Measurement and Transfer of Catastrophic Risks. A Simulation Analysis

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Abstract

When analyzing catastrophic risk, traditional measures for evaluating risk, such as the probable maximum loss (PML), value at risk (VaR), Tail VAR (TVaR), and others, can become practically impossible to obtain analytically in certain types of insurance, such as earthquake. Given the available information it can be very difficult for an insurer to measure this risk. The transfer of risk in this type of insurance is usually done through reinsurance schemes that can be of diverse types that can greatly reduce the extreme tail of the cedant's loss distribution. This effect can be assessed mathematically. The PML is defined in terms of a very extreme quantile. Also, under standard operating conditions, insurers use several "layers" of non proportional reinsurance that will be combined with some type of proportional reinsurance. The resulting reinsurance structures will then be very complicated to analyze and to evaluate their mitigation or transfer effects analytically it may be necessary to use alternative approaches, such as Monte Carlo simulation methods. This is what we do in this paper in order to measure the effect of a complex reinsurance treaty on the risk profile of an insurance company. We compute the pure risk premium, PML as well as a host of results: impact on the insured portfolio, risk transfer effect of reinsurance programs, proportion of times reinsurance is exhausted, percentage of years it was necessary to contract reinstatements, etc. Since the estimators of quantiles are known to be biased, we explore the alternative of using an Extreme Value approach to complement the analysis.

KEY WORDS: Quantile, Extreme Value, Monte Carlo Methods, PML, VAR, Reinsurance.

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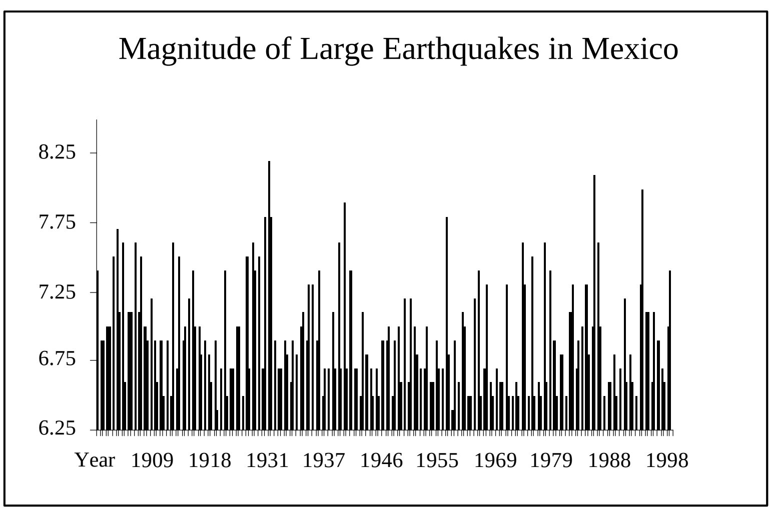
Introduction

The measurement and transfer of risk are at the essence of the insurance business. This has prompted the development of quantitative techniques to achieve both. They are important for the direct insurer, as well as for a potential reinsurer. In the case of catastrophic risk they become particularly relevant due to the magnitude of potential losses. A large earthquake or hurricane will impact losses in an extreme fashion such that if not adequately reserved it can cause the ruin of either the insurer or the reinsurer, with 'catastrophic' consequences for stockholders and society. Hence the importance of measuring risk and its transfer.

Mexico is a country with a large number of earthquakes per year. On average, there are 80 of magnitude larger than 4.3 every year. The available information and models allow us to analyze the information on earthquake intensities over the last 100 years. The National Seismological Service of Mexico, SSN (1999), has published the magnitudes of earthquakes larger than 6.5 on the Richter scale during the 20th century, see Figure 1.

Figure 1



It is clear that the correct evaluation of the potential catastrophic losses due to large earthquakes is of great importance. Mexico is one of the few countries that is also subject of large hurricane losses, in the Caribbean. However we focus on the former in this paper.

Catastrophic Risk Measurement

When analyzing catastrophic risk, traditional measures for evaluating risk, such as the probability of probable maximum loss (PML), valuate risk (VAR), Tail VAR, and others can become really impossible to obtain analytically in certain types of insurance, such as earthquake. The processes of earthquake generation, shock wave diffusion, damage to buildings, etc. are very complex and their interaction makes their analysis even more so. It is necessary to bring together the expertise of geophysicists, structure engineers, actuaries, financial experts and others in order to construct a model that represents the overall process reasonably well.

