The one-year non-life insurance risk

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Abstract

With few exceptions, the literature on non-life insurance reserve risk has been devoted to the *ultimo risk*, the risk in the full run-off of the liabilities. This is in contrast to the short time horizon in models for the total risk of the insurance company, and in particular the *one-year risk* perspective taken in the Solvency II project, and in the computation of risk margins with the Cost-of-Capital method. This paper aims at clarifying the methodology for the one-year risk; in particular we describe a simulation approach to the one-year reserve risk. We also discuss the one-year *premium* risk and the premium reserve. Finally, we initiate a discussion on the role of risk margins and discounting for the reserve and premium risk.[1]

Keywords

Reserve risk, premium risk, Solvency II, IFRS 4 phase II, risk margin, Dynamic Financial Analysis, stochastic reserving.

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[1]Any opinions expressed in this paper are the authors' own, and do not constitute policy of our employers.

1 Introduction

In most risk models, non-life insurance risk is divided into *reserve risk* and *premium risk*. Reserve risk concerns the liabilities for insurance policies covering historical years, often simply referred to as the risk in the *claims reserve*, *i.e.*, the provision for outstanding claims. Premium risk relates to future risks, some of which are already liabilities, covered by the *premium reserve*, *i.e.* the provision for unearned premium and unexpired risks; others relate to policies expected to be written during the risk period, covered by the corresponding expected premium income. (For technical reasons, *catastrophe risk* is often singled out as third part of non-life insurance risk, but that lies outside the scope of this paper.)

The mentioned risks are also involved in the calculation of *risk margins* for technical reserves; we will consider the *Cost-of-Capital* (CoC) approach, which is mandatory in the Solvency II Draft Framework Directive, EU Commission (2007). In the Discussion paper IASB (2007) on the forthcoming IFRS 4 phase II accounting standard, CoC is one of the listed possible approaches to determine risk margins.

In the Solvency II framework, the time horizon is *one year*, described by the EU Commission (2007) as follows: "all potential losses, including adverse revaluation of assets and liabilities over the next 12 months are to be assessed."

In the actuarial literature, on the other hand, reserve risk is almost exclusively discussed in terms of the risk that the estimated reserves will not be able to cover the claims payment during the *full run-off* of today's liabilities, which may be a period of several decades; we call this the *ultimo* risk. If R^0 is the reserve estimate at the beginning of the year and C^∞ are the payments over the entire run-off period, this risk is measured by studying the probability distribution of $R^0 - C^\infty$. This is the approach of the so called *stochastic claims reserving* which has been extensively discussed in the actuarial literature over the last two decades, by Mack (1993), England & Verall (2002) and many others.

Towards this background, it may not be surprising that it is noted in a study from the mutual insurers organization AISAM-ACME (2007), that "Only a few members were aware of the inconsistency between their assessment on the ultimate costs and the Solvency II framework which uses a one year horizon". It is also noted that "the use of innovative actuarial methodologies is required to replace the classical ones which are inappropriate." AISAM-ACME (2007, Press release).

This is the starting point for the present paper. Our first aim is to help in clarifying the methodological issues for the one-year approach to reserve risk, and in particular to write down a general simulation approach to the problem. A special case of this approach is the bootstrap methods in the context of Dynamic Financial Analysis which are implemented in some commercial software; a short description of that methodology was given by Björkwall, Hössjer & Ohlsson (2008), Section 3.6. In particular, it is noted that the one year reserve risk is the risk in *the run-off result*, as will be described in the next section.

Another paper on the one-year reserve risk is Wütrich, Merz & Lysenko (2007). In the special case of a pure Chain-ladder estimate, they give analytic formulae for the mean squared error of prediction of the run-off result, by them called the *claims development result*, under an extension of the classic Mack (1993) model.

After describing the simulation method for the reserve risk in the next section, we will turn to the one-year perspective for the premium risk in Section 3, followed by a discussion on the role of the risk margin for both types of risks in Section 4. To the best of our knowledge, these issues have not previously been discussed in the literature.

