14,0%	1,9%	3.200	3	3%	22,40%	31,95%
	LoB2	**9.258**	28,9%	1,9%	2.500	2	3%	64,25%	23,98%
	LoB3	**8.290**	11,2%	1,9%	6.000	8	3%	6,28%	29,51%
	LoB4	**55.658**	8,7%	1,9%	4.000	4	3%	1,88%	17,52%
	LoB5	**3.861**	13,9%	1,9%	10.000	12	3%	-7,03%	28,22%
TAUHIGH	LoB1	8.687	14,0%	1,9%	3.200	**4,5**	3%	22,40%	31,95%
	LoB2	9.258	28,9%	1,9%	2.500	**3**	3%	64,25%	23,98%
	LoB3	8.290	11,2%	1,9%	6.000	**12**	3%	6,28%	29,51%
	LoB4	55.658	8,7%	1,9%	4.000	**6**	3%	1,88%	17,52%
	LoB5	3.861	13,9%	1,9%	10.000	**18**	3%	-7,03%	28,22%
EPSILON	LoB1	**1.737**	14,0%	1,9%	3.200	3	3%	22,40%	31,95%
	LoB2	**1.852**	28,9%	1,9%	2.500	2	3%	64,25%	23,98%
	LoB3	**1.658**	11,2%	1,9%	6.000	8	3%	6,28%	29,51%
	LoB4	**11.132**	8,7%	1,9%	4.000	4	3%	1,88%	17,52%
	LoB5	**773**	13,9%	1,9%	10.000	12	3%	-7,03%	28,22%

It is to be pointed out that some crucial parameters as safety loading coefficient (λ), standard deviation of structure variable ($\sigma_{\tilde{q}}$) and expenses coefficient (*exp*) are obtained mainly by Italian market Loss Ratios and Combined Ratios (see Figure 3). This will permit a full comparison

between Internal Model results and SCR determined with undertaking-specific QIS3 Standard Formula:

Figure 3: Italian Market Combined Ratios (Source: ANIA)



Take note that the high figure shown by the safety loading for LoB Motor Damages (approx. 64,25%), will give rise to a low capital requirement for that single LoB because of high expected technical profits, taken into account in the Internal Model only because expected profits are not allowed in the QIS3 Standard Formula.
The claim size CV (c_Z) is fixed, for each LoB, on the basis of practical Italian market data. Moreover, the expected number of claims (n_0) and the expected claim cost (m_0) reported in Figure 2 for each LoB are referred to the initial year 0 and they will increase in the examined year 1 (because of a time span of 1 year is regarded), as described in previous section for the dynamic portfolio, according the annual rate of real growth of portfolio (g) as to number of claims and the annual claim inflation rate (i) as to claim size, assumed to be almost 2% and 3% respectively for all LoBs in the next simulations.
In this way the gross premium volume of year 1 will be increased for each LoB by approximately 5% and then the underwriting premium risk of year 1 is consequently estimated, with the risk measures amount compared with the initial (known) amount of gross premiums of the company.
On the basis of our simulations of the claim amount X for each LoB, it is estimated the required percentile at confidence level α and the Required Capital for the single i-th LoB is computed as:

(6) $$SCR_i^{\alpha} = CC_i^{\alpha} - \lambda_i P_i$$ [11]

where CC_i is the capital charge, obtained without safety loading:

(7) $$CC_i^{\alpha} = VaR_i^{\alpha} - P_i \ .$$

In case of independence among the claim amount of all the lines, the total aggregate amount of claims will be clearly the sum of single LoB claim amount X_i with an aggregate RBC amount

[11] All SCR is determined at the generic confidence level α. We'll cut out this indication in the next.

($SCR^{Agg,IM}$) minor than the sum of single RBC amounts of the single LoBs because of a percentile of the aggregate minor than the sum of the LoBs percentiles for the independence assumption:

(8) $$SCR^{Agg,IM} = VaR - \sum_{i=1}^{L} P_i (1+\lambda_i)$$

On the other hand, in case of full dependence among all LoBs the aggregate RBC ($SCR^{Agg,Full}$) is derived as the sum of the individual RBC.
In case of linear correlation among the claims amount of all the lines, the total aggregate RBC ($SCR^{Agg,Matr}$) is determined using an appropriate approximation formula as follows:

(9) $$SCR^{Agg,Matr} = SCR^{Agg,IM} + \frac{\left(SCR - SCR^{IND}\right)}{\left(SCR^{Full\ Corr} - SCR^{IND}\right)}\left(SCR^{Agg,Full} - SCR^{Agg,IM}\right)$$

where SCR is estimated joining the single capital charge with a correlation matrix

(10) $$SCR = \sqrt{\sum_{i=1}^{L}\sum_{j=1}^{L} Corr_{i,j} \cdot CC_i \cdot CC_j} - \sum_{i=1}^{L} \lambda_i P_i$$

and SCR^{IND} e $SCR^{Full\ Corr}$ are respectively given by (derived in either independence and full correlation hypotheses):

(11) $$SCR^{IND} = \sqrt{\sum_{i=1}^{L}\left[CC_i\right]^2} - \sum_{i=1}^{L} \lambda_i P_i$$

(12) $$SCR^{FullCorr} = \sum_{i=1}^{L}\left[CC_i\right] - \sum_{i=1}^{L} \lambda_i P_i = \sum_{i=1}^{L}\left[CC_i - \lambda_i P_i\right] = \sum_{i=1}^{L} SCR_i = SCR^{AGG,Full}$$

