ASTIN 2011

Simulation of High-Dimensional t-Student Copulas

Gerard Torrent
Josep Fortiana

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Objective

Evaluate the credit risk of a portfolio composed by assets (loans, leases, mortgages, lines of credit, bank guarantees, etc.) given to SME's using the Monte Carlo method.

Index

- Motivation
- The t-Student Copula
- Block Matrices
- Spearman's Rank

Motivation

Market Risk

We simulate stock price along time (Wiener process)

↓

Derivative value distribution at time t

↓

Derivative price

Credit Risk

We simulate obligors default times

↓

Portfolio loss distribution at time t

↓

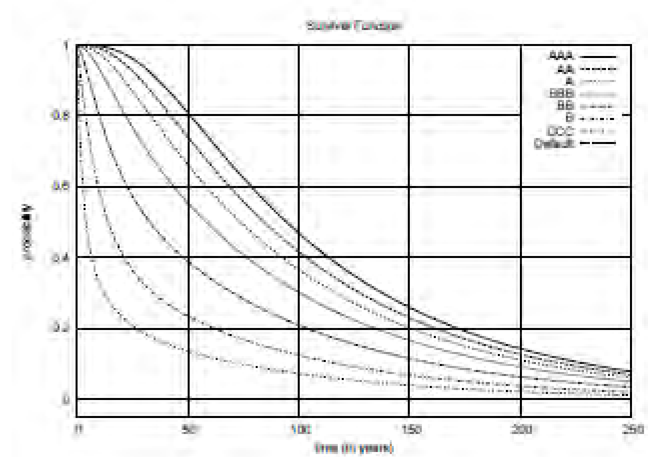
VAR, Expected Shortfall

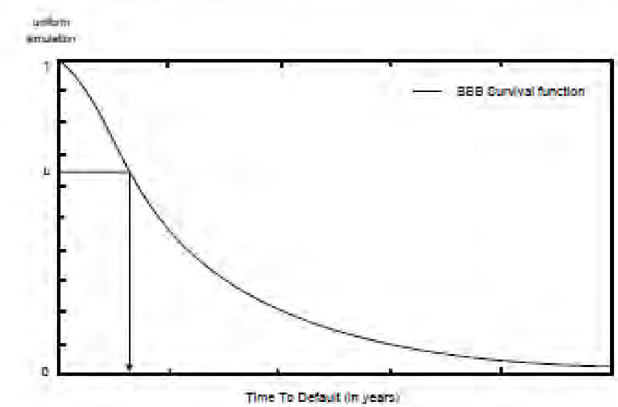
Motivation

Obligors Default Times Simulation (I)

- Rating system (AAA, BB, …)
- Survival curves (PD)

we can simulate obligors default times





Motivation

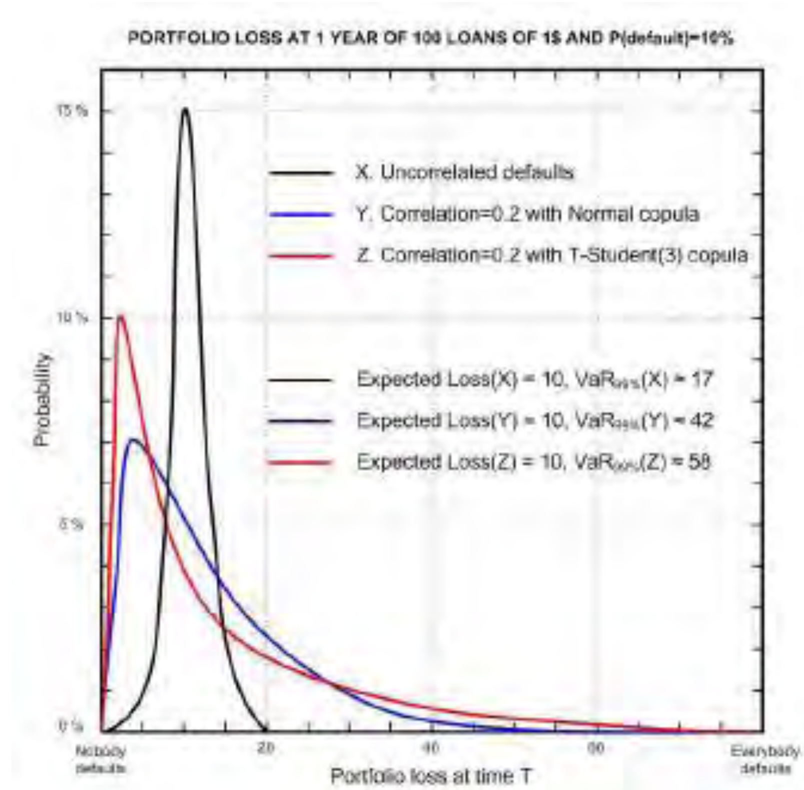
Obligors Default Times Simulation (II)

