Most elegant Premium Formulas for the most general Drop Down Excess of Loss Cover

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Abstract

The most general version of the Drop Down Excess-of-Loss Cover is considered. For that an elegant net-premium theory is derived, giving most elegant net-premium formulas. The general results are also specialized to three most important special treaties. The advantage of the new results (to the former ones of Kremer (2005a)) is that certain summations are finite instead of being infinite.

keywords: Net premium, drop down excess-of-loss cover

1 Introduction.

Nowadays reinsurance mathematics is one of the biggest fields of mathematical risk theory; one of its most important topics being the calculation of an adequate risk premium for a non-proportional reinsurance cover. So elegant premium theories were developed for all standard treaties. The author himself developed a comprehensive premium theory for generalizations of the classical largest claims reinsurance cover. For an introduction into it see e.g. Kremer (2004). In one of his basic papers the author went still a little bit further. He used in that paper (see Kremer (1985)) a more generalized model for the reinsurance cover than is taken in e.g. Kremer (2004). That generalization would not bring a lot more, in case there would not exist a practical treaty, fitting into it, and not fitting into his 2004-context. Such a miscellaneous treaty is the so-called Drop-Down-Excess-of-Loss Cover, that is subject of the present paper. But what about its premium? Not very elegant results on its premium are given in Kremer (2005a). For a quite special type very elegant premium-results were given by the author also in 2005 (see Kremer (2005b)). The question appears whether those elegant results can be extended to the general Drop-Down-Excess-of-Loss Cover. The perfect answer to that question is given in the following paper. Very elegant net-premium formulas are derived for the most general case.

2 The treaty.

Let the random variables $X_1, X_2, X_3, \ldots$ be the claims of a collective of risks and denote with N the random variable of the number of claims of the collective of risks. Suppose all is based on the probability space $(\Omega, \mathcal{A}, P)$.

Now let the claims be ordered in nonincreasing size, what shall be described by the

random variables:

$$X_{N:1} \geq X_{N:2} \geq \cdots \geq X_{N:N}.$$

Furthermore let $(f_i, i \geq 1)$ be a family of measurable mappings

$$f_i \colon [0, \infty) \to \mathbb{R}$$

with:

$$\begin{aligned} f_i(0) &= 0 \\ \sum_{i=1}^{N} f_i(y_i) &\in [0, \sum_{i=1}^{n} y_i] \\ &\text{for all } y_1 \geq y_2 \geq \cdots \geq y_n \geq 0. \end{aligned}$$

Obviously the random variable

$$R = \sum_{i=1}^{N} f_i(X_{N:i})$$

can be interpreted as the part of the total claims amount

$$S = \sum_{i=1}^{N} X_i$$

taken by a reinsurer. Consequently the family $(f_i, i \geq 1)$ defines a reinsurance treaty, called by the author in 1985 just **reinsurance treaty based on ordered claims**. A big class of such treaties is given for the more special structure

$$f_i(x) = c_i \cdot h(x)$$

with coefficients $c_1, c_2, \ldots$ and a measurable function

$$h \colon [0, \infty] \to \mathbb{R}.$$

In about 20 years the author developed for this general type of treaties a comprehensive risk theory (see for a survey Kremer (2004)). Examples of these treaties are outstanding the (classical) **Excess-of-Loss Cover**, the (classical) **Largest Claims Reinsurance Cover** and to older so-called **ECOMOR** (see for this again Kremer (2004)). Of interest are surely also practical treaties based on ordered claims that

are not contained in that bigger risk theory of the so-called **generalized largest claims reinsurance covers** (=treaties in Kremer (2004)). One such treaty is just the so-called **Drop-Down-Excess-of-Loss Cover**. For that one has with given priorities π_i, $i \geq 1$:

$$f_i(x) = \max(x - \pi_i, 0),$$

giving as its reinsurers claims amount:

$$R = \sum_{i=1}^{N} \max(X_{N:i} - \pi_i, 0). \tag{2.1}$$

Nearlying it is to assume:

$$\pi_1 \geq \pi_2 \geq \pi_3 \geq \dots . \tag{2.2}$$

Denote the resulting treaty in short by **DDXL**. Surely it is also nearlying to assume:

$$\pi_i = \pi_{k+1}, \quad \text{for } i = k+1, k+2, \dots . \tag{2.3}$$

Often one will have

a) numbers $s_1, \dots, s_k \in \mathbb{N}$

b) priorities $P_1 \geq P_2 \geq \dots \geq P_{k+1} \geq 0$

such that for:

$$t_i = \sum_{j=1}^{i} s_j$$

one has

$$\begin{aligned}
&\pi_i = P_1, \quad \forall\, i \leq t_1 \\
&\pi_i = P_2, \quad \forall\, t_1 + 1 \leq i \leq t_2 \\
&\quad \vdots \\
&\pi_i = P_j, \quad \forall\, t_{j-1} + 1 \leq i \leq t_j \\
&\quad \vdots \\
&\pi_i = P_{k+1}, \quad \forall\, i \geq t_k + 1.
\end{aligned} \tag{2.4}$$

Note that with $s_i = 1, \forall i$ one arrives again at (2.1)-(2.3). Under (2.4) one gets as the reinsurers claims amount $R =: R_{\text{DDXL}}$:

$$R_{\text{DDXL}} = \sum_{j=1}^{k} \sum_{i=t_{j-1}+1}^{t_j} \max(X_{N:i} - P_j, 0) + \sum_{i=t_k+1}^{N} \max(X_{N:i} - P_{k+1}, 0)$$

(with convention $t_0 := 0$). The corresponding cover can be regarded as being the **most general DDXL**.

It generalizes the following special cases, that are all of certain practical importance.

Example 1:

First take the case $k = 1$. With $t := t_1$ one has for the reinsurers claims amount:

$$R_{\text{DDXL}} = \sum_{i=1}^{t} \max(X_{N:i} - P_1, 0) + \sum_{i=t+1}^{N} \max(X_{N:i} - P_2, 0).$$

This is just the treaty, considered in Kremer (2005b).

Example 2:

Now take the case $k = 2$. One gets here:

$$R_{\text{DDXL}} = \sum_{i=1}^{t_1} \max(X_{N:i} - P_1, 0) + \sum_{i=t_1+1}^{t_2} \max(X_{N:i} - P_2, 0) + \sum_{i=t_2+1}^{N} \max(X_{N:i} - P_3, 0).$$

Example 3:

Finally take all $s_j = 1$, meaning that:

$$t_i = i.$$

One directly gets as the reinsurers claims amount:

$$R_{\text{DDXL}} = \sum_{j=1}^{k} \max(X_{N:i} - P_j, 0) + \sum_{j=k+1}^{N} \max(X_{N:i} - P_{k+1}, 0).$$

This is just (2.1) with (2.3).

3 Most basic result.

The topic of the present paper is the investigation of the **net premium** (on premiums in general see e.g. Kremer (1999)). The net premium of the most general DDXL is just:

$$m = E(R_{\mathrm{DDXL}}).$$

For deriving most elegant results on m, one can use results of Kremer (2003). There was considered a mixture between the (classical) excess-of-loss cover and the (classical) largest claims reinsurance treaty. With a given number t and a priority $\pi \geq 0$ its reinsurers claims amount is defined as:

$$R_1(\pi, t) = \sum_{i=1}^{t} \max(X_{N:i} - \pi, 0).$$

Denote this treaty in short with $\mathbf{XLLC}(t, \pi)$ and its net premium by

$$\nu(\pi, t) = E(R_1(\pi, t)).$$

Now remember the (classical) **excess-of-loss cover** with given priority $\pi \geq 0$ (in short $\mathbf{XL}(\pi)$). Its reinsurers claims amount is defined as:

$$R_2(\pi) = \sum_{i=1}^{N} \max(X_i - \pi, 0).$$

Denote its net premium by:

$$\mu(\pi) = E(R_2(\pi)).$$

With certain longer thinking one finds out that it holds true:

$$\begin{aligned} R_{\mathrm{DDXL}} = R_2(P_{k+1}) &- R_1(P_{k+1}, t_k) + R_1(P_k, t_k) \\ &- R_1(P_k, t_{k-1}) + R_1(P_{k-1}, t_{k-1}) \\ &\vdots \\ &- R_1(P_2, t_1) + R_1(P_1, t_1). \end{aligned}$$