We end this introduction with a discussion of the time horizon in the

QIS3 calibration[2] within the Solvency II development. In the Technical Specifications for QIS3, CEIOPS (2007), we read:

> I.3.229 Reserve risk stems from two sources: on the one hand, the absolute level of the claims provisions may be mis-estimated. On the other hand, because of the stochastic nature of future claims payments, the actual claims will fluctuate around their statistical mean value.

It is not clear from this definition what the time perspective is; it is easy to get the impression that the quotation is about the ultimo risk: the risk over the entire run-off period of the liabilities. However, at other places in the QIS definitions it is clearly stated that the perspective is strictly one-year, see e.g. CEIOPS (2006) paragraph 46, where the volatility is described as “the standard deviation of the run-off result of the forthcoming year”.

On the other hand, AISAM-ACME (2007) states that “QIS3 seems to be consistent with a full run-off approach rather than a one-year-horizon volatility.” The AISAM-ACME study shows that for long-tailed business, the full run-off (ultimo) approach gives risk estimates that are 2 to 3 times higher than the ones for the one-year run-off result. Of course, the fact the QIS3 figures are consistent with the ultimo risk level for the investigated insurance companies does not necessarily mean that the calibration has been done with the ultimo perspective in mind.

[2]When this is written, only a *draft version* of the technical specifications for QIS4 is published: we will therefore refer to QIS3 only, even though it is likely that much of what is said on QIS3 here will also be valid for QIS4.

2 The one-year reserve risk

The reserve risk over a one-year time horizon is the risk in the *one-year run-off result*. If C^1 is the amount paid during next year, R^0 the opening reserve at the beginning, and R^1 the closing reserve estimate at the end of the year, then the technical run-off result is

$$T = R^0 - C^1 - R^1 \,. \tag{2.1}$$

The one-year reserve risk is captured by the probability distribution of T, conditioned on the observations by time 0. (Note that T is also the difference between the estimate of the ultimate cost at times 0 and 1.) This is in contrast to the ultimo or full run-off risk, which was described above as the risk in $R^0 - C^\infty$.

The AISAM-ACME (2007) study distinguishes between the *shock period*, which is the one projection year when adverse events occur, and the *effect period*, which is the full length of the run-off of the liabilities. The direct effect of a shock is captured by C^1, while the variation in R^1 relates to the effect period. Hence the one-year risk is, to some extent, affected also by the risk during the entire life-span of the liabilities, but not as much as the ultimo risk is.

Note. We tacitly assume that an expenses reserve is included in the claims reserve above. The question of whether this reserve should inflate the reserve risk or not is outside the scope of this paper. □

In risk models, the insurance portfolio is divided into more or less homogenous segments, e.g. lines of business (LoB). To be able to calculate the reserve risk for a segment and combine it with other risks to the total risk of the insurer, we need the probability distribution of the run-off result T for this segment. In some cases, this may be done analytically, see the already mentioned result for the Chain ladder method in Wütrich *et al.* (2007). In most practical situations, simulation methods will be the only possibility; in the rest of this section we will describe the steps of such a simulation, in the case when we neither use discounting nor add a risk margin to the reserves. This simulation algorithm is by no means new, but, to the best of our knowledge, it has not been discussed in the literature before, except for the short description in a bootstrap context in Björkwall *et al.* (2008).

Wütrich *et al.* (2007) condition on the observed part of the claims triangle. Along the same line, we will condition on all observations up to time 0, the start of the risk year, and denote the collection of these random variables by $\mathcal{D}_0$. For simplicity, we will assume that the one-year risk is computed on January 1 for the next calendar year.