Default Times

+

- Sectors
- Correlations
- Copula type

We can simulate correlated default times using a <u>Copula</u>



Motivation

Portfolio Loss Simulation

Correlated Default Times

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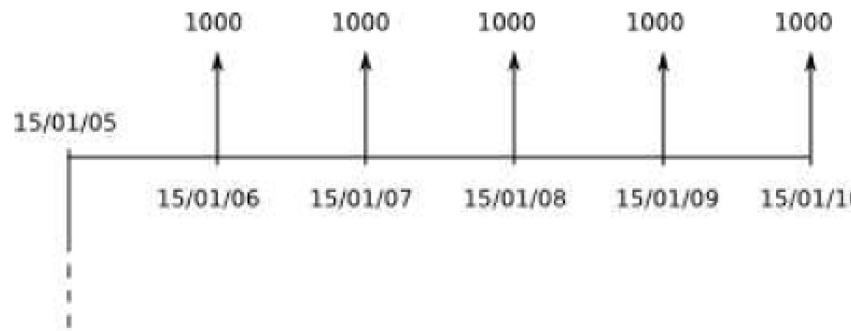
- Asset Exposures
- Asset Recoveries

We can simulate portfolio loss



$$PortfolioLoss = \sum ObligorLoss$$

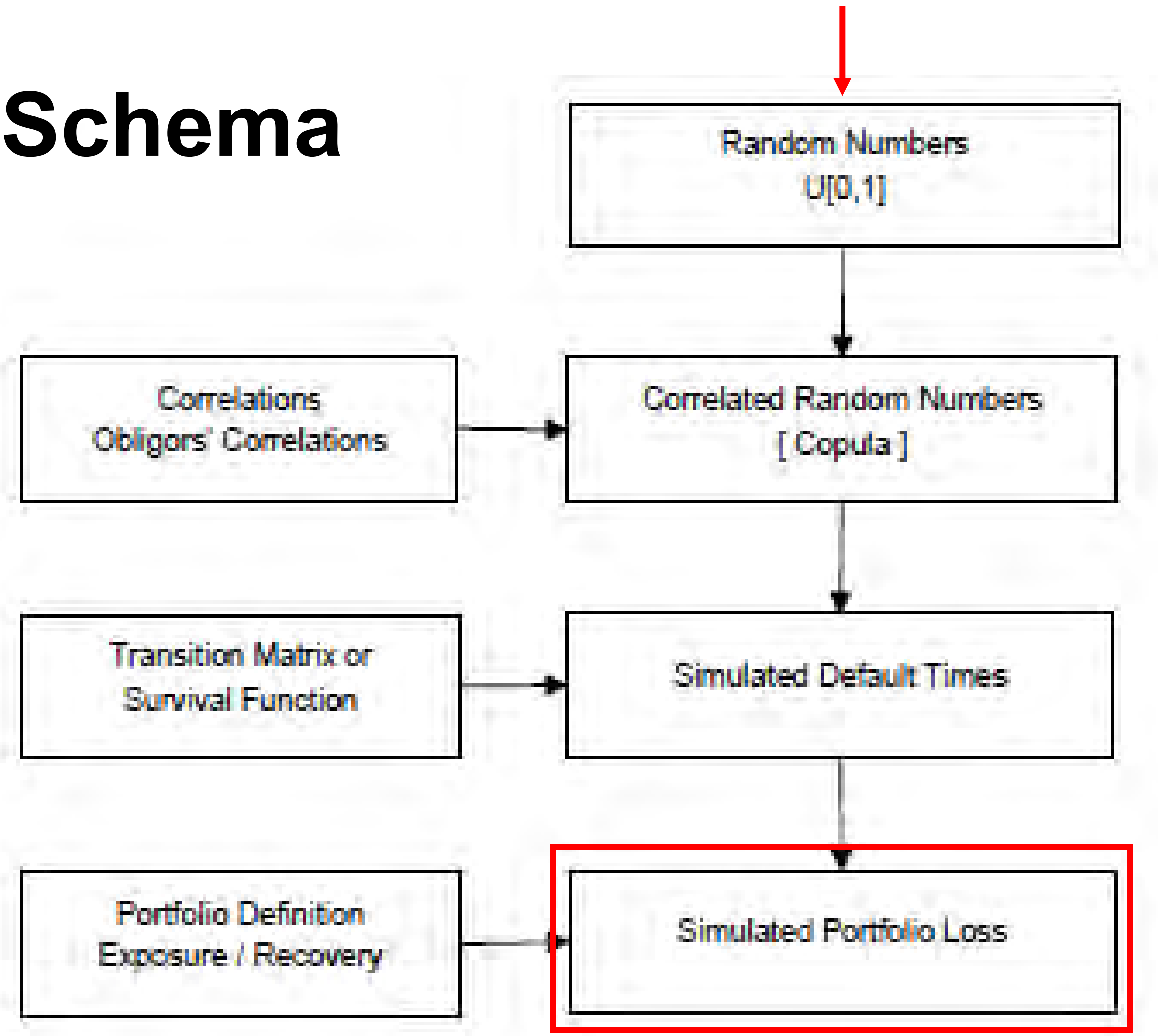
