Model Study about the Applicability of the Chain Ladder Method

Magda Schiegl

ASTIN 2011, Madrid

Fachhochschule Köln
Cologne University of Applied Sciences
Fakultät für Wirtschaftswissenschaften
ivw Köln
Institut für Versicherungswesen

Claim Reserving – P&C Insurance

Claim reserves must cover all liabilities arising from insurance contracts written in the presence and the past.

Claim reserves are calculated for homogeneous portfolios of insurance contracts

Claim reserves are needed for holistic risk management
→ Realistic modelling of the claim process !

Outline:

- **3D model**
- **3D vs. classical models (2D)**
- **Comparison: Results of MC simulation (3D / 2D)**

Claim Reserving – Classical Models

Classical Models

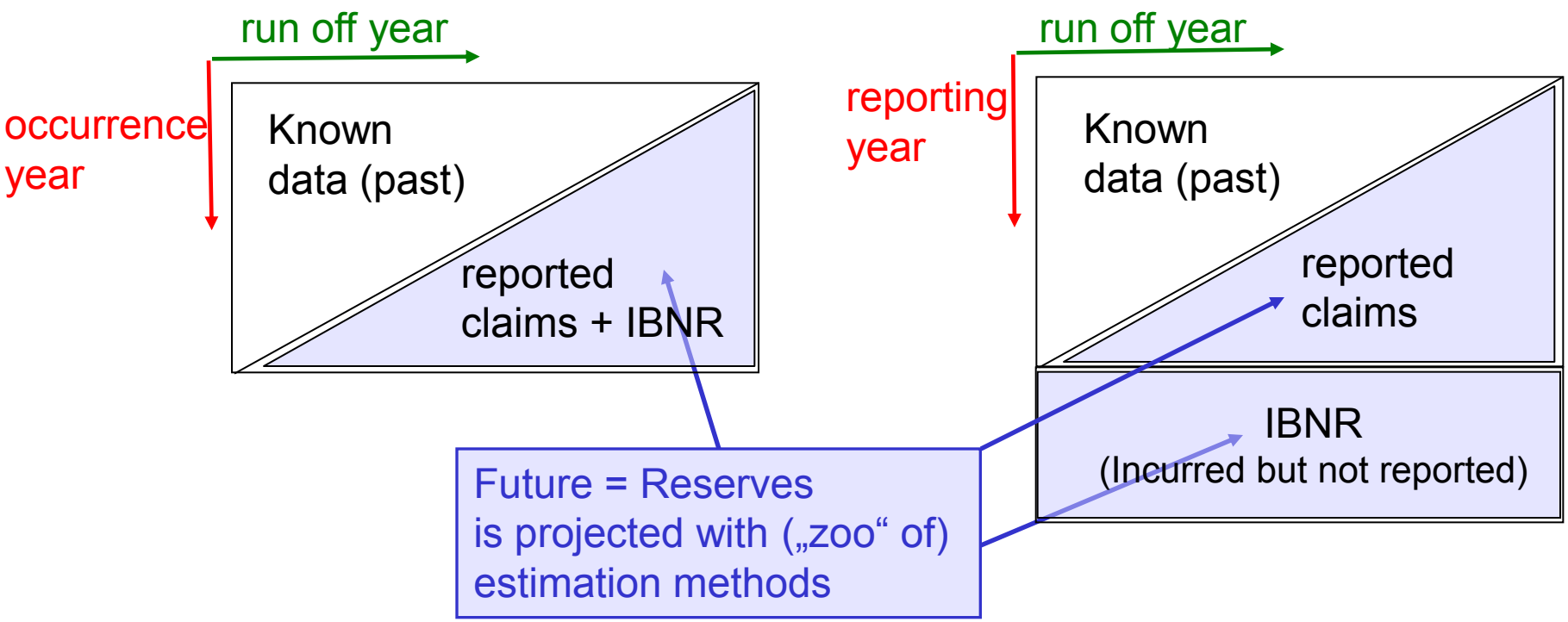
operate on a 2D data structure (cumulated data)

