Risk Margin for a Non-Life Insurance Run-Off

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| a.y. \( i \) | 0    | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | 10   | 11   | 12   | 13   | 14   | 15   | 16   |
|-------------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| 1           | 13'109 | 7'246 | 982  | 706  | 358  | 257  | 339  | 161  | 334  | 172  | 35   | 205  | 56   | 32   | 2    | 7    | 1    |
| 2           | 14'457 | 7'581 | 589  | 487  | 124  | 74   | 128  | 50   | 474  | 12   | 72   | 63   | 141  | 286  | 2    | 10   |
| 3           | 16'075 | 6'597 | 1'081 | 299  | 154  | 551  | 29   | 21   | 16   | 65   | 98   | 415  | 280  | 24   | 27   |
| 4           | 15'682 | 7'782 | 1'001 | 587  | 477  | 179  | 44   | 18   | 65   | 240  | 7    | 64   | 4    | 17   |
| 5           | 16'551 | 7'155 | 921  | 946  | 473  | 69   | 168  | 198  | 220  | 17   | 6    | 4    | 7    |
| 6           | 15'439 | 8'357 | 1'070 | 451  | 822  | 15   | 21   | 30   | 559  | 54   | 18   | 123  |
| 7           | 14'629 | 7'016 | 1'181 | 773  | 1'393 | 442  | 42   | 73   | 55   | 105  | 14   |
| 8           | 17'585 | 8'703 | 1'335 | 316  | 396  | 303  | 77   | 44   | 766  | 777  |
| 9           | 17'419 | 8'522 | 1'125 | 695  | 282  | 434  | 244  | 157  | 70   |
| 10          | 16'665 | 8'705 | 1'539 | 702  | 118  | 132  | 1'969 | 14   |
| 11          | 15'471 | 8'274 | 1'372 | 1'261 | 593  | 425  | 84   |
| 12          | 15'103 | 8'290 | 3'416 | 882  | 370  | 1'122 |
| 13          | 14'540 | 8'102 | 929  | 556  | 83   |
| 14          | 14'590 | 7'746 | 1'104 | 589  |
| 15          | 13'967 | 7'548 | 1'088 |
| 16          | 12'930 | 7'181 |
| 17          | 12'539 |

Table: Observed incremental payments \( D_I = \{ X_{i,j} ; i + j \leq I \} \) with \( I = 17 \).

- **Predict** and **value** the cash flows in \( D^c_I = \{ X_{i,j} ; i + j > I \} \)!
(Nominal) best-estimate reserves

- Model within a stochastic framework the incremental payments $X_{i,j}$.

- The (nominal) best-estimate reserves at time $I$ for the outstanding loss liabilities $X_{i,j}$, $i + j > I$ (lower triangle), are then given by

  $$R_I = \sum_{i+j>I} \mathbb{E} [X_{i,j} | D_I].$$

- These predictors $\mathbb{E} [X_{i,j} | D_I]$ have minimal prediction variance (optimal).

- For (stochastic) discounting we refer to W.-Merz [5].
Bayesian chain-ladder model

- Within the Bayesian chain-ladder model the best-estimate reserves are given by, see Bühlmann et al. [1],

\[ R_I = \sum_{i=I-J+1}^{I} C_{i,I-i} \left( \prod_{j=I-i}^{J-1} \hat{f}_j^{(I)} - 1 \right), \]

where

- \( C_{i,j} = \sum_{l=0}^{j} X_{i,l} \) cumulative payments,
- \( \hat{f}_j^{(I)} = \mathbb{E} [F_j | D_I] \) posterior chain-ladder factors given information \( D_I \),
- \( F_j \) unknown chain-ladder factors (modeled stochastically with prior distributions).

- In many cases the (nominal) best-estimate reserves \( R_I \) can be calculated analytically.

- For more details we refer to W.-Embrechts-Tsanakas [4].
**Technical provisions**

| deterministic best-estimate reserves ⟷ stochastic claims payments |

- Solvency II Directive 2009/138/EC:
  "liabilities shall be valued at the amount for which they could be transferred, or settled, between knowledgeable willing parties in an arm’s length transaction."

- The resulting amount is called **technical provisions**.

- The technical provisions are the sum of the **best-estimate reserves** and the **market-value margin (MVM)** (also called risk margin).