$$ObligorLoss = \sum AssetLoss$$

$$AssetLoss = \begin{cases} exposure(t) \cdot [1 - recovery(t)] & if\ t \le T \\ 0 & if\ t > T \end{cases}$$

Motivation

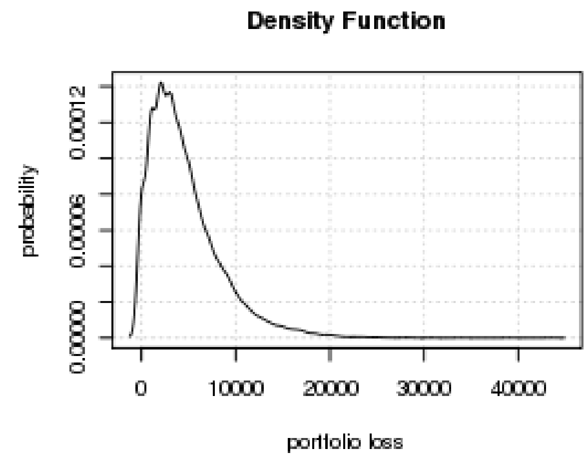
Monte Carlo Schema

We repeat *N* times this procedure to obtain the portfolio loss distribution

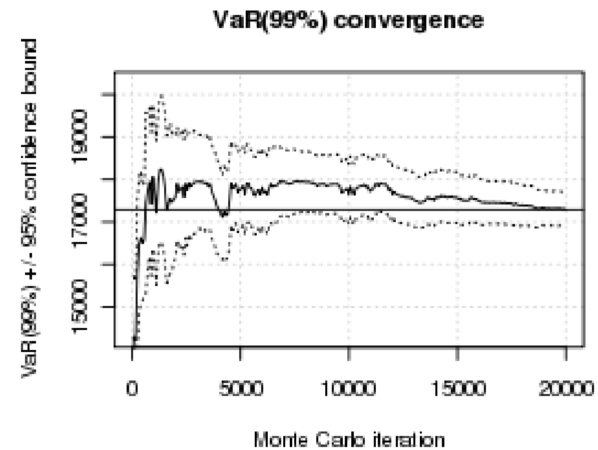


Motivation

Risk Statistics



- Portfolio Loss distribution
- VaR, Expected Shortfall
- Standard Error of risk statistics