In many countries earthquake catastrophic risk is measured in terms on the probable maximum loss (PML), an extreme quantile of the corresponding loss distribution. Given the available information it can be very difficult for an insurer to measure this risk. However, since the distribution of losses due to earthquakes for a large portfolio of risks will usually be unknown the only way to quantify risk may be through simulation. That is the process we follow here.

The PML is defined as the losses y_0 that will be observed with a probability less than or equal to some previously agreed value, say *0.002. = 1/500*, so that *Pr{losses=PML} = 1/500*, the exceedance probability. However, the PML is usually expressed in terms of a return period, the time between events of a given magnitude, and this is defined as the inverse of the exceedance probability, i.e. Return period = *1/Pr{losses = PML} = 500*. Since these probabilities are usually very small they are more difficult to compute.

Risk Transfer

Because of the magnitude of potential losses, risk transfer in this type of insurance is usually done through reinsurance schemes that can be of diverse types. In terms of risk transfer or mitigation, it is well known that in proportional reinsurance (quota share), the insurer takes a proportion of every loss, so that if X_i is the random variable that represents gross losses then the losses net of the reinsurance is $Y = \alpha X$ where α is the retention rate. Alternatively non-proportional reinsurance (e.g. excess-loss) states that for every loss exceeding a specified threshold or priority (P), the reinsurer will pay the loss up to a certain limit (L), so that for each gross loss occurrence the direct insurer will pay only Max{0, min (X-P, L)}, Booth et al. (1999). This has the effect of truncating the loss distribution.

Non-proportional reinsurance can greatly reduce the extreme tail of the cedant's loss distribution. This effect can be assessed mathematically. If the PML is being defined in terms of a very extreme quantile we argue that in simple cases, and if there is a limit to the non-proportional reinsurance, the reduction in the PML can be very unstable, depending on the relation between the limit and the PML. Also, under standard operating conditions, insurers use several "layers" of non proportional reinsurance that will be combined with some type of proportional reinsurance. The resulting reinsurance structures will then be very complicated to analyze, Veerlak and Beirlant (2003). This is further complicated if the probability distribution of losses is not known analytically. In fact most of the literature on optimal reinsurance assumes it is known. Recently, Silvestrov et al. (2006) develop criteria for evaluating alternative reinsurance contracts that are large and mathematically complex. They use a Monte Carlo based approach.

Regulatory Considerations

It has been argued that it is impossible to measure the mitigation, or transfer of risk, effect of non proportional reinsurance and so it should not be given recognition for solvency assessment. Several reasons have been put forward, among others the following:

a) The difficulty of estimating their effect
b) The inclusion of aggregate limits
c) It will often be necessary to have "reinstatement" clauses

Nevertheless, proper recognition of reinsurance is necessary in order to assess the risk reduction for the ceding company. This has implications for capital requirements to ensure effective solvency supervision. The Insurer Solvency Assessment Working Party (ISAWP) of the International Actuarial Association (IAA), ISAWP (2004), states that

The ISAWP indicates that if applied properly to evaluate the solvency of a direct insurer, reinsurance is a very efficient means of reducing risk (particularly if measured by TVaR) and hence can be a useful alternative for (solvency) capital.

"*While proper treatments and recognition of reinsurance arrangements are necessary to assess the impact of the of a ceding company's risk profile, this is a difficult task for a number of reasons.*

The first complexity comes from the tremendous diversity in the types of reinsurance contracts:

- *Typical reinsurance arrangement comprise both proportional and non-proportional covers*
- *Some contracts have variable rating terms, ... for a proportional reinsurance treaty, and reinstatements or contingent commissions for an excess-of-loss treaty*
- *Some contracts cover just one line of business, others cover multiple lines of business ...*
- *Some contracts are on an aggregate basis, with aggregate deductibles and aggregate limits*
- *Some financial type reinsurance contracts cover a hybrid of underwriting and financial risks.*

The second complexity comes from the fact that many reinsurance contracts do not bear a linear relationship with the underlying risks."

The ISAWG further indicates that *“the proper evaluation of the risk reducing impact of non-proportional reinsurance contracts is still not possible without either relatively complex mathematical transformations, which are typically beyond the of supervisory control mechanisms, or the use of simulations, which are standard routines for more complex risk modelling in internal models.”*

In addition the ISAWP indicates that if applied properly to evaluate the solvency of a direct insurer, reinsurance is a very efficient means of reducing risk (particularly if measured by TVaR) and hence can be a useful alternative for capital.