2.1 Step 1, the opening reserve

We have available the actuary's best estimate of the outstanding claims at the beginning of the year, the opening reserve R^0, which is not considered stochastic here since it is based on observed values. We assume that this reserve was computed by a documented algorithm A that could be repeated in the simulations. For a long-tailed business such an algorithm might, e.g., be to use a development factor method on paid claims, with the factors smoothed and extended beyond the observation years by some regression model; then a Generalized Cape Cod (GCC) method may be used to stabilize the latest years, while the earliest years reserves might be adjusted somehow by the claims incurred. For a description of the GCC, see Struzzieri & Hussian (1998).

The actuary's subjective judgement on individual figures can usually not be included in such an algorithm. If such judgement was used, we would have to find an approximate algorithm A, capturing the main features of the estimate, since the algorithm will be used in the simulation, see Step 3.

2.2 Step 2, generating the new year

The next step is to simulate the events of the risk year, conditional on $\mathcal{D}_0$. As a minimum, this will include claims paid during the year, for each origin year. Let C_{ij} be claims paid for origin year i and development year j and let n be the *ultimo year*, when all claims are finalized. We need to simulate a new diagonal in the development triangle; in Figure 2.1, the simulated random variables are marked with bold-face. For long-tailed business, it might be that no origin years are finalized,

	Development years					
origin years	1	2	3	$\cdots$	$n-1$	n
1	C_{11}	C_{12}	C_{13}	$\cdots$	$C_{1,n-1}$	$C_{1,n}$
2	C_{21}	C_{22}	C_{23}	$\cdots$	$C_{2,n-1}$	$\mathbf{C_{n,n}}$
3	C_{31}	C_{32}	C_{33}	$\cdots$	$\mathbf{C_{n,n-1}}$	
$\vdots$	$\vdots$	$\vdots$	$\vdots$			
$n-1$	$C_{n-1,1}$	$C_{n-1,2}$	$\mathbf{C_{n,3}}$			
n	$C_{n,1}$	$\mathbf{C_{n,2}}$				

Table 2.1: Development triangle with simulated variables in bold-face; the other variables belong to $\mathcal{D}_0$.

in which case we may (or may not) want to add an observation to year 1 as well.

There are several possibilities to get the simulated diagonal and this is really outside the scope of this paper; one possibility is bootstrapping, see for e.g. Björkwall *et al.* (2008), Section 3.6; another possibility is to simulate from a normal or log-normal distribution with mean given by the Chain-ladder and variance given by formula (3) of Mack (1993).

The sum along the new diagonal yields the claims paid during the year for historical origin years, denoted C^1 above.

For some reserving methods A, we will also need a new diagonal of claims incurred or other quantities; we will not go into details here, but just assume that all variables needed by the actuary for reserving can be simulated for the new year.

2.3 Step 3, the closing reserve

In this step we generate the closing reserve R^1, the reserve as estimated by the end of the risk year. The idea is, for each simulated outcome in Step 2, to calculate R^1 by the method A from Step 1, which is (an approximation of) the reserving method used in practice when calculating R^0.

It is easy to let the reserving method include tail estimation by regression or adjustments made by an algorithmic such as the Cape Cod method. The Bornhuetter-Ferguson method might be a problem though: are the prior ratios really fixed or are they in effect a function of observed claims and other random variables? In the latter case the functions should be included in the simulation set-up and thus the ratios would contribute to the risk; if not, they are constants which do not add anything to the risk — in our opinion, popular as it might be to view Bornhuetter-Ferguson prior ratios as fixed, this is strange from a risk perspective since, of course, the ratios do not give a perfect estimate of the liabilities.

We now have all the components needed to calculate the run-off result T in equation (2.1), for each of our, say, B simulations; the empirical distribution of these run-off results is our estimate of the probability distribution of T. The B run-off results can also be used further on in a risk model to interact with other risks in order to get the total risk and other aggregated risks. From the probability distribution of T we can, of course, get the standard deviation, any Value-at-Risk (VaR) figure, or whatever risk measure we choose.