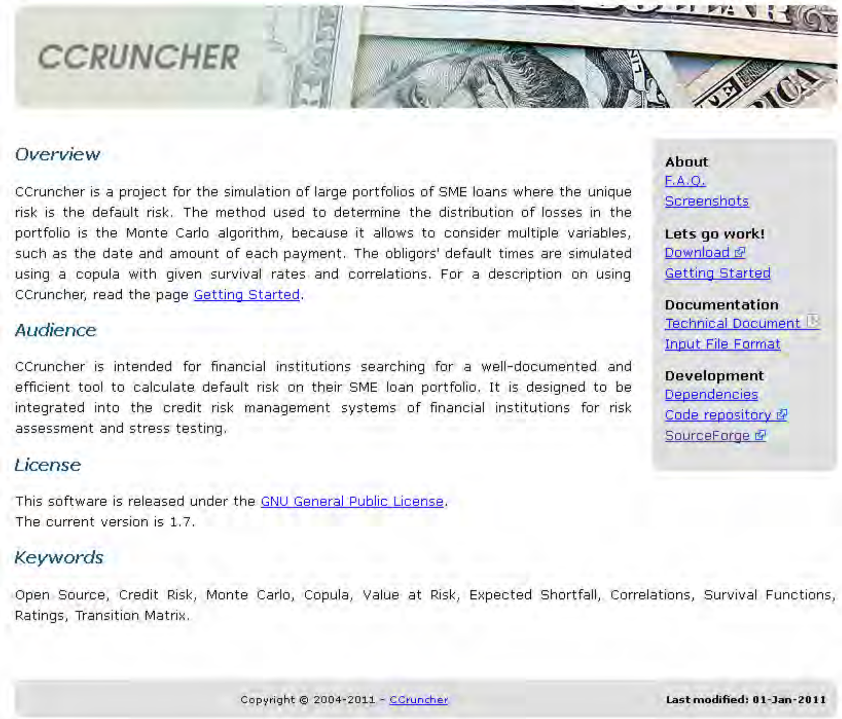


We can obtain results for any given portfolio segmentation (products, regions, etc)

Motivation

CCruncher Project

- Monte Carlo Engine
- Massive Portfolios
- Open Source
- Very Fast


CCRUNCHER

Overview

CCruncher is a project for the simulation of large portfolios of SME loans where the unique risk is the default risk. The method used to determine the distribution of losses in the portfolio is the Monte Carlo algorithm, because it allows to consider multiple variables, such as the date and amount of each payment. The obligors' default times are simulated using a copula with given survival rates and correlations. For a description on using CCruncher, read the page Getting Started.

Audience

CCruncher is intended for financial institutions searching for a well-documented and efficient tool to calculate default risk on their SME loan portfolio. It is designed to be integrated into the credit risk management systems of financial institutions for risk assessment and stress testing.

License

This software is released under the GNU General Public License.
The current version is 1.7.

Keywords

Open Source, Credit Risk, Monte Carlo, Copula, Value at Risk, Expected Shortfall, Correlations, Survival Functions, Ratings, Transition Matrix.

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Lets go work!
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Getting Started

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Technical Document
Input File Format

Development
Dependencies
Code repository
SourceForge

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> The copula simulation algorithm exposed in the paper is the CCruncher algorithm

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- The t-Student Copula
- Block Matrices
- Spearman's Rank

The t-Student Copula

Properties

- Is the copula of the multivariate t-Student
- Allows a range of dependence structures depending on v parameter
- Is a symmetrical copula (allows antithetic variance reduction method)
- Has tail dependence
- Is an elliptical copula (only depends on correlations)
- **Attention** → the correlation used to define the copula is distinct from the copula correlation

The t-Student Copula

Simulation Standard Algorithm

1. Find the Cholesky matrix of Σ → L
2. Simulate a vector of n N(0,1) iid → Y
3. Simulate a Chi-Square(v) → S
4. Compute vector $Z = \text{sqrt}(v/S) \cdot L \cdot Y$
5. Finally, $U = CDF_{t\text{-student}(v)}(Z)$

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Block Matrices

Correlations: Sectors → Obligors

	Construction	Consumer goods	Services
Construction	0.50	0.20	0.30
Consumer goods	0.20	0.60	0.34
Services	0.30	0.34	0.40