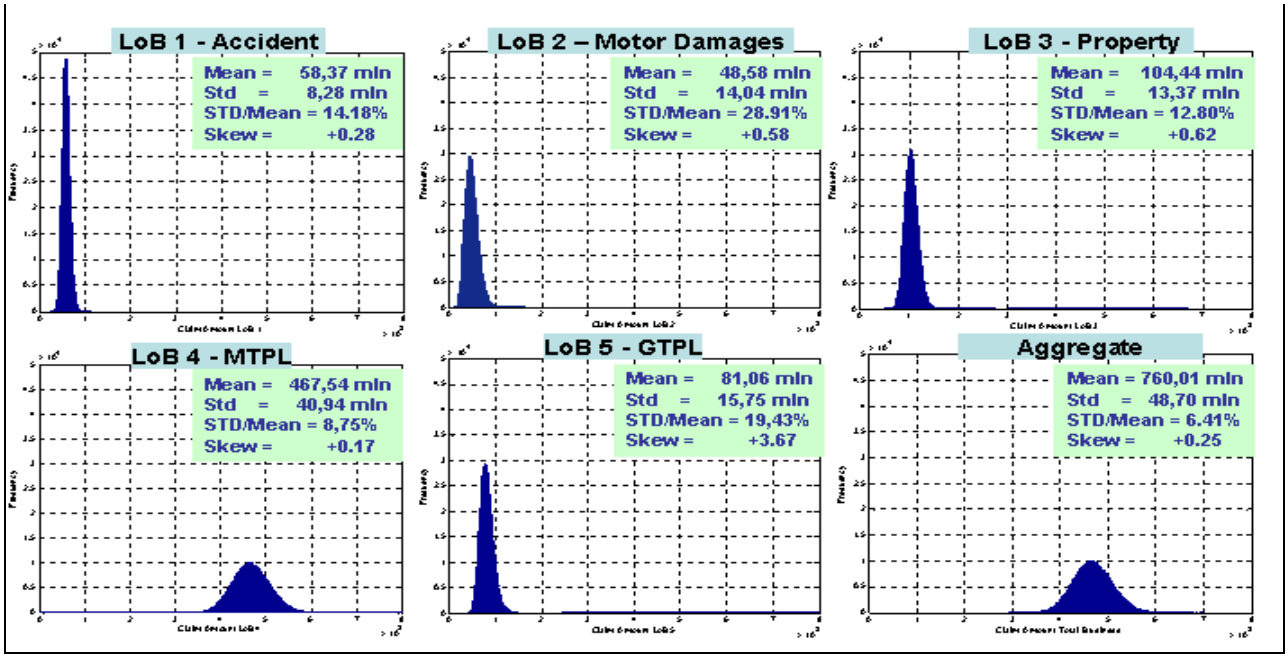
The Capital Requirement, under linear correlation assumption (see Formula (9)), is derived rescaling the RBC obtained from Internal Model in case of independence ($SCR^{Agg,IM}$). This appear necessary because SCR^{IND} gives an approximate estimation of diversification effect between different LoB. For further details, Figure 15 (in Section 5) shows the differences obtained using the Aggregate Claims Distribution and the QIS3 Aggregation Formula.

4. Some Results (Independence and Linear Correlation)

The Collective Model has permitted to obtain the main distributions and the required capital in case of independence and linear correlation. We focus here on the characteristics of the larger company OMEGA.

For the Company OMEGA the simulated distributions of the total claims amount for each LoB and the distribution of the aggregate amount (in the special case of independence among the LoBs) are reported in Figure 4 , where the main characteristics of the distributions are also figured out:

Figure 4: Company OMEGA - Simulated distributions of the Total Claim Amount for each LoB and of the Aggregate Claim Amount in case of independence (Number of simulations = 1.000.000)

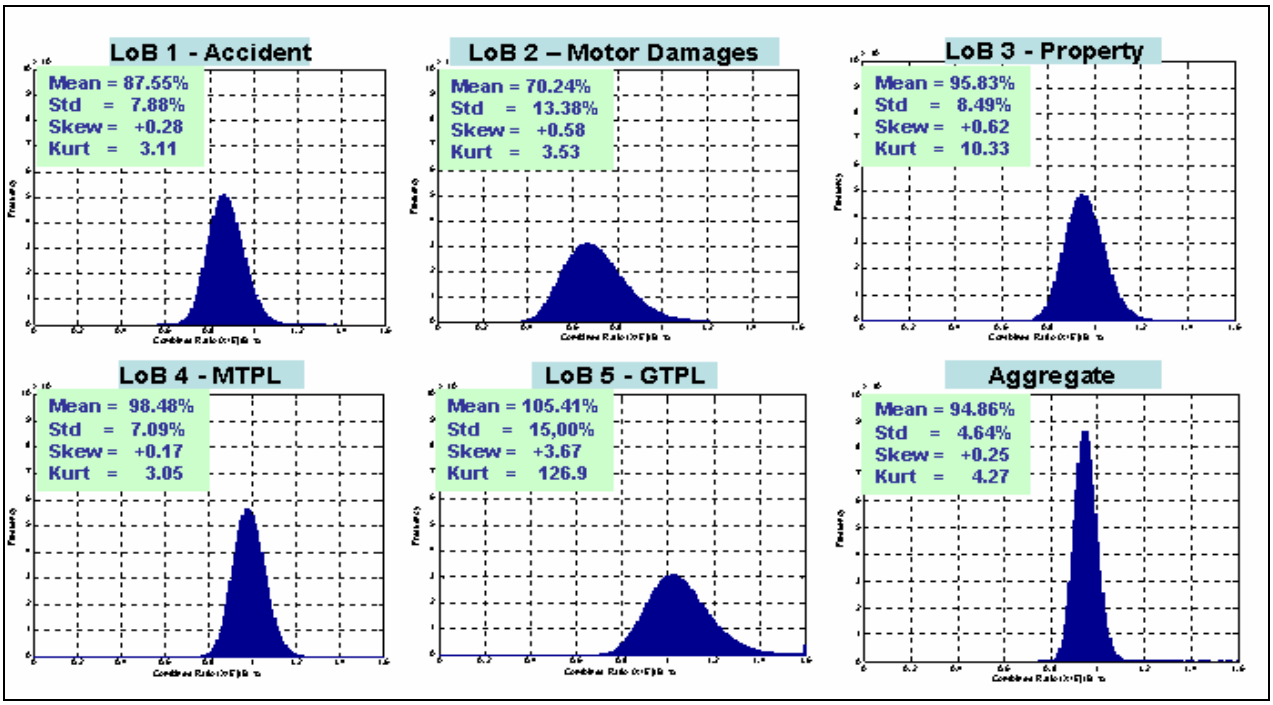


The figures show the higher positive skewness of Property and (more remarkable) GTPL, caused by the high value of claim size variability coefficient (c_z). For Motor Damages the large Aggregate Claim variability (c_x) is due to the standard deviation of the structure variabile q.

Finally the Aggregate Claim Amount in case of independence is influenced mainly by MTPL characteristics because of the relevant weight of the LoB (55% premium volume).

Further, for the same Company Omega, in Figure 5 are reported the simulated distributions of the Combined Ratios (X+E)/B. As well known, when technical results are in equilibrium this ratio (defined as claims amount plus total expenses divided by gross premiums) is equal to 100%, while if its value is under/over this threshold a technical profit/loss is occurring:

Figure 5: Company OMEGA - Simulated distributions of the Combined Ratios and RBC ratios for each LoB and for the Aggregate level in case of independence (Number of simulations = 1.000.000)



As expected, the figures show how the mean of the combined ratio (70.24%) is very low for LoB 2 (Motor Damages) denoting a large technical margin (almost 30% of gross premiums). Furthermore, the combined ratio distributions show a large expected profit for Accident too (12.5%) and small technical profit (1.5% for MTPL, 4.2% for Property) for the other LoBs.
Finally the negative safety loading for GTPL produces a mean of the combined ratio major than 100%, denoting an expected technical loss (5.41%).
The aggregate margin for all the lines is in total around 5.14%. As to variability, the standard deviation of the combined ratios are more or less in the range 7-9% for each LoB, except Motor Damages (13,4% for the highest standard deviation of q) and GTPL (15% for the high value of c_z).