→Projection of complex process to a 2D "world"

→calculate only expectation of reserves (Var under restrictive assumptions)

→No information about the tail of the reserve distribution

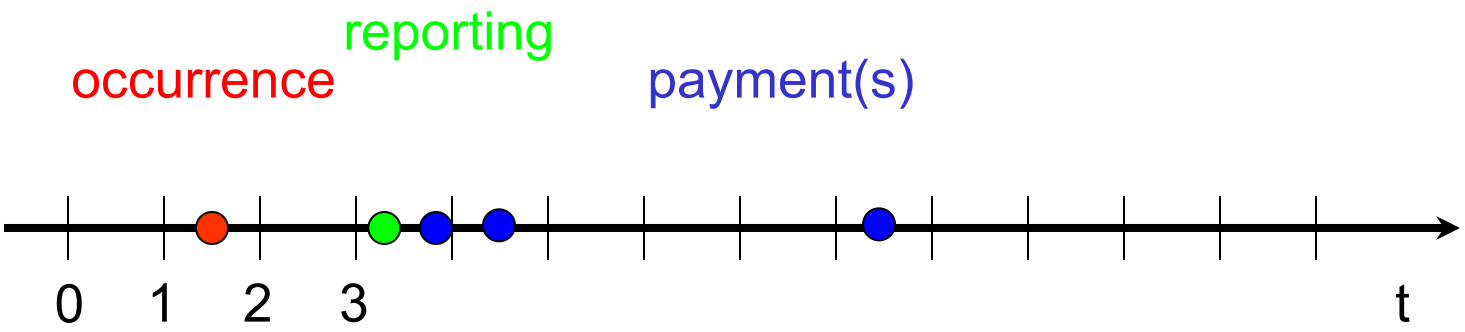
<u>Typical data structures for estimation:</u>



The 3D Model – the Claim Process

Idea: Development of claim portfolio is modeled from first principles: The single claim process

Time Dynamics of (single) claim process



3 dimensional structure:
occurrence (i) // reporting (j) // payment(s) (k)

Examine a portfolio of claims → stochastic model

The 3D Model

Modeled quantities:

1) Number of active claims N_{ijk}

occurred in ith year
reported j year after occurrence
still active k years after reporting

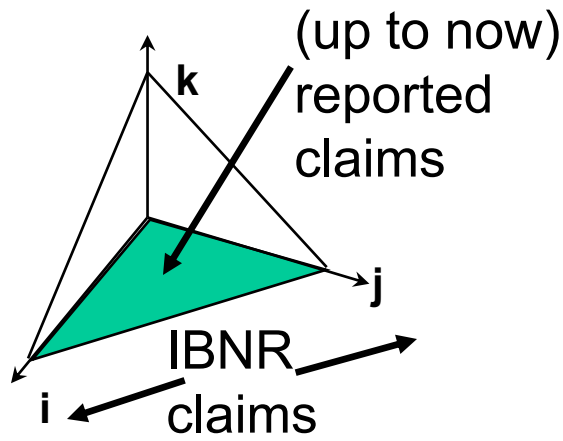
2) Total claim payments Z_{ijk}

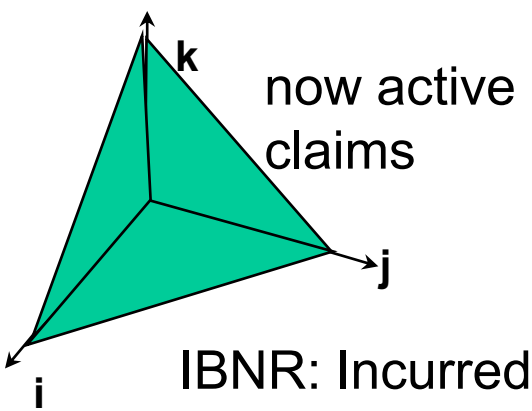
in kth year after reporting arising from all active claims with occurrence year i reported with j years delay.