This can directly be rewritten as:

$$R_{\mathrm{DDXL}} = R_2(P_{k+1}) - \sum_{j=1}^{k} R_1(P_{j+1}, t_j) + \sum_{j=1}^{k} R_1(P_j, t_j).$$

The reinsurers claims amount of the DDXL can obviously be split up into sums and differences of the claims amounts of a XL($\cdot$) and diverses XLLC($\cdot,\cdot$).
This result is most basic for the present paper, since it implies at once the important formula:

$$m = \mu(P_{k+1}) - \sum_{j=1}^{k} \nu(P_{j+1}, t_j) + \sum_{j=1}^{k} \nu(P_j, t_j). \tag{3.1}$$

4 General result.

In the above context let $Y_{ji}, i = 1, \ldots, N_j$ be the excess claims $(X_i - P_j)$ with $X_i > P_j$. N_j is the claims number of these excess claims.
Now assume for this section that:

(A) $X_1, X_2, X_3, \ldots$ are identically distributed.

(B) $N, X_1, X_2, \ldots$ are independent.

(C) $N_j, Y_{j1}, Y_{j2}, \ldots$ are independent (for each j).

Furthermore suppose that the reinsurer knows

(i) the mean claims number $\lambda = E(N)$ of the collective.

(ii) for a given $a \in (0, P_{k+1})$ the probability

$$q = P(X_i > a),$$

that a claim exceeds the a.

(iii) the distribution function G of the conditional distribution of the X_i, given the event $\{X_i > a\}$:

$$G(x) = P(X_i \leq x | X_i > a).$$

It shall hold:

$$q > 0 \quad \text{and } G(P_1) < 1.$$

One has the following main result:

Theorem 1:

In the above context with assumptions (A)-(C) and (i)-(iii) one gets for the net premium of the DDXL:

$$\begin{aligned} m = (\lambda \cdot q) \cdot & \int\limits_{[P_{k+1},\infty)} (x - P_{k+1}) G(dx) - \\ & - \sum_{i=1}^{t_k} \left(\frac{1}{q_{k+1}^i \Gamma(i)} \right) \cdot \int\limits_0^{q_{k+1}} G^{-1}(1-t) \cdot t^{i-1} \cdot M_{k+1}^{(i)}(1 - 1/q_{k+1}) dt + \\ & + \sum_{j=1}^{k} \sum_{i=t_{j-1}+1}^{t_j} \left(\frac{1}{q_j^i \Gamma(i)} \right) \cdot \int\limits_0^{q_j} G^{-1}(1-t) \cdot t^{i-1} \cdot M_j^{(i)}(1 - 1/q_j) dt + \\ & + P_{k+1} \cdot \sum_{i=1}^{t_k} \frac{1}{\Gamma(i)} \cdot \int\limits_0^1 t^{i-1} \cdot M_{k+1}^{(i)}(1-t) dt - \\ & - \sum_{j=1}^{k} P_j \cdot \sum_{i=t_{j-1}+1}^{t_j} \frac{1}{\Gamma(i)} \cdot \int\limits_0^1 t^{i-1} M_j^{(i)}(1-t) dt \end{aligned}$$

with:

$$q_j = 1 - G(P_j)$$

$$\Gamma(i) = (i-1)!$$

and the i-th derivative $M_j^{(i)}$ of the probability generating function:

$$M_j(t) = \sum_{m=0}^{\infty} P(N_j = m) \cdot t^m \quad (j = 1, \ldots, k+1).$$