Hence even when the reinsurance schemes are very sophisticated it becomes very complicated, if not impossible, to evaluate their mitigation or transfer effects analytically then it may be necessary to use alternative approaches, such as Monte Carlo simulation methods, Silvestrov et al. (2006). That is what we do in this paper in order to measure the effect of a complex reinsurance treaty on the risk profile of an insurance company.

The Model

The insurance regulatory body in Mexico (Comision Nacional de Seguros y Fianzas, CNSF) has commissioned the construction of an earthquake loss model that must be used to compute the pure risk premium as well as the PML[1]. These results are used to verify compliance with corresponding regulation and compute statutory reserves.

However the software produces additional output that can be used for simulation. Hence, based on the arguments put forth by international associations, such as the International Association of Insurer Supervisors (IAIS) and the International Actuarial Association (IAA), the latter through the ISAWP, that encourage the use of mathematical models and simulation methods, we have used this output and constructed a program that allows the actuary to generate the distributions of gross yearly losses for an insurance portfolio. The algorithm includes the possibility of simulating

a) the occurrence of earthquakes
b) their impact on the insured portfolio
c) the risk transfer effect of reinsurance programs that mix different types or reinsurance
d) descriptive statistics for, gross losses, and losses net of reinsurance
e) the proportion of times the reinsurance is exhausted
f) average cost per year of reinstatements
g) distribution of loss by reinsurance layer according to their magnitude
h) percentage of years it was necessary to contract additional reinstatements

The model consists of a series of sub-models corresponding to different aspects of the earthquake loss generation process. The initial component is earthquake occurrence. This is modeled as a spatial Poisson distribution for each of a number of potential seismic sites, i.e. space has been discretized in 3600 points. Then there is the distribution of earthquake magnitudes at each one of the sites. The exceedance rate for the *i-th* site is specified as:

[1] ERN Ingenieros Consultores, S.C. (2002), *Evaluación de Riesgo Sísmico*, Manual de Referencia.

$$n_i(y) = \int_{M_0}^{M_{ui}} -\frac{dl_i(M)}{dM} P(Y > y \mid M, f_i) dM = -\int_{M_0}^{M_{ui}} P(Y > y \mid M, f_i) dl_i(M) \qquad (1)$$

where $l_i(M)$ is the number of earthquakes of magnitude greater than M at source i. Here, $n_i(y)$ is the average number of events, by unit time, that produce losses larger than y at seismic source i, say f_i. Then the exceedance probability (Probability that losses will exceed a given value) for the whole portfolio is:

$$n(y) = \sum_{i=1}^{Nf} \int_{Mo}^{Mui} -\frac{dl_i(M)}{dM} \Pr(Y > y \mid M, f_i) \Pr(f_i) dM \qquad (2)$$

where M_{oi} and M_{ui} are the lower and upper bounds on the magnitudes at site i, respectively; and N_f is the number of sites. So that if $n(0) =$ average number of events by unit time, that produce losses greater than y, then the probability distribution of the proportion of losses for the whole portfolio, given an earthquake of magnitude M, at site i is

$$F(y \mid M, f_i) = \Pr(Y \le y \mid M, f_i) = 1 - \frac{n(y)}{n(0)} \ . \qquad (3)$$

There is one for each ‘site-magnitude’ combination. These distributions are produced by the software, ERN (2002). The corresponding densities are specified as:

$$f_B(b) = P_0 d(b) + (1 - P_0 - P_1) B(b; a, b \mid M, f_i) + P_1 d(b - 1) \qquad 0 \le b \le 1 \qquad (4)$$

where b = relative loss, as a proportion of the amount exposed, i.e. $b = \frac{Y}{S_{\exp}}$, Y are the losses, P_0 is the probability of zero losses, P_1 the probability of total losses, $Beta(y;a,b|M,f_i)$ refers to a beta density with parameters a,b, that are conditional on the ‘site-magnitude’ combination, and δ is the Dirac delta. These distributions are obtained using information on the construction characteristics for each insured building combined with shock wave diffusion and local effects from earthquakes at the given ‘site-magnitude’; they lead to a ratio damage distribution for each building. The individual loss proportion distributions are then aggregated over all the portfolio to obtain (4). For a detailed description see ERN (2003).