The 3D Model

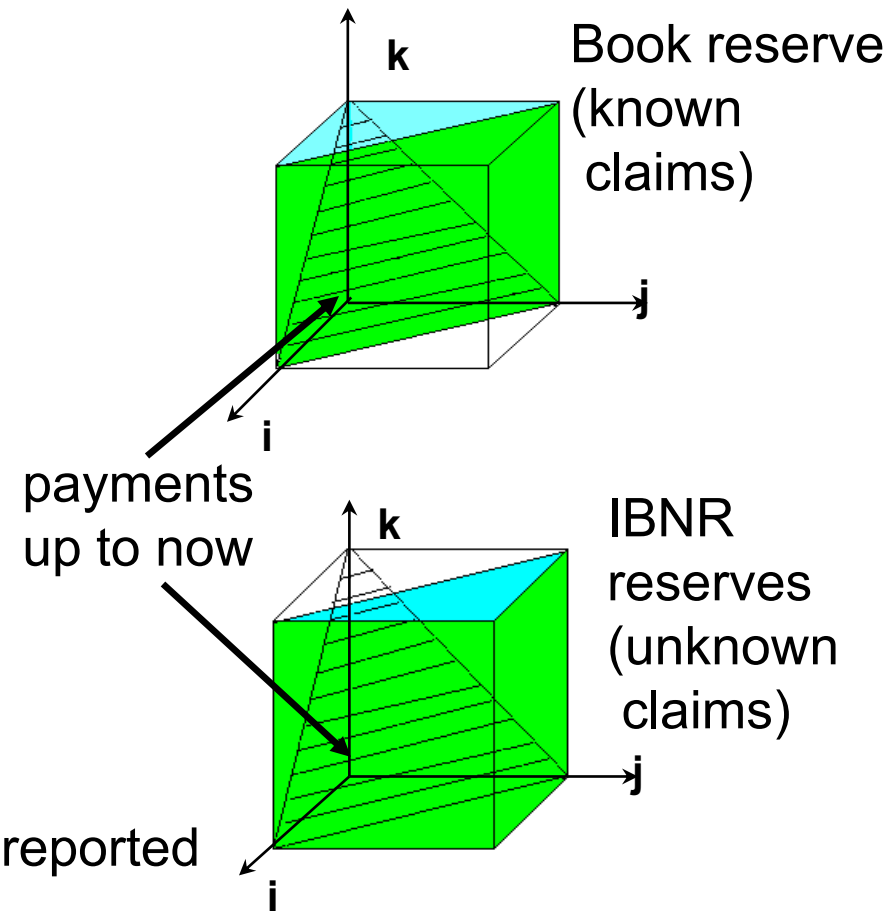
Visualising the model

Number of active claims N_{ijk}





Payments Z_{ijk}



IBNR: Incurred but not reported

Connection 3D and 2D Model

Up to now reported claims: N_{ij0} with $i + j \leq I_{\max}$

Number of IBNR Claims: $$\hat{N}_{IBNR} = \sum_{\{i,j \mid i+j > I_{\max}\}} \hat{N}_{ij0}$$

IBNR Reserve: $$\hat{R}_{IBNR} = \sum_{\{i,j,k \mid i+j+k > I_{\max} \vee i+j < I_{\max}\}} \hat{Z}_{ijk}$$

Total Reserve: $$\hat{R}_{total} = \hat{R}_{IBNR} + \sum_{\{i,j,k \mid i+j > I_{\max}\}} \hat{Z}_{ijk}$$