G^{-1} denotes the pseudo-inverse of G:

$$G^{-1}(u) = \inf\{x : G(x) \geq u\}.$$

Proof. It is well-known that

$$\mu(P_{k+1}) = (\lambda \cdot q) \cdot \int\limits_{[P_{k+1},\infty)} (x - P_{k+1}) G(dx).$$

Furthermore one knows from theorem 2 in Kremer (2003) that:

$$\nu(P_j,t) = \sum_{i=1}^{t}\left(\frac{1}{q_j^i\Gamma(i)}\right)\cdot\int_0^{q_j} G^{-1}(1-t)\cdot t^{i-1}M_j^{(i)}(1-t/q_j)dt$$
$$- P_j\cdot\sum_{i=1}^{t}\left(\frac{1}{\Gamma(i)}\right)\cdot\int_0^{1} t^{i-1}\cdot M_j^{(i)}(1-t)dt.$$

One inserts these formulas into the rhs of (3.1) and gets:

$$m = (\lambda\cdot q)\cdot\int_{[P_{k+1},\infty)}(x-P_{k+1})G(dx)$$
$$-\sum_{j=1}^{k}\sum_{i=1}^{t_j}\left(\frac{1}{q_{j+1}^i\Gamma(i)}\right)\cdot\int_0^{q_{j+1}} G^{-1}(1-t)\cdot t^{i-1}M_{j+1}^{(i)}(1-t/q_{j+1})dt$$
$$+\sum_{j=1}^{k}\sum_{i=1}^{t_j}\left(\frac{1}{q_j^i\Gamma(i)}\right)\cdot\int_0^{q_j} G^{-1}(1-t)\cdot t^{i-1}M_j^{(i)}(1-t/q_j)dt$$
$$+\sum_{j=1}^{k}P_{j+1}\cdot\sum_{i=1}^{t_j}\frac{1}{\Gamma(i)}\cdot\int_0^{1} t^{i-1}M_{j+1}^{(i)}(1-t)dt$$
$$-\sum_{j=1}^{k}P_j\cdot\sum_{i=1}^{t_j}\frac{1}{\Gamma(i)}\cdot\int_0^{1} t^{i-1}\cdot M_j^{(i)}(1-t)dt$$

With certain subtractions one arrives at the result of the theorem. □

For illustration the following special cases.

Example 1 (continued):

For the special DDXL of example 1 one gets from Theorem 1 for its net premium:

$$m = (\lambda\cdot q)\cdot\int_{[P_2,\infty)}(x-P_2)G(dx)-$$
$$-\sum_{i=1}^{t}\left(\frac{1}{q_2^i\Gamma(i)}\right)\cdot\int_0^{q_2} G^{-1}(1-t)\cdot t^{i-1}M_2^{(i)}(1-t/q_2)dt+$$

$$+\sum_{i=1}^{t}\left(\frac{1}{q_1^i\Gamma(i)}\right)\cdot\int_0^{q_1} G^{-1}(1-t)\cdot t^{i-1}M_1^{(i)}(1-t/q_1)dt+$$

$$+P_2\cdot\sum_{i=1}^{t}\frac{1}{\Gamma(i)}\cdot\int_0^{1} t^{i-1}M_2^{(i)}(1-t)dt-$$

$$-P_1\cdot\sum_{i=1}^{t}\frac{1}{\Gamma(i)}\cdot\int_0^{1} t^{i-1}M_1^{(i)}(1-t)dt.$$

This is just the result of theorem 1 in Kremer (2005b).

Example 2 (continued):