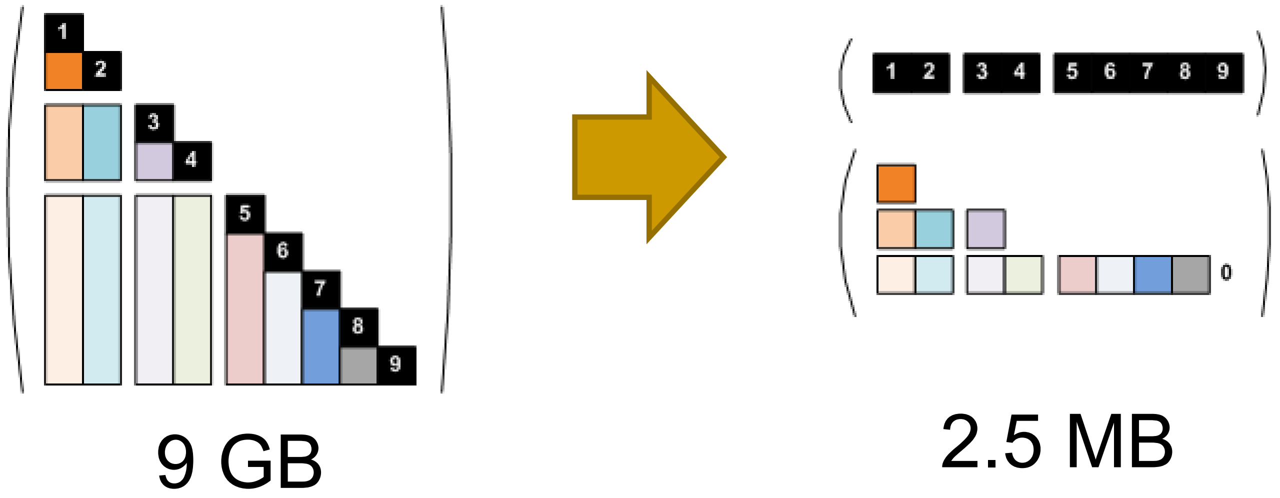
Sectors (inter/intra)

$$\Sigma = \left(\begin{array}{cc|cc|ccccc}
1.00 & 0.50 & 0.20 & 0.20 & 0.30 & 0.30 & 0.30 & 0.30 & 0.30 \\
0.50 & 1.00 & 0.20 & 0.20 & 0.30 & 0.30 & 0.30 & 0.30 & 0.30 \\
\hline
0.20 & 0.20 & 1.00 & 0.60 & 0.34 & 0.34 & 0.34 & 0.34 & 0.34 \\
0.20 & 0.20 & 0.60 & 1.00 & 0.34 & 0.34 & 0.34 & 0.34 & 0.34 \\
\hline
0.30 & 0.30 & 0.34 & 0.34 & 1.00 & 0.40 & 0.40 & 0.40 & 0.40 \\
0.30 & 0.30 & 0.34 & 0.34 & 0.40 & 1.00 & 0.40 & 0.40 & 0.40 \\
0.30 & 0.30 & 0.34 & 0.34 & 0.40 & 0.40 & 1.00 & 0.40 & 0.40 \\
0.30 & 0.30 & 0.34 & 0.34 & 0.40 & 0.40 & 0.40 & 1.00 & 0.40 \\
0.30 & 0.30 & 0.34 & 0.34 & 0.40 & 0.40 & 0.40 & 0.40 & 1.00
\end{array}\right)$$

Obligors (correlations)