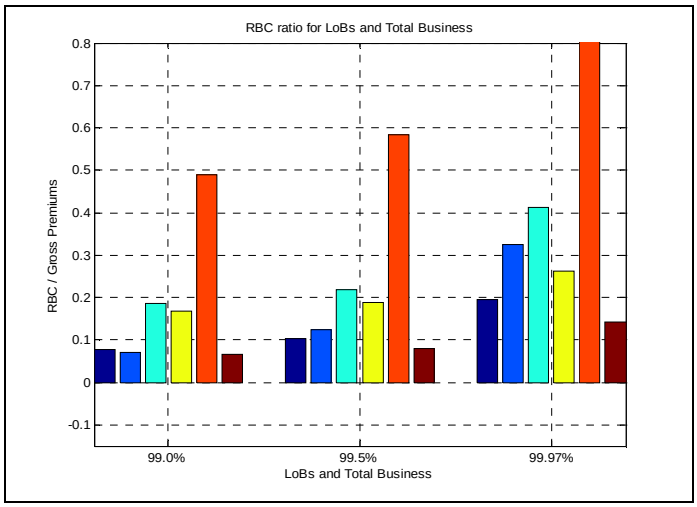
The RBC ratios (given by the RBC amount divided by initial Gross Premiums) is related to three examined confidence levels (see Figure 6):

- 99.00% (corresponding to a S&P rating BB approx.)
- 99.50% (adopted by QIS3, and roughly equivalent to a S&P rating BBB-)[12]
- 99.97% (corresponding to a S&P rating AA).

and regarding 99.5% level as our benchmark:

[12] The declared risk measure target in QIS3 standard formula assessment is the VaR approach with a 99.5% confidence level

Figure 6: Company OMEGA – RBC ratio for LoBs and Total Business in case of independence for different confidence levels (Number of simulations = 1.000.000)



As expected the highest ratio is registered for the line GTPL (58.4%) due mainly to its large claim size CV. Property Line shows a high ratio too (21.8%), while Line MTPL (18.8%) and Accident (10.2%) has lower ratios. Motor Damage has a 12.5% ratio, notwithstanding the large safety loading λ, because of the already mentioned large standard deviation of q.
The total capital requirement for the whole company OMEGA is then equal to 7.96% of gross premiums in case of independence (almost 77 million of Euro). Always in case of independence, if an alternative confidence level is adopted, the requirement is obviously decreasing (to 6.5%) for the 99.0% level, and increasing (to 14.2%) for the 99.97% level.

In Figure 7 it is shown the total capital requirement with linear correlation between LoBs, derived applying formula (9), using Internal Model results (above illustrated) in case of independence and the QIS3 matrix in case of correlation (reported in Figure 8):

Figure 7: Company OMEGA – RBC ratio

	RBC ratio		
	99%	99,50%	99,97%
No Corr.	6,51%	7,96%	14,21%
Corr. QIS3	11,63%	13,96%	25,87%
Corr. QIS2	9,61%	11,49%	19,43%
Full Corr.	18,33%	21,76%	40,81%

Figure 8: QIS3 Correlation Matrix[13]

LoB	Accident	Mot. Damages	Property	MTPL	GTPL
Accident	1,00	0,25	0,25	0,25	0,25
Mot. Damages	0,25	1,00	0,25	0,5	0,25
Property	0,25	0,25	1,00	0,25	0,25
MTPL	0,25	0,50	0,25	1,00	0,50
GTPL	0,25	0,25	0,25	0,5	1,00

[13] QIS3 correlation matrix shows linear coefficient higher than QIS2. However some coefficient (for example Property-Motor Damage) was reduced from QIS3.

For those 5 lines the capital requirement is increasing to 13.96% in case the QIS3 matrix correlation is assumed, and 11.49% with QIS2 correlation.
Finally in the extreme case of a positive full correlation the ratio is rising to 21.76% (18.33% and 40.81% for the other two confidence levels).

In Figure 9 are summed up the 99.5% RBC ratios obtained for all four companies for each LoB and the aggregate in case of independence. As to the Company TAU, having half dimension of the company OMEGA but identical parameters, for the 99.5% confidence level the capital requirement does not receive a large improvement and under independence assumptions the ratio increase to 8.68% (it was 7.96% for company OMEGA):

Figure 9: RBC ratio (99.5%) for 4 different Companies for LoB and Total Business in case of independence (Number of simulations = 1.000.000)

LoB	OMEGA	TAU	TAUHIGH	EPSILON
Accident	10,40%	10,78%	11,71%	13,91%
Mot. Damages	12,47%	12,69%	12,99%	13,04%
Property	21,82%	26,35%	37,35%	55,34%
MTPL	18,84%	18,99%	19,52%	20,78%
GTPL	58,39%	76,51%	106,53%	159,08%
Aggregate	**7,96%**	**8,68%**	**10,53%**	**14,76%**

Regarding Company TAUHIGH, with the only difference from company TAU of the claim size CV (values multiplied for 1.5 for each LoB), the requirement becomes more significant (10.53%).
This ratio is not so far to that one (14.76%) obtained for the smallest company (EPSILON), having the only difference of dimension with companies OMEGA and TAU.
In particular, it is to be pointed out the relevant RBC ratios obtained for the single LoB General Third Party Liability, caused by its higher c_Z (equal to 12) and negative technical profit, with values major than 100% for companies TAU HIGH and EPSILON and over 350% for the same two companies in case a AA rating target is desired.
Finally, it is important to analyse how the 99.5% RBC ratio is moving for all the examined companies according the (linear) correlations assumptions (see Figure 10):