In very broad terms the simulation algorithm is as follows:

a) Choose an earthquake site at random
b) Given the site, generate a magnitude at random from the corresponding distribution
c) Use the distribution of proportion of losses for the site-magnitude combination to generate a random loss proportion (damage) for each insured building
d) Combine, using suitable assumptions, the individual proportion of losses into a loss proportion fro the portfolio
e) Multiply the proportion resulting in d) by the total value insured for the portfolio y obtain a loss amount.
f) Apply any reinsurance that is in effect.

This process is applied as many times as there are earthquakes in a year to derive a figure of total losses for a year. As many yearly replications are generated according to the required precision.

We apply the algorithm to (disguised) portfolio from a real Mexican insurance company. The portfolio consists of 25,000 buildings. The reinsurance scheme (in thousands of dollars) is as shown in Table 1:

Table 1

Layers	Priority	Cover	Reinstatement Premium	RoI	Reins
1	$ 7,500	$ 7,500	$ 1,586	21.15%	2
2	$ 15,000	$ 15,000	$ 1,890	12.60%	2
3	$ 30,000	$ 30,000	$ 2,268	7.56%	1
4	$ 60,000	$ 40,000	$ 1,548	3.87%	1
5	$ 100,000	$ 130,000	$ 2,574	1.98%	1
Superior	$ 230,000	None	NA	NA	NA

The insurance company also has a quota share with 10% retention for losses below 7,500, the priority. Applying the algorithm described above and through simple statistical analysis we evaluate the mitigation effect of the reinsurance contract. In Table 1 we show some statistics for the gross losses (without any reinsurance) and for losses net of all reinsurance.

Table 2

	MEAN	ST. D.	MINIMUM	Q1	MEDIAN	Q3	MAXIMUM
GROSS LOSS	$ 4,873	$ 18,881	$ 6	$ 754	$ 1,565	$ 3,682	$ 1,213,000
NET LOSS	$ 526	$ 5,970	$ 1	$ 75	$ 157	$ 368	$ 762,500
NET LOSS W.O. REINS	$ 516	$ 5,815	$ 1	$ 75	$ 157	$ 368	$ 752,600
RETENTION	10.80%	31.62%	10.00%	10.00%	10.00%	10.00%	62.86%

Table 3 shows gross and net losses for several return periods. This is relevant for complying with the regulatory authority with respect to solvency. Further analysis yields the results in Table 4, where we can see if the reinsurance strategy is what the company needs. One can evaluate that insurance was insufficient in only an extremely low percentage of the total number of years simulated (150,000). The retention levels after reinsurance are clearly smaller than would be attained under a quota share with 10% retention.

Table 3

Gross Losses	Net Losses	Net Losses without Reinstatement Premiums	% Reduction	Fn	Return Period
$ 300,710	$ 14,278	$ 9,094	95.25%	0.999333333	1500
$ 231,938	$ 10,968	$ 8,448	95.27%	0.999	1000
$ 143,763	$ 9,313	$ 7,827	93.52%	0.998	500
$ 90,040	$ 8,038	$ 7,587	91.07%	0.995	200
$ 78,011	$ 7,684	$ 7,537	90.15%	0.99	100

We also use the Monte Carlo results to show that for a large portfolio and a complicated reinsurance contract non-proportional reinsurance can be compared with the effect of proportional reinsurance, Figure 2[2].

Figure 2

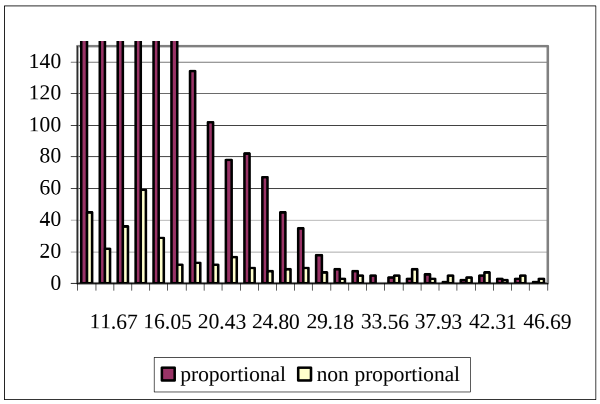


Table 4

Layers	% years all reinstatements were used	% years only one reinstatement was used	% years the second reinstatement was used	Distribution of events by layer
(1)	(2)	(3)	(4)	(5)
Priority	NA	NA	NA	99.9937%
1	0.00%	0.49%	0.00%	0.0040%
2	0.18%	0.18%	0.00%	0.0012%
3	0.09%	0.09%	NA	0.0002%
4	0.07%	0.07%	NA	0.0002%
5	0.05%	0.05%	NA	0.0002%
Sup	NA	NA	NA	0.0005%