In case of example 2 one has:

$$m=(\lambda\cdot q)\cdot\int_{[P_3,\infty)}(x-P_3)G(dx)-$$

$$-\sum_{i=1}^{t_2}\left(\frac{1}{q_3^i\Gamma(i)}\right)\cdot\int_0^{q_3} G^{-1}(1-t)\cdot t^{i-1}M_3(1-t/q_3)dt+$$

$$+\sum_{i=t_1+1}^{t_2}\left(\frac{1}{q_2^i\Gamma(i)}\right)\cdot\int_0^{q_2} G^{-1}(1-t)\cdot t^{i-1}M_2(1-t/q_2)dt+$$

$$+\sum_{i=1}^{t_1}\left(\frac{1}{q_1^i\Gamma(i)}\right)\cdot\int_0^{q_1} G^{-1}(1-t)\cdot t^{i-1}M_1^{(i)}(1-t/q_1)dt+$$

$$+P_3\cdot\sum_{i=1}^{t_2}\frac{1}{\Gamma(i)}\cdot\int_0^{1} t^{i-1}M_3^{(i)}(1-t)dt-$$

$$-P_2\cdot\sum_{i=t_1+1}^{t_2}\frac{1}{\Gamma(i)}\cdot\int_0^{1} t^{i-1}M_2^{(i)}(1-t)dt-$$

$$-P_1\cdot\sum_{i=1}^{t_1}\frac{1}{\Gamma(i)}\cdot\int_0^{1} t^{i-1}M_1^{(i)}(1-t)dt.$$

Example 3 (continued):

Finally for example 3 one gets as net-premium formula for the DDXL:

$$
\begin{aligned}
m = (\lambda \cdot q) \cdot & \int\limits_{[P_{k+1},\infty)} (x - P_{k+1}) G(dx) - \\
& - \sum_{i=1}^{k} \left(\frac{1}{q_{k+1}^{i} \Gamma(i)} \right) \cdot \int\limits_{0}^{q_{k+1}} G^{-1}(1-t) \cdot t^{i-1} M_{k+1}^{(i)}(1 - t/q_{k+1}) dt + \\
& + \sum_{i=1}^{k} \left(\frac{1}{q_{i}^{i} \Gamma(i)} \right) \cdot \int\limits_{0}^{q_{i}} G^{-1}(1-t) \cdot t^{i-1} M_{i}^{(i)}(1 - t/q_{i}) dt + \\
& + P_{k+1} \cdot \sum_{i=1}^{k} \frac{1}{\Gamma(i)} \cdot \int\limits_{0}^{1} t^{i-1} M_{k+1}^{(i)}(1-t) dt - \\
& - \sum_{i=1}^{k} P_i \cdot \frac{1}{\Gamma(i)} \cdot \int\limits_{0}^{1} t^{i-1} M_{i}^{(i)}(1-t) dt.
\end{aligned}
$$

5 Special results.

Take the context of section 4 with more special:

(D) N_j is **Poisson-distributed** with mean $\lambda_j > 0$, i.e.:

$$P(N_j = n) = \frac{\lambda_j^n}{n!} \cdot \exp(-\lambda_j), \quad (j = 1, 2, \ldots, k+1)$$

(E) G is **generalized Pareto-distributed** with threshold a and parameters $g \in (0,1)$, $s > 0$, i.e.:

$$G(x) = 1 - \left(1 + (x - a) \cdot \left(\frac{g}{s} \right) \right)^{-1/g}, \quad \text{for } x \geq a.$$

Remember, that this model was first introduced in reinsurance mathematics by the author in 1986 (see Kremer (1986), (1998)).

It is obvious that the following holds true:

$$\lambda_j = \lambda \cdot q \cdot q_j$$

with:

$$q_j = \left(1 + (P_j - a) \cdot \left(\frac{g}{s}\right)\right)^{-1/g}.$$

As the most important result of the present paper one gets the following:

Theorem 2:

Under the conditions of section 4 with additionally the conditions (D), (E) one gets for the net premium of the DDXL:

$$\begin{aligned} m = (\lambda q)) \cdot \left(\frac{s}{1-g}\right) \cdot \left(1 + (P_{k+1} - a) \cdot \left(\frac{g}{s}\right)\right)^{1-1/g} - \\ - (\lambda q)^g \cdot \left(\frac{s}{g}\right) \cdot \left[\sum_{i=1}^{t_k} \frac{\Gamma_{\lambda_{k+1}}(i-g)}{\Gamma(i)} - \sum_{j=1}^{k} \sum_{i=t_{j-1}+1}^{t_j} \frac{\Gamma_{\lambda_j}(i-g)}{\Gamma(i)}\right] + \\ + \left(P_{k+1} - \left(a - \left(\frac{s}{g}\right)\right)\right) \cdot \sum_{i=1}^{t_k} \frac{\Gamma_{\lambda_{k+1}}(i)}{\Gamma(i)} - \\ - \sum_{j=1}^{k} \left(P_j - \left(a - \left(\frac{s}{g}\right)\right)\right) \cdot \sum_{t_{j-1}+1}^{t_j} \frac{\Gamma_{\lambda_j}(i)}{\Gamma(i)}. \end{aligned}$$

with the incomplete gamma function:

$$\Gamma_\lambda(s) = \int_0^\lambda t^{s-1} \exp(-t) dt.$$

Proof. The result follows with routine calculations from Theorem 1. More concretely one gets with here

$$\begin{aligned} G^{-1}(u) &= (1-u)^{-g} \cdot \left(\frac{s}{g}\right) + \left(a - \left(\frac{s}{g}\right)\right) \\ M_j^{(i)}(t) &= \lambda_j^i \cdot \exp(\lambda(t-1)) \end{aligned}$$

after standard manipulations:

$$\int_0^1 t^{i-1} M_j^{(i)}(1-t) dt = \Gamma_{\lambda_j}(i)$$

$$\int_0^{q_j} G^{-1}(1-t) \cdot t^{i-1} \cdot M_j^{(i)}(1 - t/q_j)) dt =$$

$$= \lambda_j^g \cdot q_j^{i-g} \cdot \left(\frac{s}{g}\right) \cdot \Gamma_{\lambda_j}(i-g) + q_j^i \left(a - \left(\frac{s}{g}\right)\right) \Gamma_{\lambda_j}(i).$$

Finally one gets from Kremer (1986):

$$\int\limits_{[P_{k+1},\infty)} (x - P_{k+1})G(dx) = \left(\frac{s}{1-g}\right)\left(1 + (P_{k+1} - a)\left(\frac{g}{s}\right)\right)^{1-1/g}.$$

□

Example 1 (continued):

Under the above conditions one gets as net-premium formula for the DDXL of example 1:

$$\begin{aligned} m = {} & (\lambda q) \cdot \left(\frac{s}{1-g}\right) \cdot \left(1 + (P_2 - a) \cdot \left(\frac{g}{s}\right)\right)^{1-1/g} \\ & - (\lambda q)^g \cdot \left(\frac{s}{g}\right) \cdot \left[\sum_{i=1}^{t} \frac{\Gamma_{\lambda_2}(i-g)}{\Gamma(i)} - \sum_{i=1}^{t} \frac{\Gamma_{\lambda_1}(i-g)}{\Gamma(i)}\right] \\ & + \left(P_2 - \left(a - \left(\frac{s}{g}\right)\right)\right) \cdot \sum_{i=1}^{t} \frac{\Gamma_{\lambda_2}(i)}{\Gamma(i)} \\ & - \left(P_1 - \left(a - \left(\frac{s}{g}\right)\right)\right) \cdot \sum_{i=1}^{t} \frac{\Gamma_{\lambda_1}(i)}{\Gamma(i)}. \end{aligned}$$

Example 2 (continued):

For the DDXL of example 2 one has under the conditions of Theorem 2:

$$\begin{aligned} m = {} & (\lambda q)) \cdot \left(\frac{s}{1-g}\right) \cdot \left(1 + (P_3 - a) \cdot \left(\frac{g}{s}\right)\right)^{1-1/g} - \\ & - (\lambda q)^g \cdot \left(\frac{s}{g}\right) \cdot \left[\sum_{i=1}^{t_2} \frac{\Gamma_{\lambda_3}(i-g)}{\Gamma(i)} - \sum_{i=t_1+1}^{t_2} \frac{\Gamma_{\lambda_2}(i-g)}{\Gamma(i)} - \sum_{i=1}^{t_1} \frac{\Gamma_{\lambda_1}(i-g)}{\Gamma(i)}\right] + \\ & + \left(P_3 - \left(a - \left(\frac{s}{g}\right)\right)\right) \cdot \sum_{i=1}^{t_2} \frac{\Gamma_{\lambda_3}(i)}{\Gamma(i)} - \\ & - \left(P_2 - \left(a - \left(\frac{s}{g}\right)\right)\right) \cdot \sum_{i=t_1+1}^{t_2} \frac{\Gamma_{\lambda_2}(i)}{\Gamma(i)} - \\ & - \left(P_1 - \left(a - \left(\frac{s}{g}\right)\right)\right) \cdot \sum_{i=1}^{t_1} \frac{\Gamma_{\lambda_1}(i)}{\Gamma(i)}. \end{aligned}$$

Example 3 (continued):

Finally take the DDXL of example 3. One arrives under the above conditions at:

$$
\begin{aligned}
m = (\lambda q)) \cdot \left(\frac{s}{1-g}\right) \cdot \left(1 + (P_{k+1} - a) \cdot \left(\frac{g}{s}\right)\right)^{1-1/g} - \\
- (\lambda q)^g \cdot \left(\frac{s}{g}\right) \cdot \left[\sum_{i=1}^{k} \frac{\Gamma_{\lambda_{k+1}}(i-g)}{\Gamma(i)} - \sum_{i=1}^{k} \frac{\Gamma_{\lambda_i}(i-g)}{\Gamma(i)}\right] + \\
+ \left(P_{k+1} - \left(a - \left(\frac{s}{g}\right)\right)\right) \cdot \sum_{i=1}^{k} \frac{\Gamma_{\lambda_{k+1}}(i)}{\Gamma(i)} - \\
- \sum_{i=1}^{k} \left(P_i - \left(a - \left(\frac{s}{g}\right)\right)\right) \cdot \frac{\Gamma_{\lambda_i}(i)}{\Gamma(i)}.
\end{aligned}
$$

Finally replace the asumption (E) through:

(E*) G is (classical) **Pareto-distributed** with threshold a and parameter $\alpha > 1$, i.e.

$$G(x) = 1 - \left(\frac{x}{a}\right)^{-\alpha}, \quad \text{for } x \geq a.$$

One gets as final result:

Corollary:

Under the conditions of section 4 with additionally the conditions (D) and (E*) one gets for the net premium of the general DDXL:

$$
\begin{aligned}
m = (\lambda \cdot q) \cdot a^{\alpha} \cdot \left(\frac{P_{k+1}^{1-a}}{\alpha - 1}\right) \\
- (\lambda q)^{1/\alpha} \cdot a \cdot \left[\sum_{i=1}^{t_k} \frac{\Gamma_{\lambda_{k+1}}(i - 1/\alpha)}{\Gamma(i)} - \sum_{j=1}^{k} \sum_{i=t_{j-1}+1}^{t_j} \frac{\Gamma_{\lambda_j}(i - 1/\alpha)}{\Gamma(i)}\right] \\
+ P_{k+1} \cdot \sum_{i=1}^{t_k} \frac{\Gamma_{\lambda_{k+1}}(i)}{\Gamma(i)} - \sum_{j=1}^{k} P_j \cdot \sum_{i=t_{j-1}+1}^{t_j} \frac{\Gamma_{\lambda_j}(i)}{\Gamma(i)}.
\end{aligned}
$$

Proof. The model in (E*) results from the model in (E) by putting $g = 1/\alpha$, $s = a \cdot g$. Consequently the results of the Corollary follow directly from the results in Theorem 3 by putting there $g = 1/\alpha$, $s = a \cdot g$. □

The specialization of this Corollary to the examples 1-3 the author leaves for the interested reader. In case of example 1 one gets the result of Theorem 2 in Kremer (2005b).

6 Final remark.

As far as the author knows, the Drop Down Excess of Loss Cover is not used in practice yet. Certainly it is quite senseful to offer lim on the reinsurance market. The above premium theory should encourage reinsurance companies to do this.

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