Note. In many cases one would wish to include (claims) inflation in the above calculations by initially adjusting the paid claims triangle for historic inflation and at the end of the calculations recalculate the result in running prices, by using some assumed future inflation. The outcome of the inflation for the risk year, as well as the assumed future inflation at the end of that year, should preferably be stochastic. We will not go into further detail on inflation here. □

2.4 Discussion

A problem with the one-year is risk that the reserves for long-tailed business might change so little over one year that "/. . . / it should not be a surprise that some long tail business – where adverse movements in claims provisions emerge slowly over many years – require less solvency capital than some short tail business /. . . / ". (AISAM-ACME, 2007). In our experience, this really happens in practice.

This is a general problem with the one-year horizon of the Solvency II frame-work, which relates to risks that could appear in the financial statements over one year and does not take the long-term nature of insurance into account, right or wrong. Of course, mixing an ultimo perspective for liabilities with a one-year perspective for assets is not an alternative, if we are interested in the combined total risk of the company. These problems indicate that for internal risk models, it might be a good idea to consider a longer risk period, say three or five years. Note that it is straight-forward to extend the simulation method described above to two or more years.

3 The one-year perspective on premium risk

In the QIS3 technical specifications, CEIOPS (2007), premium risk is introduced as follows.

> I.3.228 Premium risk relates to policies to be written (including renewals) during the period, and to unexpired risks on existing contracts.

In the specification for input calculations in I.3.231, the historic loss ratios, from which the volatility is calculated, are given by the estimated cost at the end of the first development year, divided by earned premiums. This indicates that the premium risk here is the risk in the cost $(\tilde{C}^1 + \tilde{R}^1)$, where $\tilde{C}^1$ are first year payments for the current origin year and $\tilde{R}^1$ is the (closing) claims reserve for the same year. In a simulation model, risks are formulated in terms of a profit/loss result; if $\tilde{P}$ is the earned premium expected for the year and E the operating expenses, the result is

$$\tilde{T} = \tilde{P} - E - (\tilde{C}^1 + \tilde{R}^1)\,, \tag{3.1}$$

which gives a premium risk of the above type, if $\tilde{P}$ and E are non-random.

While we agree with the fact that the technical results for one year are mainly affected by the first years payment and the initial claims reserve, we would also like to clarify the role of the premium reserve in this context.

In today's accounting, the premium reserve is computed *pro rata temporis* and then an additional provision for unexpired risks is added, if so required by the outcome of a liability adequacy test. From an economic perspective, however, this reserve is not very different from the claims reserve, only that it relates to claims that have not yet occurred, but for which we have a contractual liability; another difference is that it only covers a part of the expected liabilities for next origin year, in our experience 10-45 % depending on the LoB. By this view, the premium reserve should only cover expected claim costs (including handling expenses). In particular, the insurance company might recognize part of the profit at the inception of an insurance contract, while the rest of the profit is recognized as the liabilities are run off.[3]

If this perspective is taken on the premium reserve, one-year risk models should ideally include the risk in that reserve for existing contracts plus the risk in contracts we expect to write/renew during next year, the latter with expected premium income P. Let $\tilde{U}^t; t = 0, 1$ be the opening and closing premium reserve (the reserve for unexpired risks on existing contracts). Then the technical result for the current year is

$$\tilde{T} = \tilde{U}^0 + P - E - (\tilde{C}^1 + \tilde{R}^1) - \tilde{U}^1 \,. \tag{3.2}$$

If premium cycle variation is modeled, P is stochastic. We will consider it as fixed here, *i.e.* we model the risk inherent in the premium volume the company expects to receive; we also assume that expenses will equal the budgeted value so that E is fixed.

There are several ways to simulate the claim cost $(\tilde{C}^1 + \tilde{R}^1)$, one possibility being to simulate the corresponding loss ratio, with volatility estimated as in QIS3, as discussed above. Another possibility suggested by Kaufmann, Gadmer & Klett (2001) is to simulate the ultimo loss ratio (from historical loss ratios) and thus get the total claims cost, then split that cost into paid the first year and claims reserve, by using a beta distribution for the proportion paid the first year, see their Equation (2.27). Yet another possibility is to simulate frequency and severity separately.