Block Matrices

Cholesky Decomposition

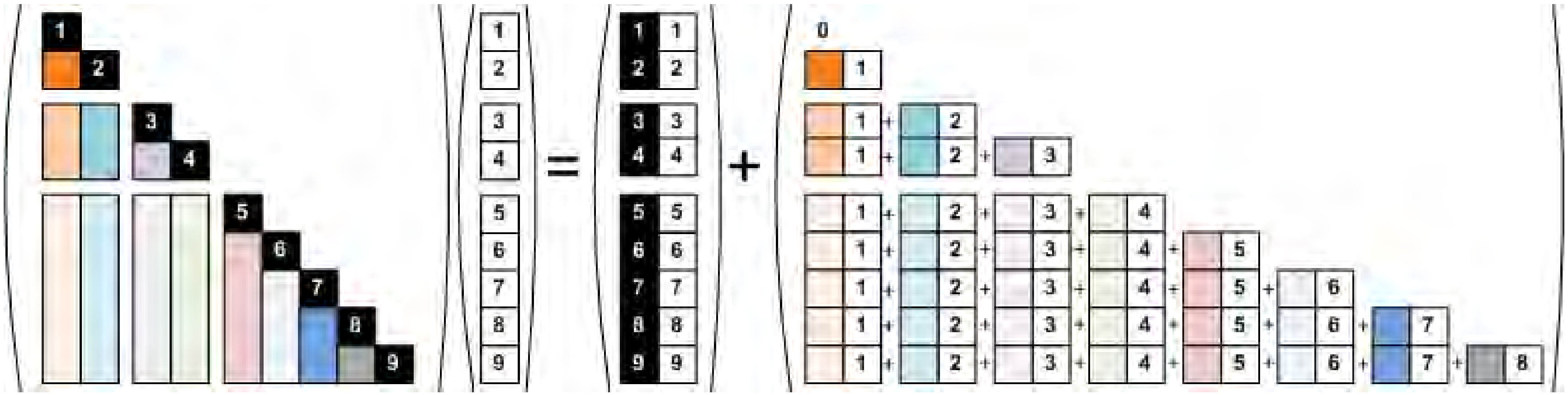


Storage required: $N^2 \rightarrow N \cdot (K+1)$ (great!!)

Number of operations: $N^3 \rightarrow N^2$

Block Matrices

Cholesky Matrix Multiplied by a Vector



Number of operations: $N^2 \rightarrow \sim K \cdot N$ (great!!)

Block Matrices

Eigenvalues

They are used to:

- Check if Σ is definite positive
- Assert stability of the Cholesky decomposition ($\acute{L}=L+\varepsilon$)
- Fit the numerical error of $L\cdot(x+\varepsilon)$

In spite of dimensionality, the numerical stability is maintained (great!!)

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Spearman's Rank

… with a given correlation matrix

$$\rho_s(X,Y) = \rho(F_1(X),F_2(Y)) = \rho(U_1,U_2)$$

Exist function g that → $\rho_s = g_v(\rho)$

If we use a correlation equal to $g^{-1}_v(\rho)$ to construct a t-Student copula, then this copula has correlation ρ.

$$\rho(U_1,U_2) = \rho_s(X,Y) = g_v(g^{-1}_v(\rho)) = \rho$$

Spearman's Rank

Approximation (I)

Normal copula = t_∞-Student copula

$$g_\infty(\rho) = 6/\pi \cdot \arcsin(\rho/2)$$

Based on this we propose:

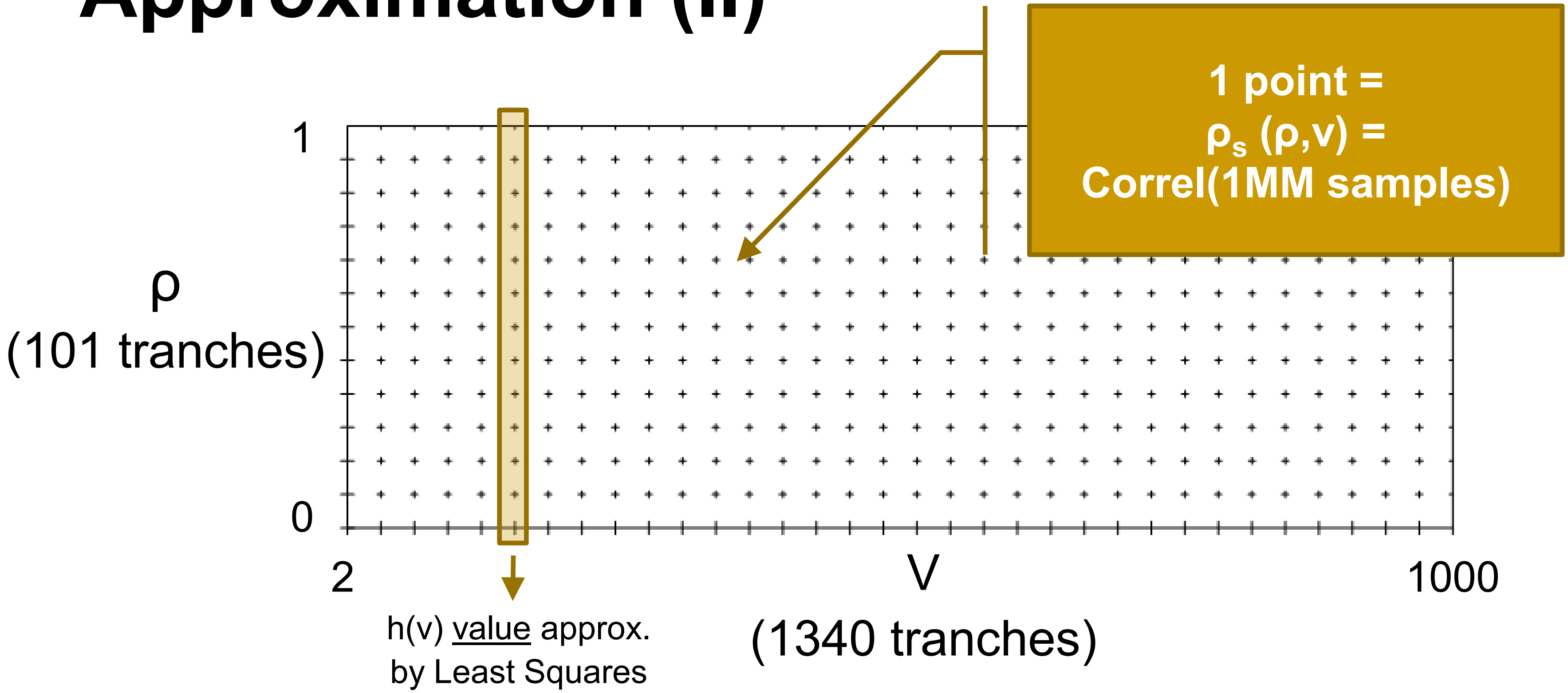
$$g_v(\rho) = \arcsin(\rho \cdot \sin(h(v))/h(v)$$

