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RiskLab

# Risk Margin for a Non-Life Insurance Run-Off

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# Claims reserving in non-life insurance

| a.y. | development year $j$ | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $i$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| 1 | 13'109 | 7'246 | 982 | 706 | 358 | 257 | 339 | 161 | 334 | 172 | 35 | 205 | 56 | 32 | 2 | 7 | 1 |
| 2 | 14'457 | 7'581 | 589 | 487 | 124 | 74 | 128 | 50 | 474 | 12 | 72 | 63 | 141 | 286 | 2 | 10 | |
| 3 | 16'075 | 6'597 | 1'081 | 299 | 154 | 551 | 29 | 21 | 16 | 65 | 98 | 415 | 280 | 24 | 27 | | |
| 4 | 15'682 | 7'782 | 1'001 | 587 | 477 | 179 | 44 | 18 | 65 | 240 | 7 | 64 | 4 | 17 | | | |
| 5 | 16'551 | 7'155 | 921 | 946 | 473 | 69 | 168 | 198 | 220 | 17 | 6 | 4 | 7 | | | | |
| 6 | 15'439 | 8'357 | 1'070 | 451 | 822 | 15 | 21 | 30 | 559 | 54 | 18 | 123 | | | | | |
| 7 | 14'629 | 7'016 | 1'181 | 773 | 1'393 | 442 | 42 | 73 | 55 | 105 | 14 | | | | | | |
| 8 | 17'585 | 8'703 | 1'335 | 316 | 396 | 303 | 77 | 44 | 766 | 777 | | | | | | | |
| 9 | 17'419 | 8'522 | 1'125 | 695 | 282 | 434 | 244 | 157 | 70 | | | | | | | | |
| 10 | 16'665 | 8'705 | 1'539 | 702 | 118 | 132 | 1'969 | 14 | | | | | | | | | |
| 11 | 15'471 | 8'274 | 1'372 | 1'261 | 593 | 425 | 84 | | | | | | | | | | |
| 12 | 15'103 | 8'290 | 3'416 | 882 | 370 | 1'122 | | | | | | | | | | | |
| 13 | 14'540 | 8'102 | 929 | 556 | 83 | | | | | | | | | | | | |
| 14 | 14'590 | 7'746 | 1'104 | 589 | | | | | | | | | | | | | |
| 15 | 13'967 | 7'548 | 1'088 | | | | | | | | | | | | | | |
| 16 | 12'930 | 7'181 | | | | | | | | | | | | | | | |
| 17 | 12'539 | | | | | | | | | | | | | | | | |

To be **predicted** and **valued**!

**Table:** Observed incremental payments $\mathcal{D}_I = \{X_{i,j};\ i+j \leq I\}$ with $I = 17$.

- **Predict** and **value** the cash flows in $\mathcal{D}_I^c = \{X_{i,j};\ i+j > I\}$!

# (Nominal) best-estimate reserves

- Model within a stochastic framework the incremental payments $X_{i,j}$.
- The (nominal) best-estimate reserves at time $I$ for the outstanding loss liabilities $X_{i,j}$, $i+j>I$ (lower triangle), are then given by

$$\mathcal{R}_I = \sum_{i+j>I} \mathbb{E}\left[X_{i,j} | \mathcal{D}_I\right].$$

- These predictors $\mathbb{E}\left[X_{i,j} | \mathcal{D}_I\right]$ have minimal prediction variance (optimal).
- For (stochastic) discounting we refer to W.-Merz [5].

# Bayesian chain-ladder model

- Within the Bayesian chain-ladder model the best-estimate reserves are given by, see Bühlmann et al. [1],

$$\mathcal{R}_I = \sum_{i=I-J+1}^{I} C_{i,I-i} \left( \prod_{j=I-i}^{J-1} \widehat{f}_j^{(I)} - 1 \right),$$

where

  - $C_{i,j} = \sum_{l=0}^{j} X_{i,l}$ cumulative payments,
  - $\widehat{f}_j^{(I)} = \mathbb{E}\left[ F_j \,\middle|\, \mathcal{D}_I \right]$ posterior chain-ladder factors given information $\mathcal{D}_I$,
  - $F_j$ unknown chain-ladder factors (modeled stochastically with prior distributions).

- In many cases the (nominal) best-estimate reserves $\mathcal{R}_I$ can be calculated analytically.

- For more details we refer to W.-Embrechts-Tsanakas [4].