Since we condition on $\mathcal{D}_0$, $\tilde{U}^0$ is non-random. Similar to the case with the claims reserve, we suggest that $\tilde{U}^1$ is computed by the same rule as $\tilde{U}^0$, but on the data including the simulated year. (That rule might be similar to a liability adequacy test.)

Until the new accounting rules are clear, one might choose to stick to the simplified premium risk in (3.1).

[3]In fact, the name *premium reserve* is not really adequate under this interpretation; indeed, we might just have one reserve for all liabilities, where the former premium reserve corresponds to an extra origin year in the claims reserve, with yet no observation. For convenience, we stick to the term *premium reserve* in this paper, though.

4 Risk margins and discounting

In the Solvency II frame-work as well as the IASB discussion paper on the forthcoming accounting standards IFRS 4, phase II, reserves are discounted by a yield curve of risk-free interest rates. Furthermore, a *risk margin* is added to the reserves. In this section, we shall investigate the relation between the risk margin and the one-year insurance risk. The reader should note that this section is tentative and not in all parts based on our own risk modelling practice.

For reserve risk, let $R_d^t; t = 0, 1$ be the discounted best estimate and $M_d^{Rt}; t = 0, 1$ the discounted risk margin of the opening and closing claims reserve, respectively. Then the discounted run-off result with risk margin is

$$T_d^R = (R_d^0 + M_d^{R0}) - C^1 - (R_d^1 + M_d^{R1}) \,, \tag{4.1}$$

where C^1 is the payments during year 1.

For premium risk, let $\tilde{R}_d^1$ be the discounted claims reserve for the current origin year and $M_d^{\tilde{R}1}$ the corresponding discounted risk margin. Furthermore, let $U_d^t; t = 0, 1$ be the discounted premium reserve and $M_d^{Ut}; t = 0, 1$ the corresponding discounted risk margin. Then the discounted technical result for the current year, including risk margins, is

$$T_d^P = (U_d^0 + M_d^{U0}) + P - E - (\tilde{C}^1 + \tilde{R}_d^1 + M_d^{\tilde{R}1}) - (U_d^1 + M_d^{U1}) \,. \tag{4.2}$$

By adding (4.1) and (4.2) we get the result for the segment, from which we find the risk of the segment, conditional on $\mathcal{D}_0$.

Note that, due to discounting, the expected result from any segment would tend to be negative; a remedy is to add an *investment income transferred from financial operations*, called I here. In our opinion, this quantity should be determined to meet the discounting, so that if all other things were equal (in particular the cash-flows equal their expected values), the run-off result would be zero.

Conceptually the risk margin is the extra amount, besides the expected value, a third party would demand for a transfer of the liabilities. (Note that the risk margin is sometimes called the *market value margin.*) If reserves where traded on a liquid market, the probability distribution of any risk margin could be estimated from observations of that market. In reality such markets do not exist and some proxy must be used. In the Solvency II preliminaries, the preferred method is *Cost-of-Capital* (CoC), which is also one of the options set up by IASB (2007) for IFRS 4, phase II.

In QIS3, the risk margin is computed per segment and diversification effects between segments are not taken into account when aggregating; we will use the same approach. Within segments, we will assume that only one risk margin is computed, *i.e.* diversification effects between the two reserves *are* taken into account. If separate margins are still required, these can be derived by some special method for allocating the diversification effect to the two reserves, such as the Shapley method, see e.g. Land, M., Vogel, C. & Gefeller, O. (2001).

Technically this means that we use a combined risk margin $M_d^t \doteq M_d^{Rt} + M_d^{Pt}$ for the premium reserve and claims reserve.