We suggest to use:

$$h(v) = \pi/6 + 1/(a+b \cdot v)$$

Spearman's Rank

Approximation (II)



Simulated 135,340 ρ_s values

Spearman's Rank

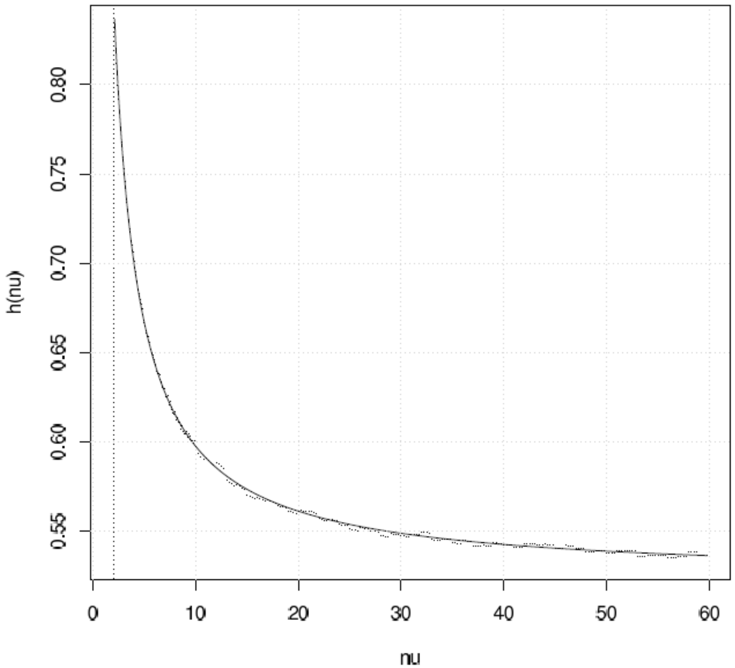
Approximation (III)

h(v) function approximated by Least Squares

$\rho_s = \arcsin(\rho \cdot \sin(h(v))/h(v)$

where,

$h(v) = \pi/6 + 1/(0.45+1.31 \cdot v)$



Error less than 0.005

Summary

- Portfolio Credit Risk
- Large Portfolios
- Monte Carlo
- Open Source
- CCruncher

Thank you !

Contact us:

- Gerard Torrent (gtorrent@ccruncher.net)
- Josep Fortiana (fortiana@ub.edu)

Useful links:

- http://www.ccruncher.net
- http://www.ccruncher.com