Model Study about the Applicability of the Chain Ladder Method

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Claim Reserving – P&C Insurance

Claim reserves must cover all liabilities arising from insurance contracts written in the presence and the past.

Claim reserves are calculated for homogeneous portfolios of insurance contracts

Claim reserves are needed for holistic risk management → Realistic modelling of the claim process!

Outline:

• 3D model
• 3D vs. classical models (2D)
• Comparison: Results of MC simulation (3D / 2D)
Claim Reserving – Classical Models

Classical Models
operate on a 2D data structure (cumulated data)
→ Projection of complex process to a 2D “world”
→ calculate only expectation of reserves (Var under restrictive assumptions)
→ No information about the tail of the reserve distribution

Typical data structures for estimation:

Future = Reserves is projected with ("zoo" of) estimation methods
The 3D Model – the Claim Process

Idea: Development of claim portfolio is modeled from first principles: The single claim process

Time Dynamics of (single) claim process

3 dimensional structure:
occurrence (i) // reporting (j) // payment(s) (k)

Examine a portfolio of claims \( \rightarrow \) stochastic model
The 3D Model

Modeled quantities:

1) Number of active claims $N_{ijk}$
   occurred in $i$th year
   reported $j$ year after occurrence
   still active $k$ years after reporting

2) Total claim payments $Z_{ijk}$
   in $k$th year after reporting arising from all active claims with
   occurrence year $i$ reported with $j$ years delay.
The 3D Model

Visualising the model

Number of active claims $N_{ijk}$

Payments $Z_{ijk}$

IBNR: Incurred but not reported

Reported claims

Now active claims

Now active claims up to now

Book reserve (known claims)

IBNR reserves (unknown claims)
Connection 3D and 2D Model

Up to now reported claims: \( N_{ij0} \) with \( i + j \leq I_{\text{max}} \)

Number of IBNR Claims: 
\[
\hat{N}_{\text{IBNR}} = \sum_{i,j | i+j > I_{\text{max}}} \hat{N}_{ij0}
\]

IBNR Reserve: 
\[
\hat{R}_{\text{IBNR}} = \sum_{i,j,k | i+j+k > I_{\text{max}} \lor i+j < I_{\text{max}}} \hat{Z}_{ijk}
\]

Total Reserve: 
\[
\hat{R}_{\text{total}} = \hat{R}_{\text{IBNR}} + \sum_{i,j,k | i+j > I_{\text{max}}} \hat{Z}_{ijk}
\]

The run off triangle “payments: occurrence versus run off year”:
\[
S_{mn} = \sum_{j,k \mid j+k = n \lor m \land j+k \leq I_{\text{max}}} Z_{mjk}
\]

The run off triangle “payments: reporting versus run off year”:
\[
S_{mn} = \sum_{i,j \mid i+ j = m \lor i+j+n \leq I_{\text{max}}} Z_{ijm}
\]
The 3D Model

Modeling the number of active claims

\[ N_i = \sum_j N_{ij0} \quad \text{(number of claims with occurrence year } = i) \]

\[ N_i \sim \text{Poisson} \left( N_i \right) \]

Claim numbers multinomially distributed along reporting years

with parameters \( \lambda_j \quad j \in 1, 2, \ldots, I_{\text{max}} \)

and \[ \sum_j \lambda_j = 1 \]

Closing the claims along years after reporting (Binomial process):

\[ \eta_k \quad k \in 1, 2, \ldots, K_{\text{max}} \]

with \( \eta_0 = 1 \) and \( \eta_{k+1} < \eta_k \)

\[ N_{ijk} \sim \text{BiNom} \left( N_{ijk-1}, \eta_k \right) \]
The 3D Model

Modeling the total claim payments: Collective Model

\[ Z_{ijk} = \sum_{l=1}^{N_{ijk}} X_l \]

Number of single payments (r.v.)

Size of single payment (r.v.)

Probability of payment for an active claim

\[ \nu_{ijk} \sim Binom \left( N_{ijk}, p_{ik} \right) \]
Chain – Ladder and 3D Model

Condition for the appropriate use of the CL method applied to the 3D model:

1) occurrence vs. run – off matrix → Estimate total reserve

\[
E \left[ s^{(1)}_{in+1} \right] = \xi_n s^{(1)}_{in} \quad \text{with} \quad s^{(1)}_{in} = \sum_{y=1}^{n} \sum_{j+k=y} Z_{ijk} \\

s^{(1)}_{in+1} = s^{(1)}_{in} + \sum_{j+k=n+1} Z_{ijk}
\]