[2] Insurer Solvency Assessment Working Party (2004), *A Global Framework for Insurer Solvency Assessment*, International Actuarial Association

Although the paper is written in the context of earthquake catastrophe insurance, it is applicable to others, such as hurricane, provided the hurricane model is available. It is shown how a relatively simple simulation model can provide a wealth of information not obtainable by analytic procedures. In fact, the Regulatory Authority has also commissioned a model to evaluate hurricane risk, along the lines of the earthquake model. We have begun to carry out simulation exercises similar to those presented here.

Extreme Value Analysis

The previous analysis provides a large amount of information to the insurer. Yet, there are some technical details that can be explored further. For example, it is know that quantile estimates obtained by simulation are biased, Inui et al. (2005). And the bias tends to zero as the sample size increases. We have used large sample sizes so that the bias should be small. Nevertheless we will carry out additional analyses in order to fine tune the results. In particular the quantile corresponding to the 1500 year return period, which must be used to compute the PML and hence required reserves for earthquake catastrophe risk.

Since PML estimation is essentially an exercise in estimating a large quantile it falls in the field of extreme values. We use the Peaks Over Threshold (POT) method and proceed along the lines set out in Embrechts et al. (1997). We assume that the losses from earthquakes are $X_1, X_2, \ldots, X_n$, i.i.d. with distribution $F(x)$, a Generalized Extreme Value Distribution (GEV). Then we choose a high threshold u, and so the corresponding excesses are denoted by $Y_i = X_i - u, \quad i = 1, \ldots, n,$; and N_u is the number of exceedances of u by $X_1, \ldots, X_n$. The exceedances follow a generalized Pareto distribution (GPD). This GPD, denoted by $G_{x,b}$, with parameters x Î IR and $b > 0$ has distribution tail

$$\overline{G}_{x,b}(x) = \begin{cases} \left(1 + x\dfrac{x}{b}\right)^{-1/x} & \text{if} \quad x \neq 0, \\ e^{-x/b} & \text{if} \quad x = 0, \end{cases} \qquad x \in D(x,b), \qquad (5)$$

where

$$D(x,b) = \begin{cases} [0,\infty) & \text{if} \quad x \geq 0, \\ [0, -b/x] & \text{if} \quad x < 0. \end{cases},$$

Embrechts et al. (1997). We must choose a high threshold u, although there are no clear criteria. It should not be too small so as to not produce biased estimators and it should not be too high because it will produce high variance estimators. With our sample of 150,000 we decided to use u = 66.22 (in million dollars), which yields N_u = 1000. This value has the added advantage that (empirically) $Prob\{ X > 66.22 \} = 1/150$, and can easily be used to compute our 1500 year return period quantile, as will be shown below. We used the program “ExtRemes” to carry out the analysis, Gilleland and Katz (2005). We then generate the sample of 1000 exceedances that used for fitting the tail of the GPD. Figure 3 shows the histogram, which indicates a good fit.

Figure 3

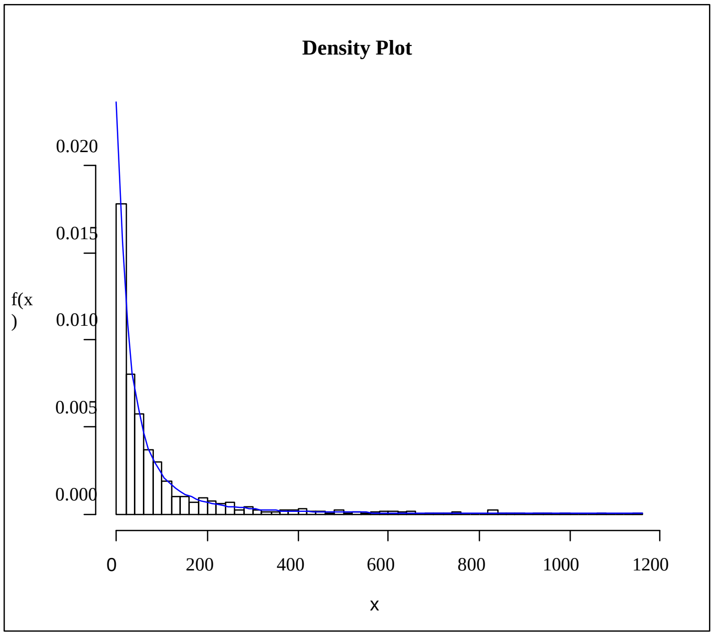


Figure 4 shows the corresponding fit diagnostics. Since we are fitting to the exceedances the two graphs that involve return periods must be viewed with care. In order to compare these results with those obtained by simulation we compute the required quantiles from the GPD. These are given by