Figure 10: RBC ratio (99.5%) for 4 different Companies

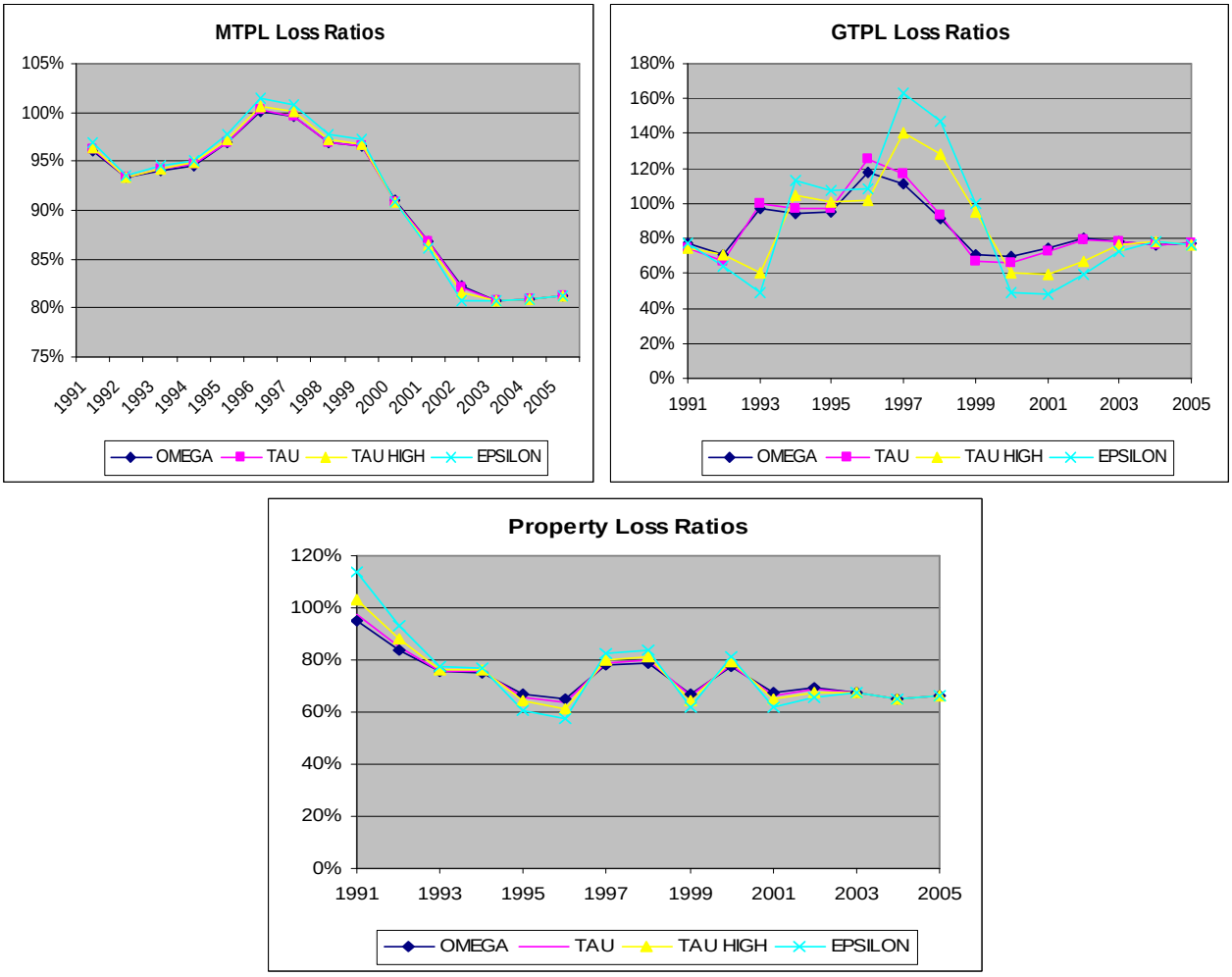
	OMEGA	TAU	TAUHIGH	EPSILON
NoCorr	7,96%	8,68%	10,53%	14,76%
Corr QIS3	13,96%	15,53%	18,69%	24,73%
Corr QIS2	11,49%	12,32%	14,36%	18,74%
Full Corr	21,76%	24,39%	29,46%	38,34%

For all companies the ratio is increasing by roughly 75% under QIS3 dependence assumptions. The positive effect of aggregation of different lines is very clear, that in case of not full correlation allow the companies a significant saving of required capital. For instance, in case of these two extreme dependencies the required capital ratio is decreasing from 21.8% to 7.8% of premiums for the largest company and from 38.3% to 14.8% in case of the smallest company.

5. Internal Model vs Standard Formula

In the next, some analyses to assess the impact of the standard formula proposed in QIS3 are performed restricted to the Non-Life Underwriting Risk, including only Premium Risk.
At this regard we refer to the 4 theoretical companies mentioned in the previous Section 4 having 5 LoBs, with their parameters summed up in Figure 2. To those data, related mainly to premium and claims, some data are now added concerning the historical series of the loss ratios.
For each company and for each LoBs a different historical pattern of loss ratios is determined.
It is to be emphasized that different historical patterns are assumed for these 4 insurers with the aim to compare consistently Internal Model results and undertaking-specific approach of Standard Formula and to consider that a smaller company would obviously report a more volatile distribution of the loss ratios.
In particular, line by line loss ratio patterns for each company (see Figure 11 for MTPL, GTPL and Property) are determined with the double assumptions that the mean of last 3 Loss Ratios and the standard deviation of last 15 Loss Ratios coincide with the exact mean and standard deviation obtained under the Compound Mixed Poisson Process assuming as parameters those reported previously in Figure 2.
Hence, we have four different patterns for each LoB, where OMEGA Company has the behaviour more similar to Italian market data:

Figure 11: Loss Ratios patterns for last 15 years for 4 different Companies (LoBs: MTPL,GTPL and Property)



It is worth to emphasize as all companies have the same average Loss Ratio, because we have assumed identical safety loading and expenses loading, both derived from market data. On the other hand the standard deviation is slightly modified according to the dimension (see Figure 12):

Figure 12: Standard Deviation of Loss Ratios for 4 different Companies

	Accident	Mot. Damages	Property	MTPL	GTPL
OMEGA	7,89%	13,40%	8,49%	7,09%	15,04%
TAU	8,00%	13,42%	9,45%	7,16%	18,39%
TAU HIGH	8,24%	13,47%	11,48%	7,32%	24,82%
EPSILON	8,84%	13,60%	15,07%	7,70%	35,13%

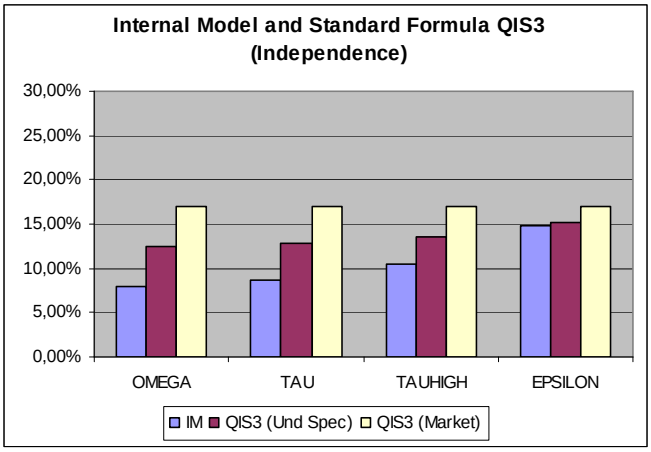
As shown in Figure 13 standard deviations obtained using the undertaking-specific approach (QIS3 Standard Formula) are different from those reported in Figure 12; in effect standard deviation is determined weighting each loss ratio with earned premium of the same year and obtaining the company-specific estimate of the expected value by the premium-weighted average of historic loss ratios. Finally the standard deviation estimate is obtained with a credibility mix based on market-wide volatility factor too (see formula (4)):

Figure 13: Standard Deviation for Premium Risk obtained by QIS3 undertaking-specific approach