2) reporting vs. run – off matrix → Estimate reserve of known claims

\[
E \left[ s^{(2)}_{xn+1} \right] = \xi_n s^{(2)}_{xn} \quad \text{with} \quad s^{(2)}_{xn} = \sum_{k=1}^{n} \sum_{i+j=x} Z_{ijk} \\

s^{(2)}_{xn+1} = s^{(2)}_{xn} + \sum_{i+j=x} Z_{ijn+1}
\]
Chain – Ladder and 3D Model

Condition for the appropriate use of the CL method applied to the 3D model:

1) occurrence vs. run – off matrix $\rightarrow$ Estimate total reserve

\[
E \left[ \begin{array}{c} \xi_{in+1}^{(1)} \\ \xi_{in}^{(1)} \end{array} \right] \xi_n S_{in} \xi_n \quad \text{with} \quad S_{in+1}^{(1)} = S_{in}^{(1)} + \sum_{j,k \mid j+k=n+1} Z_{ijk}
\]

2) reporting vs. run – off matrix $\rightarrow$ Estimate reserve of known claims

\[
E \left[ \begin{array}{c} \xi_{nxn+1}^{(2)} \\ \xi_{xn}^{(2)} \end{array} \right] \xi_n S_{xn} \xi_n \quad \text{with} \quad S_{xn+1}^{(2)} = S_{xn}^{(2)} + \sum_{t,j \mid i+j=x} Z_{ijn+1}
\]

Structure of claim data $Z_{ijk}$ is decisive for applicability of CL
It is a matter of structure of the claim process – and its realisations in claim data – if the CL method is an appropriate estimation method for the reserves.

Empirical analysis, MC Simulation
Numerical Examples and MC Simulation

Systematic of numerical calculations

We compare examples with three different structures:

Example 1: \[ E \{ \bar{z}_{ijk} \} = \alpha_i \beta_j \gamma_k \]

Example 2: \[ E \{ \bar{z}_{ijk} \} = \alpha_i \mu_{jk} \]

Example 3: \[ E \{ \bar{z}_{ijk} \} = \mu_{ij} \gamma_k \]
Numerical Examples and MC Simulation

Systematic of numerical calculations

Model Parameters

MC Simulation

1000 scenarios of claim portfolio

Calculation of 2D triangles as basis for CL

Occurrence year triangle

Reporting year triangle

CL Method
- Total Reserve

CL Method
- Res. known claims

Expectation Value
- Total Reserve / Res. known claims (3D)
- Triangles → CL → Reserves (2D)

MC simulated Reserves (3D)
- Total Reserve
- Res. known claims
### Comparison of estimation and empirical mean values.

<table>
<thead>
<tr>
<th>Example</th>
<th>Total Reserve</th>
<th>Reserve of known claims</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example 1</td>
<td>$E_{\beta_{ij}} = \alpha_i \beta_j \gamma_k$</td>
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### Numerical Examples and MC Simulation – Results

Comparison of estimation and empirical mean values.

**Result for mean values: Applicability of CL depends on data structure**

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**Numerical Examples and MC Simulation – Results**

Comparison of estimation and empirical mean values.

**Result:** 3D empirical mean fits better than CL estimation // CL positive bias

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Numerical Examples and MC Simulation – Results

MC reserve distributions

Example 1
Total reserve

Result: 3D model distribution is narrow → efficient capital use
Numerical Examples and MC Simulation – Results

MC reserve distributions

Example 3
Total Reserve

Chain Ladder

3D model
Example 3
Reserve of known claims

MC reserve distributions

Chain Ladder

3D model
Thank you for your attention!

I look forward to your questions and our discussion!
Structure and Applicability

What does this mean for the 3D Model?

1) Occurrence – run – off matrix:

\[
\frac{S_{in+1}^{(1)}}{S_{in}^{(1)}} = 1 + \frac{\sum_{y=1}^{n} \sum_{\substack{i, j \mid j + k = n + 1}} Z_{ijk}}{\sum_{k=1}^{n} \sum_{\substack{i, j \mid i + j = x}} Z_{ijn+1}}
\]

Independence on \( i \) \( \Rightarrow \) CL is appropriate forecast for total reserve

2) Reporting – run – off matrix:

\[
\frac{S_{xn+1}^{(2)}}{S_{xn}^{(2)}} = 1 + \frac{\sum_{\substack{i, j \mid i + j = x}} Z_{ijn+1}}{\sum_{k=1}^{n} \sum_{\substack{i, j \mid i + j = x}} Z_{ijk}}
\]

Independence on \( x = i+j \) \( \Rightarrow \) CL is appropriate forecast for reported claims, reserve