Now, by adding (4.1) and (4.2) we get

$$T_d = (U_d^0 + R_d^0 + M_d^0) + P + I - E - C^1 - (U_d^1 + R_d^1 + \tilde{R}_d^1 + M_d^1)\,, \quad (4.3)$$

where we have chosen to let C^1 now denote *all* payments made during year 1. In the CoC method, M_d^t is the cost of the solvency capital required for running off the liabilities completely, *i.e.* of the sum of the consecutive capital amounts required to run the business for each run-off year until the ultimate year. Since that capital for each year is the risk in the one-year result, it may look as though there is a circular reference here: the risk depends on the risk margin while the risk margin depends on the risk. However, this is not the case, as will be shown in the next subsection, where we go into details for the CoC approach.

4.1 The CoC method and the one-year risk perspective

In theory, the CoC method requires a risk calculation for all years until complete run-off, so the time scale will now be $t = 0, 1, 2, \ldots, n$, where n is the ultimate year of the run-off of the company's liabilities at time $t = 0$.

Denote the collection of the random variables that are observed up to and including calendar year t by $\mathcal{D}_t$. This generalizes our previous notation $\mathcal{D}_0$ denoting the history up to the point in time at which we do our calculations. To compute the risk margin, we consider the company to be in run-off at the beginning of year 1; in particular, no premiums are written. Let $\hat{R}_d^t$ be the discounted best estimate of the *entire* liabilities for the segment at the closing of year t; M_d^t is the corresponding risk marginal.

The run-off result T_d^t for year t is

$$T_d^t = \hat{R}_d^{t-1} + M_d^{t-1} + I - C^t - \hat{R}_d^t - M_d^t \,. \tag{4.4}$$

Note that the payments C^t during the year now relate to the contractual liabilities only, since there are no new policies written in the run-off situation.

Let VaR(L) denote the Value-at-Risk for the loss L at the chosen level — in Solvency II the level is 99.5%, and so VaR is the 99.5% quantile of the loss distribution: if L is continuous then $Pr\{L \leq \mathrm{VaR}(L)\} = 99.5\%$. Let SCR^{t-1} denote the solvency capital set up at the closing of year $t-1$, required to run the business during year t.

Let us try to unwind this risk backwards. Since liabilities are completely run off by year n, $\hat{R}_d^n = M_d^n = 0$ and

$$\begin{aligned} T_d^n &= \hat{R}_d^{n-1} + M_d^{n-1} + I - C^n; \\ SCR^{n-1}|\mathcal{D}_{n-1} &= \mathrm{VaR}(-T_d^n|\mathcal{D}_{n-1}) \\ &= \mathrm{VaR}(C^n|\mathcal{D}_{n-1}) - \hat{R}_d^{n-1} - M_d^{n-1} - I\,, \end{aligned}$$

where we have used the fact that $\hat{R}_d^{n-1}$ and M_d^{n-1} are non-random when we condition on $\mathcal{D}_{n-1}$. Note that, since the best estimate is unbiased, and I is the capital income that meets the discounting, we have $E(T_d^n) = M^{n-1}$, which is exactly the cost of providing the capital SCR^{n-1}, so that after this capital has been withdrawn, the expectation is zero. From now on, we draw this annual CoC from the one-year result. In the current case of year n, the result is

$$SCR^{n-1}|\mathcal{D}_{n-1} = \mathrm{VaR}(-T_d^n|\mathcal{D}_{n-1}) = \mathrm{VaR}(C^n|\mathcal{D}_{n-1}) - \hat{R}_d^{n-1} - I\,, \quad (4.5)$$

where I is not the same amount as before, but still defined to meet the discounting for the terms left in the equation.