	Accident	Mot. Damages	Property	MTPL	GTPL
OMEGA	6,35%	9,91%	7,38%	7,16%	11,67%
TAU	6,42%	9,92%	7,88%	7,20%	13,87%
TAU HIGH	6,58%	9,95%	8,98%	7,29%	18,20%
EPSILON	6,96%	10,02%	11,08%	7,51%	25,31%

In Figure 14 it is reported the 99.5% RBC ratio derived from the QIS3 Standard Formula under the independence assumption. As expected, all Insurers have the same ratio with the market-wide approach (17.0%), because of the lack of a size factor. Therefore, for a more interesting comparison it would be preferable refer to the undertaking-specific approach results. From the comparison between Internal Model and Standard Formula (based on undertaking-specific approach) for Premium Risk, it seems that Standard Formula overestimates the requirement of large-medium size companies with medium variability (OMEGA e TAU).
For those insurers the use of Internal Model may allow to reduce the required capital more than 30% (36.5% for OMEGA and 32.4% for TAU) while for the smallest insurer no significant reduction of the required capital will take place:

Figure 14: RBC ratio (99.5%) for 4 different Companies with Internal Model and Standard Formula (QIS3) under independence assumptions



However the undertaking-specific approach (under QIS3 Standard Formula) gives a result consistent with Internal Models output.

The differences are due to some assumptions of the Standard Formula. In particular it is to be observed:

a) the credibility factor, with 15 loss ratios, is less than 100%. Then the market-wide higher volatility factors have some impact on the undertaking-specific approach too;
b) SCR QIS3 formula does not take into account the technical expected profits/ losses[14] while Internal Models regard safety loading in Risk Based Capital (see formula (6)) as a reducing factor;
c) QIS3 Aggregation Formula considers less than Internal Model the diversification effect (see Figure 15) under independence assumptions. In fact, Internal Model determines the Capital Requirement on the Aggregate Claims distribution, while QIS3 derives it, joining single-line Capital Charges by an approximation formula (see formula (11));
d) moreover the "QIS3 ρ(x) transformation" is calibrated with the assumption of a LogNormal distribution for the aggregate claims. This assumption produces a standard deviation multiplier underestimated for small or highly variable LoBs (see Figure 16).

As to assumption c), in Figure 15 is figured out how QIS3 Aggregation Formula presents higher RBC ratios than Internal Model for all Companies and confidence levels describing only approximately the diversification effect. For instance, regarding the target confidence level of 99,50% as to OMEGA Company the Standard Formula obtains a required ratio of 8.54% instead of 7,96% by Internal Model :

Figure 15: RBC ratio for 4 different Companies obtained, under independence assumption, from Aggregate Claims Distribution and joining single Capital Charge by QIS3 Aggregation formula

		OMEGA	TAU	TAUHIGH	EPSILON
99%	$SCR^{Agg,Im}$	6,51%	7,06%	8,32%	11,21%
	SCR^{IND}	6,87%	7,57%	9,04%	12,34%
99,50%	$SCR^{Agg,Im}$	7,96%	8,68%	10,53%	14,76%
	SCR^{IND}	8,54%	9,59%	11,97%	16,83%
99,97%	$SCR^{Agg,Im}$	14,21%	18,82%	34,79%	51,97%
	SCR^{IND}	17,84%	23,24%	38,72%	56,51%

[14] In QIS2 Standard Formula, the overall BSCR was reduced (or increased) by the next-year expected profits (or losses) for Premium Risk

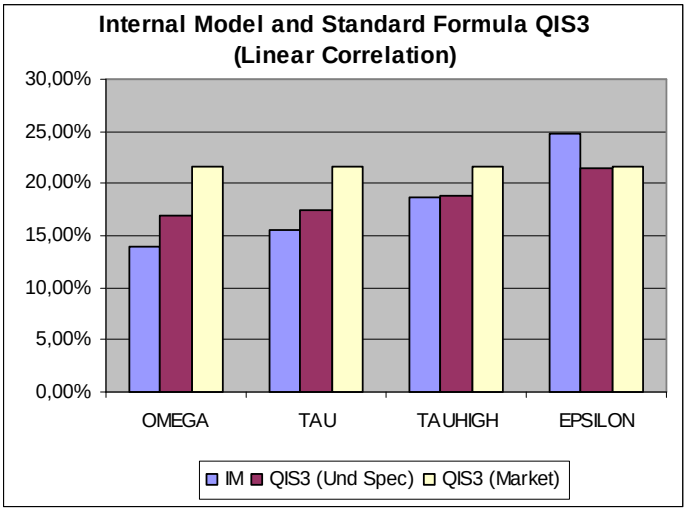
As to the multiplier mentioned at the assumption d), in the next Figure 16 this multiplier obtained from the Internal Model results and under the assumption of LogNormal distribution of aggregate claims is figured out. For OMEGA Company LogNormal assumption is not so far from frequency-severity results (i.e. Internal Model) while in case of EPSILON Company the LogNormal assumption underestimates by far the skewness of aggregate claims obtained by simulations (0.25 against an exact skewness of 3.68) and it drives to a multiplier lower than Internal Model (2,82 instead of 3,13):

Figure 16: Standard Deviation Multiplier (for OMEGA and EPSILON Companies), Derived by Internal Model and under LogNormal assumptions

	OMEGA	EPSILON	OMEGA	EPSILON
	$(RBC+\lambda^*P)/\sigma(X)$		$(RBC_{LogN}+\lambda^* P)/\sigma(X)$	
Accident	2,84	2,91	2,99	3,04
Mot. Damage	3,11	3,10	3,43	3,45
Property	2,94	3,84	2,95	3,23
MTPL	2,74	2,77	2,83	2,85
GPTL	3,35	4,19	3,15	3,91
Aggregate	2,74	3,13	2,76	2,82

In case of linear correlation, Internal Model results show again (but not for EPSILON Company) a lower ratio than Standard Formula (see Figure 17).
The Internal Model capital reduction is less than the independence case: OMEGA Company obtains a decreasing of 17% respect to undertaking-specific approach (it was 36.5% in case of independence) and TAU Company reduces the requirement of only 11%. The different way to consider diversification effect, between Internal Model and Standard Formula, has a lower impact in correlation case.

Figure 17: RBC ratio (99.5%) for 4 different Companies with Internal Model and Standard Formula (QIS3) under QIS3 dependence assumptions



Moreover, formula (9), used with the aim to determine RBC ratio by Internal Model under dependence assumptions, represents indeed an approximation formula defined in a similar way than QIS3 Aggregation Formula. Under these assumptions (dependence), Internal Model and QIS3

Standard Formula aggregation structures describe almost in the same way the diversification impact.
However, some analyses show that formula (9) gives Capital Requirement not so far from the RBC obtained using the multivariate aggregate claims distribution with a dependence structure described by a Gaussian Copula[15]. Obviously, if another copula function is used, the differences arising from aggregation will result quite different (and then less comparable) because of the tail dependence.

