A Dynamic Model of a Non-Life Insurance Portfolio

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Abstract

There are two major problems with most actuarial models used in risk theory: (i) they view an insurance company's operation as static, i.e., policyholders remains constant, and (ii) they assume the demand for insurance is perfectly inelastic, i.e., potential policyholders are insensitive to the size of the premium charged. These assumptions are clearly false in practice. As a step toward more realistic mathematical models, this paper introduces a simple dynamic model of an insurance portfolio where number of insured policyholders is a stochastic process and the rate of sale of new policies depends on the premium charged. We assume the insurer is a pure monopolist facing various types of costs (to obtain and maintain policies, and to process claims) and a linear demand function for its insurance policies. The insurer's profit maximizing premium is determined and is compared to the traditional actuarial premium based on the equivalence principle. It turns out that the optimal premium is the average of the equivalence principle premium and the maximum premium a policyholder would be willing to pay, and that the equivalence principle premium priced portfolio is expected to be twice as large as the profit maximized portfolio. In addition, expressions are provided for the portfolio's value, value-at-risk, and tail value-at-risk. Finally, we consider the case where there are several insurers with different costs selling identical policies and operating within a Cournot oligopoly framework and provide an expression for each insurer's profit maximizing premium.

Key words and phrases: *demand function, new sales, profit maximization, value-at-risk, tail value-at-risk, monololy, Cournot oligopoly*