For $t = 1, 2, \ldots, n-1$, first note that the risk margin in the accounts at $t-1$ is the one we expect to need at the closing of year t, plus the CoC for year t, *i.e.*

$$M^{t-1} = E(M^t|\mathcal{D}_{t-1}) + \alpha\, SCR^{t-1}\,, \quad (4.6)$$

where α is the CoC rate above risk-free interest rate, often chosen to be 6%. The term $\alpha\, SCR^{t-1}$ is again drawn from the result, as discussed above. Then

$$\begin{aligned} &SCR^{t-1}|\mathcal{D}_{t-1} = \mathrm{VaR}(-T^t|\mathcal{D}_{t-1}) = \quad (4.7) \\ &= \mathrm{VaR}(C^t + \hat{R}_d^t + M_d^t|\mathcal{D}_{t-1}) - \hat{R}_d^{t-1} - E(M_d^t|\mathcal{D}_{t-1}) - I\,. \end{aligned}$$

Together with (4.6), we get an equation system that is, in principle, solvable by backwards recursion, where the starting value is the SCR^{n-1} that was found in (4.5).

This shows that there really is no circular reference in letting the risk margin enter into the SCR calculation, as claimed earlier. However, solving the above equations is impracticable, even with simulation: let us look at the next-to-last year and try to find SCR^{n-2}:

$$\begin{aligned} SCR^{n-2}|\mathcal{D}_{n-2} &= \mathrm{VaR}[C^{n-1} + \hat{R}_d^{n-1} + \alpha\,\mathrm{VaR}(-T^n|\mathcal{D}_{n-1})|\mathcal{D}_{n-2}] \\ &- \hat{R}_d^{n-2} - \alpha\,E[\mathrm{VaR}(-T^n|\mathcal{D}_{n-1})] - I\,. \end{aligned}$$

If the computation of $\mathrm{VaR}(-T^n|\mathcal{D}_{n-1})$ requires simulation, then for each outcome of the random variables in year $n-1$, we must do a complete simulation of year n. So if we use B simulations in general, we here need $B \times B$ iterations. Going one more year backwards requires $B \times B \times B$ simulations, etc. This calls for simplification.

4.1.1 Simplified CoC method, using duration

A common simplification of the CoC calculation is to assume that the SCR in each year is run off at the same expected rate as the reserve. For some c, the risk margin before discounting is then $M^0 = \alpha \times c \times SCR^0$. Here the CoC rate α is fixed (say 6%) and the calculation of SCR^0 will be discussed below. The factor c is the grossing up factor for all future SCR. It is assumed that there is a known payment pattern $p_1, p_2, \ldots, p_n$, with $\sum_t p_t = 1$; in practice this would be estimated from the claims triangle, but it is not subject to random fluctuation in this model. Since the capital required in any year t is the sum of all future SCR, it is readily seen that

$$c = \sum_{t=1}^{n} \sum_{s=t}^{n} p_s = \sum_{s=1}^{n} \sum_{t=1}^{s} p_s = \sum_{s=1}^{n} s p_s\,, \tag{4.8}$$

which is simply the (average) duration of the reserve[4]. Hence another way of looking at this well-known approximation of the SCR needed for CoC is to say that we are holding the entire initial SCR for as long as the (average) duration of the liabilities.

The simplified approach implies that M_d^t is not stochastic and so the two terms containing it in (4.7) cancel and we could leave the risk

[4]There may be a difference of 0.5, if we assume payments to be made at July 1.

margin out of the SCR-calculation, as is done in QIS3. The initial SCR^0 that we need for the CoC calculation would then result from

$$SCR^0 = \mathrm{VaR}(C^1 + \hat{R}_d^1) - \hat{R}_d^0 - I\,, \tag{4.9}$$

where we have deliberately used a notation without the default conditioning on the claims history $\mathcal{D}_0$. The simulation approach to this kind of risk calculation was discussed in the first two sections of the paper. This finishes the specification of the simplified CoC approach.

Note. While the risk margin can be left out of the SCR-calculation, it must be taken into account when calculating the own funds in the balance sheet, since it is a fundamental part of the approximation of a market value of the reserves. □

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