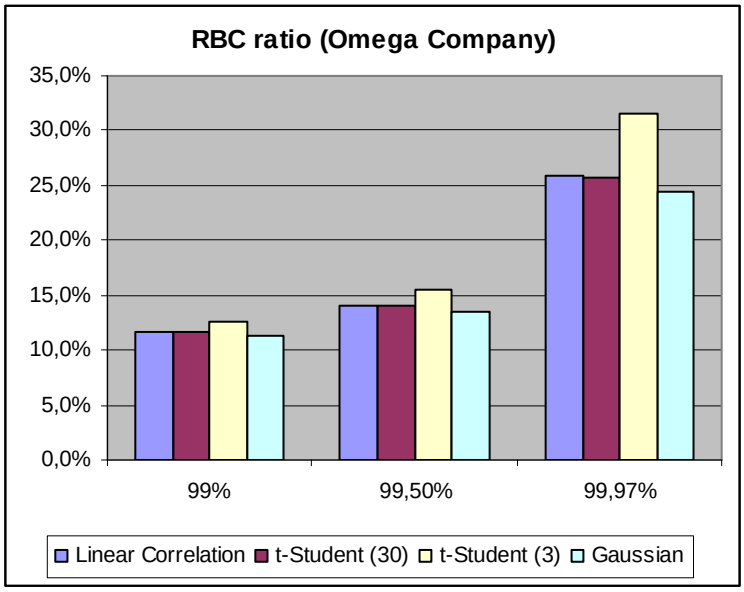
Finally it is worth to point out how Internal Model approach obtains a higher Capital Requirement than by the undertaking-specific for EPSILON Company, mainly due to the inappropriate use of the LogNormal distribution in the Standard Formula for a small size company.

6. Copulas and Correlation Sensitivity

Copulas functions give the possibility to obtain Capital Requirement under dependence assumption without using approximation formula. Actually this functions allow to model the stochastic dependence structure between lines of business and to describe aggregate claims distribution using Monte Carlo simulations under dependence assumptions. We present RBC ratio obtained by Internal Model where single LoBs are aggregated through elliptical copulas (Gaussian and T-Student).
In dimension N, these parametric Copulas are more adequate than the often used Archimedean Copulas. Archimedean Copulas have a major disadvantage: they represent the dependence structure with only one parameter (two in some cases). They are not able to take account of all the complexity of the observed structures in dimensions higher than two.
Elliptical Copulas are here implemented using correlation coefficients derived from the QIS3 correlation matrix (see Figure 8).
In Figure 18 it is compared RBC ratio obtained by Internal Model and by different dependence assumptions. For the OMEGA Company, it can be observed how Gaussian Copula gives rise to a lower Capital Requirement than linear correlation. T-Student, with few degrees of freedom (3), presents a high dependence on both tails, causing a skewed Aggregate Claims (0.45 against 0.33 under independence) and the highest RBC ratio (15,5% at 99,5% level). These results confirm the positive tail dependence[16] assumed by Student Copula.
Finally a Student Copula, with 30 degrees of freedom, has a distribution rather close to a Gaussian (tail dependence decreases for raising degrees of freedom) and shows results similar to linear correlation:

[15] At this regard, see Savelli and Clemente (2008), section 10.
[16] The tail dependence coefficients are asymptotic measures of the dependence in tails of the bivariate distributions. These measurements depend on the copula. Interesting cases are those where the coefficients are strictly larger than zero, since that indicates a propensity to jointly generate extreme events.

Figure 18: RBC ratio (OMEGA Company) under different dependence assumptions (Elliptical Copulas and Linear Correlation)



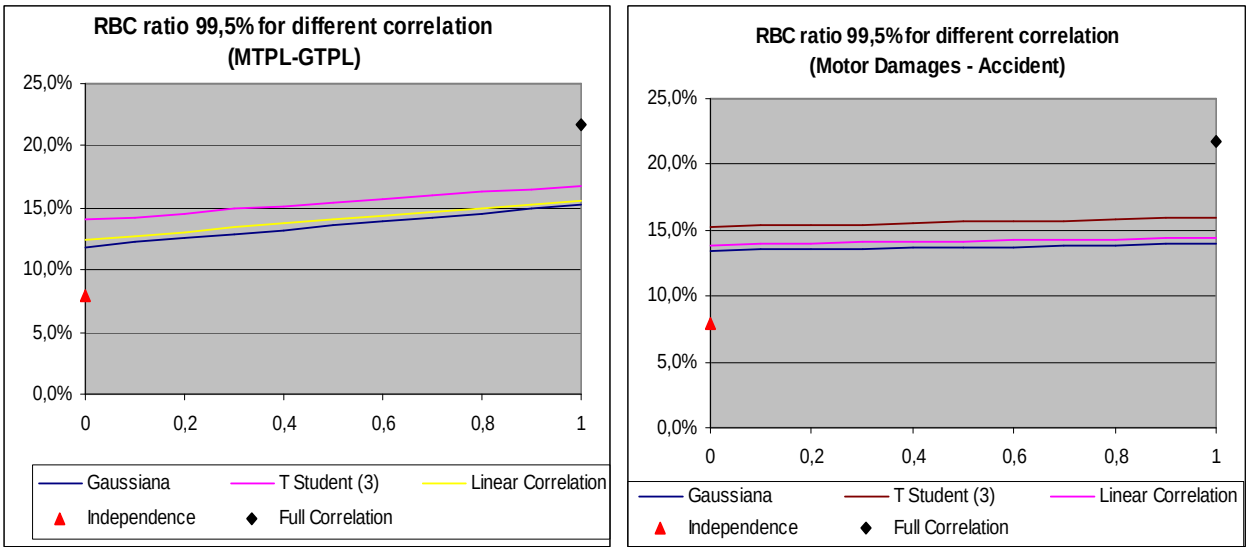
If a 99,97% confidence level is adopted, differences between T-Student (3 degrees) and Gaussian are increasing. This is explained by the fact that the simultaneous occurrences of large claims amount are much more frequent for Student than for Normal Copula.
Moving to the others companies, the results are almost the same. Gaussian shows the lowest Capital and Student Copula with 3 degrees has the highest RBC ratio. T-Student, with 30 degrees of freedom, presents the same RBC ratio of linear correlation for medium-large size companies, while it has lower Capital Requirements than linear correlation for small or highly variable Companies (EPSILON and TAU HIGH)

Figure 19: RBC ratio (only 99,5% confidence level) for 4 different Companies under different dependence assumptions (Elliptical Copulas and Linear Correlation)

	Linear Correlation	t-Student (30)	t-Student (3)	Gaussian
OMEGA	14,0%	14,0%	15,5%	13,5%
TAU	15,5%	15,5%	17,1%	14,9%
TAU HIGH	18,7%	18,3%	20,5%	17,9%
EPSILON	24,7%	24,1%	26,8%	23,8%