$$\hat{x}_p = u + \frac{\hat{b}}{\hat{x}}\left(\left(\frac{n}{N_u}(1-p)\right)^{-\hat{x}} - 1\right), \tag{6}$$

Embrechts et al. (1997), where $\hat{b}$ *and* $\hat{x}$ are the parameter estimates. The value of *p* must be determined such that

$$\overline{F}(u+y) = \overline{F}(u)\cdot\overline{F}_u(y) = 1-p\,,$$

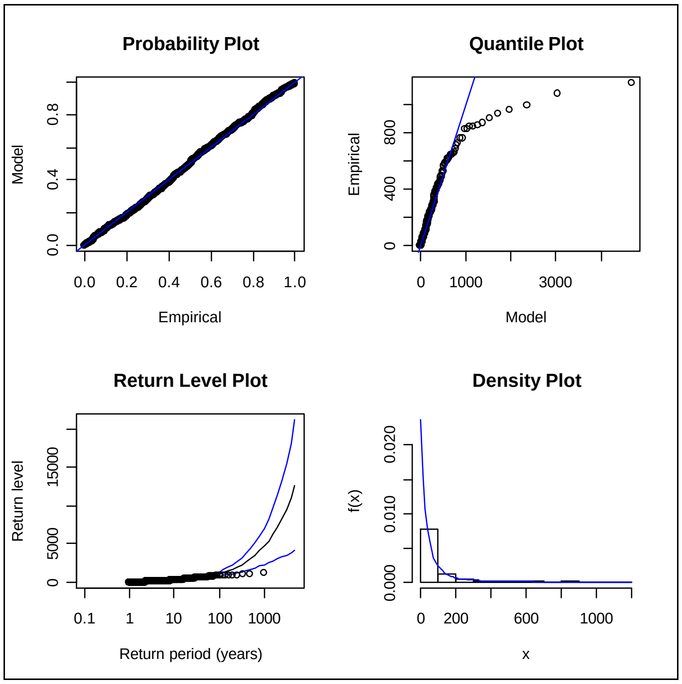
where $\overline{F}(x) = 1 - F(x)$, and $\overline{F}(u)$ is estimated by N_u / n = 1/150. For example, to get the quantile for the 1500 year return we take *p* = .999333 and we use an approximation based on (5) to estimate $\overline{F}_u(y)$ and hence the quantile. But from these specifications we end up with

$$0.000667 = 1 - 0.999333 = (1/150)\cdot\overline{F}_u(y)$$

Now we use (6) to obtain a p'-quantile for the exceedances (the *Y's*) and where *p'* = 1-0.000667*150 = 1-0.10 = 0.9. This is done with the "extRemes Toolkit", Gilleland and Katz (2005), by obtaining the 10 year return level. Table 5, shows the return levels, along with their confidence intervals and the simulation results for the return periods 500, 1000 and 1500. These would be the corresponding PML's for this insurance company using the

Mexican earthquake data. Note they are fairly close and in all cases the simulation results are within the 95% confidence intervals.

Figure 4



The result obtained from the ENR program, which is supposed to be exact turns out to be PML = $ 284,718.06. This is clearly very close to the one obtained using an Extreme Value approach. However the simulation approach yields a wealth of information not obtainable via the ERN program.

Table 5

RETURN PERIODS	SIMULATION	EXTREME VALUE	LOWER LIMIT	UPPER LIMIT
1500	$ 300,710	$ 280,076	$ 256,524	$ 309,758
1000	$ 231,938	$ 217,937	$ 203,170	$ 235,519
500	$ 143,763	$ 141,439	$ 134,747	$ 148,824

We have shown how Monte Carlo methods can be used to analyze the effect of complicated reinsurance treaties on a heterogeneous portfolio. Simulation also allows the evaluation of large quantiles although the results may be biased if we do not have a large sample. However this may be further explored via extreme values.

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