# Technical provisions

**deterministic best-estimate reserves $\Longleftrightarrow$ stochastic claims payments**

- Solvency II Directive 2009/138/EC:
  "liabilities shall be valued at the amount for which they could be transferred, or settled, between knowledgeable willing parties in an arm's length transaction."
- The resulting amount is called **technical provisions**.
- The technical provisions are the sum of the **best-estimate reserves** and the **market-value margin (MVM)** (also called risk margin).
- The **MVM** is a **reward for risk bearing** of the (non-hedgeable) run-off risks of the outstanding loss liabilities.

# Market-value margin (MVM)

- The **technical provisions** (market-consistent value) for the outstanding loss liabilities are then given by

$$\mathcal{R}_I^+ \;=\; \mathcal{R}_I \;+\; \mathrm{MVM}_I.$$

- **How should we calculate $\mathrm{MVM}_I$?**

- It should reflect the uncertainties in the prediction of $\sum_{i+j>I} X_{i,j}$ using the predictor $\mathcal{R}_I$.

- A risk averse agent asks for a **reward (MVM)** for bearing possible shortfalls during the run-off of the outstanding loss liabilities.

# Different MVM approaches

- Full **cost-of-capital approach** is rather complex. Leads to nested simulations. Therefore, approximations are used:
  - expected run-off scaling approach: is **NOT risk-based**, but currently used in Solvency II,
  - split of total uncertainty approach (see Salzmann-W. [2]).
- **Expected utility theory approach**
- **Probability distortion approach**, see W.-Embrechts-Tsanakas [4], W.-Merz [5]
  - straightforward,
  - well-known in life insurance.

# Probability distortion approach

The **technical provisions** at time $I$ for the outstanding loss liabilities $X_{i,j}$, $i+j>I$ (lower triangle), are then given by

$$\mathcal{R}_I^+ \;=\; \sum_{i+j>I} \frac{1}{\varphi_I}\, \mathbb{E}\left[\varphi_{i+j}\; X_{i,j} \,|\, \mathcal{D}_I\right],$$

where $(\varphi_k)_{k\geq 0}$ is a **probability distortion** satisfying:

1. $(\varphi_k)_{k\geq 0}$ is a density process, i.e.
   - $\varphi_k$ is strictly positive, $\mathbb{P}$-a.s.,
   - $(\varphi_k)_{k\geq 0}$ is $(\sigma\{\mathcal{D}_k\})_{k\geq 0}$-adapted,
   - $(\varphi_k)_{k\geq 0}$ is a martingale, i.e. $\mathbb{E}\left[\,\varphi_{k+1}\,|\,\mathcal{D}_k\right] = \varphi_k$, with $\varphi_0 \equiv 1$ (normalization).
2. The sequence $\frac{1}{\varphi_k}\,\mathbb{E}\left[\varphi_{i+j}\; X_{i,j} \,|\, \mathcal{D}_k\right]$, for $k\geq 0$, is a super-martingale.

These assumptions imply

$$\mathcal{R}_I^+ \;\geq\; \mathcal{R}_I \qquad \text{and} \qquad \mathrm{MVM}_I \;\stackrel{\text{def.}}{=}\; \mathcal{R}_I^+ - \mathcal{R}_I \;\geq\; 0.$$

# Explicit probability distortion choice

- In W.-Embrechts-Tsanakas [4] we provide an explicit choice for the probability distortion $(\varphi_k)_{k\geq 0}$ in the Bayesian chain-ladder model.

- It provides technical provisions

$$\mathcal{R}_I^+ \;=\; \sum_{i=I-J+1}^{I} C_{i,I-i}\left(\prod_{j=I-i}^{J-1} \widehat{f}_j^{(I+)} - 1\right),$$

where

$$\widehat{f}_j^{(I+)} \;=\; \left(\widehat{f}_j^{(I)} - 1\right) h_j^{(I)}(\alpha_1,\alpha_2) + 1 \;\geq\; \widehat{f}_j^{(I)},$$

with

- $h_j^{(I)}(\alpha_1,\alpha_2) \geq 1$ (distortion function) with
- $\alpha_1$ risk-aversion parameter for process uncertainty,
- $\alpha_2$ risk-aversion parameter for parameter uncertainty.