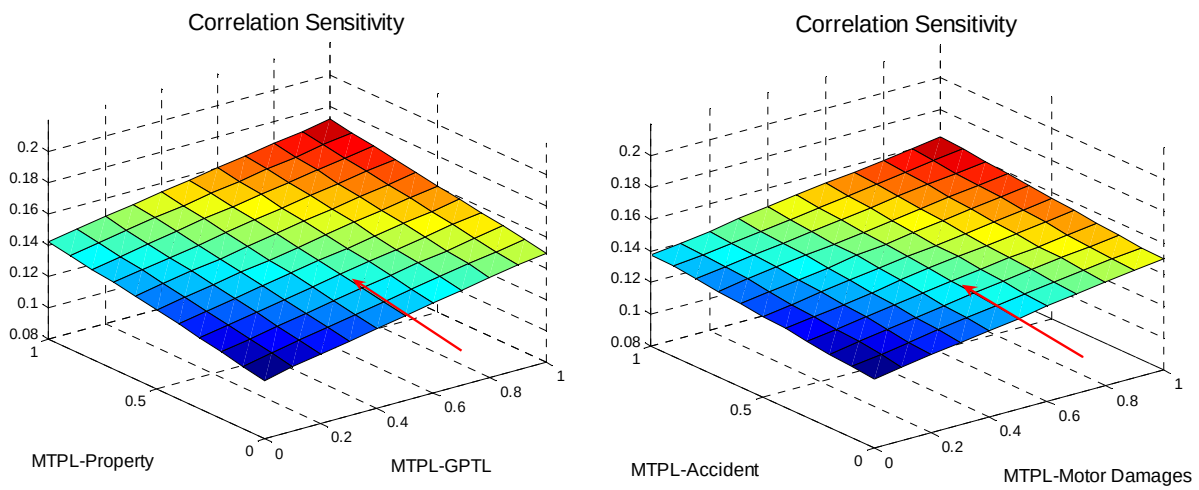
It can be interesting to analyse how RBC ratio can change moving some coefficients of QIS3 correlation matrix.
At this regard in Figure 20 it is compared RBC ratio obtained by linear correlation and by elliptical copulas, changing some coefficients. It can be observed how Capital Requirement is very sensitive to MTPL/GTPL correlation (RBC moves from 12,4%, if the two Lobs are independent, to 15,5% in case of full correlation). In fact, MTPL has a not negligible RBC ratio and it is representing 55% of the total Business whereas GTPL has the highest RBC ratio by LoB (58,4%).
RBC ratio moves very slightly from 13,9% to 14,4% changing correlation between Motor Damages and Accident from independence to full correlation. In this case the correlation has not a great impact on the Capital Requirement.
Finally in both cases elliptical copulas have a similar behaviour: the correlation coefficient choice is a major problem than which copula to use.

Figure 20: Correlation coefficient sensitivity (OMEGA Company) between MTPL and GTPL (default 0.5) and between Motor Damages and Accident (default 0.25)



Finally, Figures 21 show the 99.5% RBC ratio behaviour, changing the correlation coefficient between MTPL and the other LoBs. A full correlation between MTPL and GTPL gives the highest ratio (15,5%) and it can be obtained a Capital Requirement equal to 17,1% of gross premium, combining with a full correlation between MTPL and Property.
Red arrows show the 99,5% RBC ratio obtained with QIS3 correlation coefficient.
The correlations with the others two LoBs (Motor Damages and Accident) have a lower effect on Capital Level: RBC moves from 12,3% in case of independence to almost 16% with full correlation.

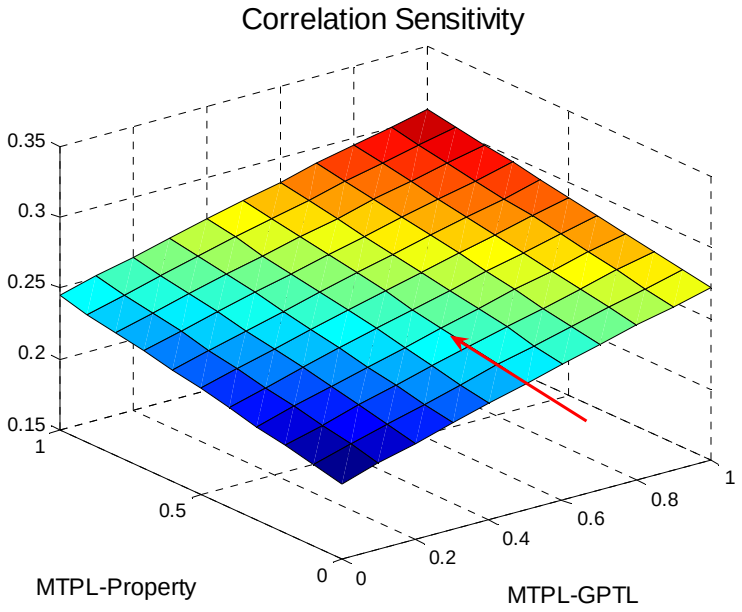
Figure 21: RBC ratio (99,5% OMEGA Company) with correlation coefficient sensitivity



Moving to the smallest Company (EPSILON), we can observe that the RBC ratio at 99,5% moves from 24,7% (RBC ratio with QIS3 correlations) to 20% under independence assumption between MTPL and the other two main LoBs (Property and GTPL).

The assumptions of full correlation between MTPL and the other two LoBs lead to a Capital Requirement equal to 30% of gross premium (see Figure 22).
MTPL and GTPL correlation plays a primary role for EPSILON Company too: using QIS3 correlation matrix and setting only the coefficient between this two LoBs equal to -1, RBC ratio is equal to 12,98%.
This ratio is lower than Capital ratio (14,76%) obtained, for the same confidence level (99,5%), using aggregate claims distribution under independence assumption between all five LoBs.