- The **MVM** is a **reward for risk bearing** of the (non-hedgeable) run-off risks of the outstanding loss liabilities.
Market-value margin (MVM)

- The **technical provisions** (market-consistent value) for the outstanding loss liabilities are then given by

\[ R^+_I = R_I + \text{MVM}_I. \]

- **How should we calculate** $\text{MVM}_I$?

- It should reflect the uncertainties in the prediction of \( \sum_{i+j>I} X_{i,j} \) using the predictor $R_I$.

- A risk averse agent asks for a **reward (MVM)** for bearing possible shortfalls during the run-off of the outstanding loss liabilities.
Different MVM approaches

- Full **cost-of-capital approach** is rather complex. Leads to nested simulations. Therefore, approximations are used:
  - expected run-off scaling approach: is **NOT risk-based**, but currently used in Solvency II,
  - split of total uncertainty approach (see Salzmann-W. [2]).

- **Expected utility theory approach**

- **Probability distortion approach**, see W.-Embrechts-Tsanakas [4], W.-Merz [5]
  - straightforward,
  - well-known in life insurance.
Probability distortion approach

The technical provisions at time $I$ for the outstanding loss liabilities $X_{i,j}$, $i + j > I$ (lower triangle), are then given by

$$R_I^+ = \sum_{i+j>I} \frac{1}{\varphi_I} \mathbb{E} [\varphi_{i+j} X_{i,j} | D_I],$$

where $(\varphi_k)_{k \geq 0}$ is a probability distortion satisfying:

1. $(\varphi_k)_{k \geq 0}$ is a density process, i.e.
   - $\varphi_k$ is strictly positive, $\mathbb{P}$-a.s.,
   - $(\varphi_k)_{k \geq 0}$ is $(\sigma \{D_k\})_{k \geq 0}$-adapted,
   - $(\varphi_k)_{k \geq 0}$ is a martingale, i.e. $\mathbb{E} [\varphi_{k+1} | D_k] = \varphi_k$, with $\varphi_0 \equiv 1$ (normalization).

2. The sequence $\frac{1}{\varphi_k} \mathbb{E} [\varphi_{i+j} X_{i,j} | D_k]$, for $k \geq 0$, is a super-martingale.

These assumptions imply

$$R_I^+ \geq R_I \quad \text{and} \quad \text{MVM}_I \overset{\text{def.}}{=} R_I^+ - R_I \geq 0.$$
Explicit probability distortion choice

- In W.-Embrechts-Tsanakas [4] we provide an explicit choice for the probability distortion \((\varphi_k)_{k \geq 0}\) in the Bayesian chain-ladder model.

- It provides technical provisions

\[
\mathcal{R}_I^+ = \sum_{i=I-J+1}^{I} C_{i,I-i} \left( \prod_{j=I-i}^{J-1} \hat{f}_{j}^{(I+)} - 1 \right),
\]

where

\[
\hat{f}_{j}^{(I+)} = \left( \hat{f}_{j}^{(I)} - 1 \right) h_{j}^{(I)}(\alpha_1, \alpha_2) + 1 \geq \hat{f}_{j}^{(I)},
\]

with
- \(h_{j}^{(I)}(\alpha_1, \alpha_2) \geq 1\) (distortion function) with
- \(\alpha_1\) risk-aversion parameter for process uncertainty,
- \(\alpha_2\) risk-aversion parameter for parameter uncertainty.
Interpretation

- Chain-ladder factors $\hat{f}_j^{(I)}$ correspond to a second order life table,
- risk-aversion-adjusted factors $\hat{f}_j^{(I+)} \geq \hat{f}_j^{(I)}$ to a first order life table.
- Technical provisions satisfy

$$R_I^+ = \sum_{i=I-J+1}^{I} C_{i,I-i} \left( \prod_{j=I-i}^{J-1} \hat{f}_j^{(I+)} - 1 \right)$$

$$\geq \sum_{i=I-J+1}^{I} C_{i,I-i} \left( \prod_{j=I-i}^{J-1} \hat{f}_j^{(I)} - 1 \right) = R_I.$$

- Market-value margin is given by

$$MVM_I = \sum_{i=I-J+1}^{I} C_{i,I-i} \left( \prod_{j=I-i}^{J-1} \hat{f}_j^{(I+)} - \prod_{j=I-i}^{J-1} \hat{f}_j^{(I)} \right).$$
Expected run-off of reserves: private liability insurance

MVM(1) = Solvency II approach
MVM(2) = split of total uncertainty approach, Salzmann-W. [2]
MVM(3) = probability distortion approach, W.-Embrechts-Tsanakas [4]
References