# Interpretation

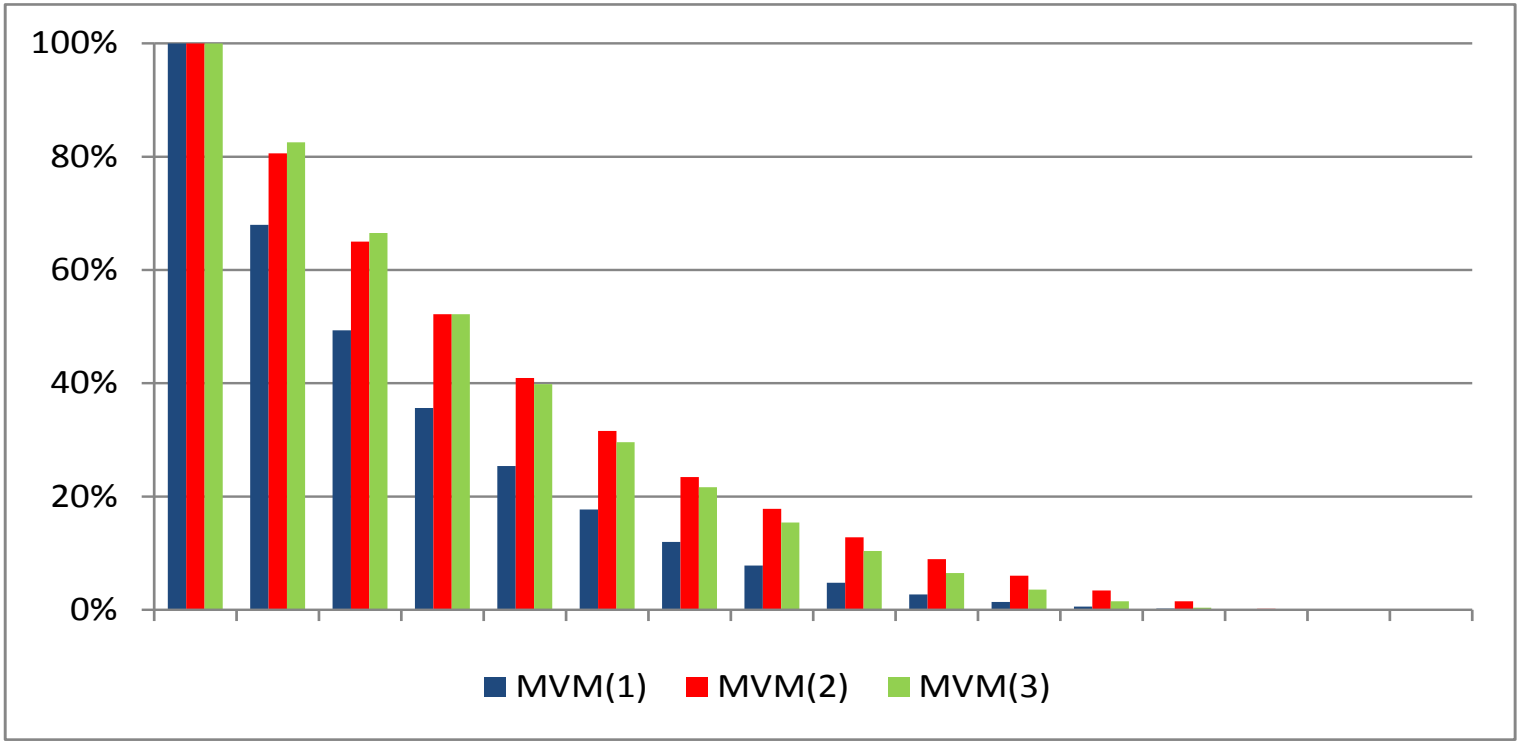
- Chain-ladder factors $\widehat{f}_j^{(I)}$ correspond to a second order life table,
- risk-aversion-adjusted factors $\widehat{f}_j^{(I+)} \geq \widehat{f}_j^{(I)}$ to a first order life table.
- Technical provisions satisfy

$$\begin{aligned}\mathcal{R}_I^+ &= \sum_{i=I-J+1}^{I} C_{i,I-i} \left( \prod_{j=I-i}^{J-1} \widehat{f}_j^{(I+)} - 1 \right) \\ &\geq \sum_{i=I-J+1}^{I} C_{i,I-i} \left( \prod_{j=I-i}^{J-1} \widehat{f}_j^{(I)} - 1 \right) = \mathcal{R}_I.\end{aligned}$$

- Market-value margin is given by

$$\mathrm{MVM}_I = \sum_{i=I-J+1}^{I} C_{i,I-i} \left( \prod_{j=I-i}^{J-1} \widehat{f}_j^{(I+)} - \prod_{j=I-i}^{J-1} \widehat{f}_j^{(I)} \right).$$

# Expected run-off of reserves: private liability insurance




MVM(1) = Solvency II approach
MVM(2) = split of total uncertainty approach, Salzmann-W. [2]
MVM(3) = probability distortion approach, W.-Embrechts-Tsanakas [4]

# References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