Figure 22: RBC ratio (99,5% EPSILON Company) with correlation coefficient sensitivity



7. Some remarks on the forthcoming QIS4

Following the publication of QIS3 report, CEIOPS has developed a first draft of QIS4 Technical Specifications under tight deadlines (December 2007). Then another document, published on 31 March 2008, sets out the technical specifications to be used for the Fourth Quantitative Impact Study.
Focusing our attention only above Underwriting Risk Non-Life module, QIS4 has developed a modular structure like QIS3. Some Premium Risk parameters and some formulas has been modified.
LoBs segmentation is the same as the segmentation applied in QIS3 valuation, excluding health and accident, which for the purpose of SCR calculation are treated in Underwriting Risk Health module with the same formulation as Non-Life LoBs.
Premium Risk Capital Requirement can be still estimated according two different approaches (market-wide and undertaking-specific estimate) like in QIS3 Standard Formula.
In both approaches, premiums and provisions should be allocated between different geographical areas. It's calculated, for each line of business, the Herfindahl[17] index quantifying the

[17] The Herfindahl index is calculated as follows:

diversification effect only if undertaking has less than 95% of its non-life activities in the same geographical area.
The market-wide estimate of the standard deviation for premium risk in the individual LoB is determined as the following volatility factor:

Figure 23: Volatility factor (QIS3 and QIS4 – Premium Risk)

Volatility Factor (Premium Risk)	Accident	Mot. Damages	Property	MTPL	GTPL
QIS3	5%	10%	10%	10%	10%
QIS4	5%	9%	10%	9%	12,5%

Figure 23 shows how QIS4 has reduced volatility factor for MTPL and Motor Damages and it has increased GTPL standard deviation.
The undertaking-specific estimate of the standard deviation for premium risk is determined on the basis of the volatility of historic loss ratio patterns with the same credibility formula as in QIS3 (see formula (4)).
The maximum number of Loss Ratios (n_{lob}) is obtained according to the line of business: MTPL and GTPL consent to use 15 loss ratios like in QIS3.
Companies can not use any more than five loss ratios for Property, Motor Damages and Accident.
Furthermore the credibility factor depends on number of Loss Ratios[18], but the volatility will never be determined only using the standard deviation of loss ratio.
In fact, if undertaking has the maximum number of Loss Ratios, it should calculate the standard deviation using a credibility factor equal to 0.79 as in QIS3.
Finally QIS4 has introduced a new correlation matrix for Non-Life, but CEIOPS chooses the same correlations coefficients for the following LoB (see Figure 24).
Accident is correlated to this LoBs using the aggregation formula between Health and Non-Life with correlation coefficient equal to 0.25 (like in QIS3).

Figure 24: QIS4 Correlation Matrix

	Mot. Damages	**Property**	**MTPL**	**GTPL**
Mot. Damages	1,00	0,25	0,50	0,25
Property	0,25	1,00	0,25	0,25
MTPL	0,50	0,25	1,00	0,50
GTPL	0,25	0,25	0,50	1,00

In the next, some analyses to assess the impact of the standard formula proposed in QIS4 are performed restricted to the Non-Life Underwriting Risk, including only Premium Risk.

$$DIV_{pr,lob} = \frac{\sum_j (V_{(prem,j,lob)} + V_{(res,j,lob}))^2}{\left(\sum_j (V_{(prem,j,lob)} + V_{(res,j,lob)})\right)^2}$$

where $V_{(prem,j,lob)}$ and $V_{(res,j,lob})$ are the volume measure in geographical area j for single lob for premium and reserve risk.

[18] See Credibility Factor Table in Technical Specifications QIS4 (pag. 204)

We refer to the 4 theoretical companies mentioned in the previous Sections having 5 LoBs, with their parameters summed up in Figure 2.
We assume the same Loss Ratios historical series with the aim to calculate undertaking-specific approach. The historical series are here considered taking into account the maximum number of Loss Ratios fixed by QIS4 (five for Motor Damages, Property and Accident, fifteen for MTPL and GTPL).
The reductions of volatility factor lead to a lower Capital Requirement with the market-wide approach. Figure 25 shows RBC ratio for only Premium Risk obtained by QIS3 and QIS4 Standard Formulas. As in the QIS3 the RBC ratios, determined with a market-wide approach, are the same for all Companies.

Figure 25: RBC ratio for Premium Risk only (Market-Wide Approach) by Standard Formula (QIS3 and QIS4)

	Accident	Mot. Damages	Property	MTPL	GTPL	Aggregate
QIS3 Premium Risk	14,30%	30,14%	30,14%	30,14%	30,14%	21,57%
QIS4 Premium Risk	14,30%	26,85%	30,14%	26,85%	38,66%	20,25%

Using undertaking-specific approach, QIS4 gives lower standard deviation than QIS3, except for GTPL line. In fact, the credibility mix with lower volatility factor and the behaviour of last five Loss Ratios reduce undertaking-specific standard deviation and Capital Requirement.
Figure 27 shows the RBC ratios for all 4 Companies with QIS3 and QIS4 Standard Formula. For large and middle-size QIS4 shows again higher Capital Requirement than Internal Model. Companies with small dimension or with high variability gives RBC ratio higher than Internal Model.

Figure 26: Standard Deviation for Premium Risk obtained by undertaking-specific approach (OMEGA Company)

	Accident	**Mot. Damages**	**Property**	**MTPL**	**GTPL**
QIS3	6,35%	9,91%	7,38%	7,16%	11,67%
QIS4	3,44%	4,41%	4,74%	6,87%	12,17%

However it could be observed that it is not appropriate a full comparison between QIS4 Standard Formula and Internal Model results. In fact undertaking-specific approach uses only 5 five Loss Ratios for some LoBs, while Internal Model parameters have been calibrated considering all 15 Loss Ratios with a higher standard deviation.

Figure 27: RBC ratio forPremium Risk only (undertaking-specific approach)

	QIS3	QIS4
OMEGA	16,88%	14,63%
TAU	17,51%	15,14%
TAU HIGH	18,85%	16,22%
EPSILON	21,37%	18,25%

Finally, QIS4 Technical Specifications have confirmed the calculations of a total capital requirement for the Group too, specifically addressed for the first time in QIS3. A particular section provides specifications for calculating and reporting group capital requirements and group own funds.
The total capital requirement for the group is calculated using different methods based on the Standard SCR formula applied to the consolidated group position (Accounting Consolidation Method) or it's determined as the sum of the solo SCRs properly adjusted with the aim to consider deduction and aggregation effects.

8. Conclusions

In this paper, a Collective Risk Model is applied with the aim to quantify the Solvency Capital Requirement for the Premium Risk only. In particular, having paid a large care to choose different (theoretical) insurers to be representative of Italian market, we presents an interesting comparison between Internal Model and Standard Formula.
The use of Internal Models show significant reduction of required capital for large and medium size companies. This aspect is confirmed by QIS3 Final Report.
It can be observed how some Standard Formula assumptions can conduct at a similar requirement between Undertaking Specific and Internal Model for small and highly variable companies.
It should be emphasized that the Risk Theoretical Model is only a simplified version of the complex practical risk management process and furthermore all valuations have been made without considering reinsurance treaties. That could have a high impact on aggregate claims variability and on safety loading, with a more flat scale of capital requirements according to company size.

Finally, it is worth to reminding that when simulation models are used great attention need to be paid to avoid as much as possible the three classical modelling risks (model, parameter and process risk).
In particular the risk of assessing inappropriate parameters used in the model assumes a relevant importance for the high impact of some parameters on Capital Requirement. For example, it could be useful to introduce a structure variable on claim size distribution too with the aim to consider the parameter uncertainty. This should have a relevant impact on systematic risk and then producing higher capital requirements for large insurers.

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