The run off triangle “payments: occurrence versus run off year” :

$$S_{mn}^{C} = \sum_{\{j,k \mid j+k=n \vee m+j+k \leq I_{\max}\}} Z_{mjk}$$

The run off triangle “payments: reporting versus run off year ” :

$$S_{mn}^{C} = \sum_{\{i,j \mid i+j=m \vee i+j+n \leq I_{\max}\}} Z_{ijn}$$

The 3D Model

Modeling the number of active claims

$$N_i = \sum_j N_{ij0}$$ (number of claims with occurrence year =i)

$$N_i \propto Poisson(\bar{N}_i)$$

Claim numbers multinomially distributed along reporting years

with parameters $\lambda_j \quad j \in \{1,2,..,I_{max}\}$ and $\sum_j \lambda_j = 1$

Closing the claims along years after reporting (Binomial process):

$\eta_k \quad k \in \{1,2,..,K_{max}\}$ with $\eta_0 = 1$ and $\eta_{k+1} < \eta_k$

$$N_{ijk} \propto BiNom(N_{ijk-1}, \eta_k)$$

The 3D Model

Modeling the total claim payments: Collective Model

Number of single payments (r.v.)

$$Z_{ijk} = \sum_{l=1}^{\nu_{ijk}} X_l^{(jk)}$$

Size of single payment (r.v.)

Probability of payment for an active claim

$$\nu_{ijk} \propto Binom\left(N_{ijk}, p_{ik}\right)$$

Chain – Ladder and 3D Model

Condition for the appropriate use of the CL method applied to the 3D model:

1) occurrence vs. run – off matrix → Estimate total reserve

$$E\left[S^{(1)}_{in+1} \middle| S^{(1)}_{in}\right] = \xi_n S^{(1)}_{in} \quad \text{with} \quad S^{(1)}_{in} = \sum_{y=1}^{n} \sum_{\{j,k \mid j+k=y\}} Z_{ijk}$$

$$S^{(1)}_{in+1} = S^{(1)}_{in} + \sum_{\{j,k \mid j+k=n+1\}} Z_{ijk}$$

2) reporting vs. run – off matrix → Estimate reserve of known claims

$$E\left[S^{(2)}_{xn+1} \middle| S^{(2)}_{xn}\right] = \zeta_n S^{(2)}_{xn} \quad \text{with} \quad S^{(2)}_{xn} = \sum_{k=1}^{n} \sum_{\{i,j \mid i+j=x\}} Z_{ijk}$$

$$S^{(2)}_{xn+1} = S^{(2)}_{xn} + \sum_{\{i,j \mid i+j=x\}} Z_{ijn+1}$$

Chain – Ladder and 3D Model

Condition for the appropriate use of the CL method applied to the 3D model:

1) occurrence vs. run – off matrix → Estimate **total reserve**

$$E\left[S^{(1)}_{in+1} \middle| S^{(1)}_{in}\right] = \xi_n S^{(1)}_{in} \quad \text{with} \quad S^{(1)}_{in} = \sum_{y=1}^{n} \sum_{\{j,k \mid j+k=y\}} Z_{ijk}$$

$$S^{(1)}_{in+1} = S^{(1)}_{in} + \sum_{\{j,k \mid j+k=n+1\}} Z_{ijk}$$

2) reporting vs. run – off matrix → Estimate **reserve of known claims**

$$E\left[S^{(2)}_{xn+1} \middle| S^{(2)}_{xn}\right] = \zeta_n S^{(2)}_{xn} \quad \text{with} \quad S^{(2)}_{xn} = \sum_{k=1}^{n} \sum_{\{i,j \mid i+j=x\}} Z_{ijk}$$

$$S^{(2)}_{xn+1} = S^{(2)}_{xn} + \sum_{\{i,j \mid i+j=x\}} Z_{ijn+1}$$

Structure of claim data Z_{ijk} is decisive for applicability of CL

Structure and Applicability

It is a matter of structure of the claim process – and its realisations in claim data –

if the CL method is an appropriate estimation method for the reserves.

↓

Empirical analysis, MC Simulation

Numerical Examples and MC Simulation

Systematic of numerical calculations

We compare examples with three different structures:

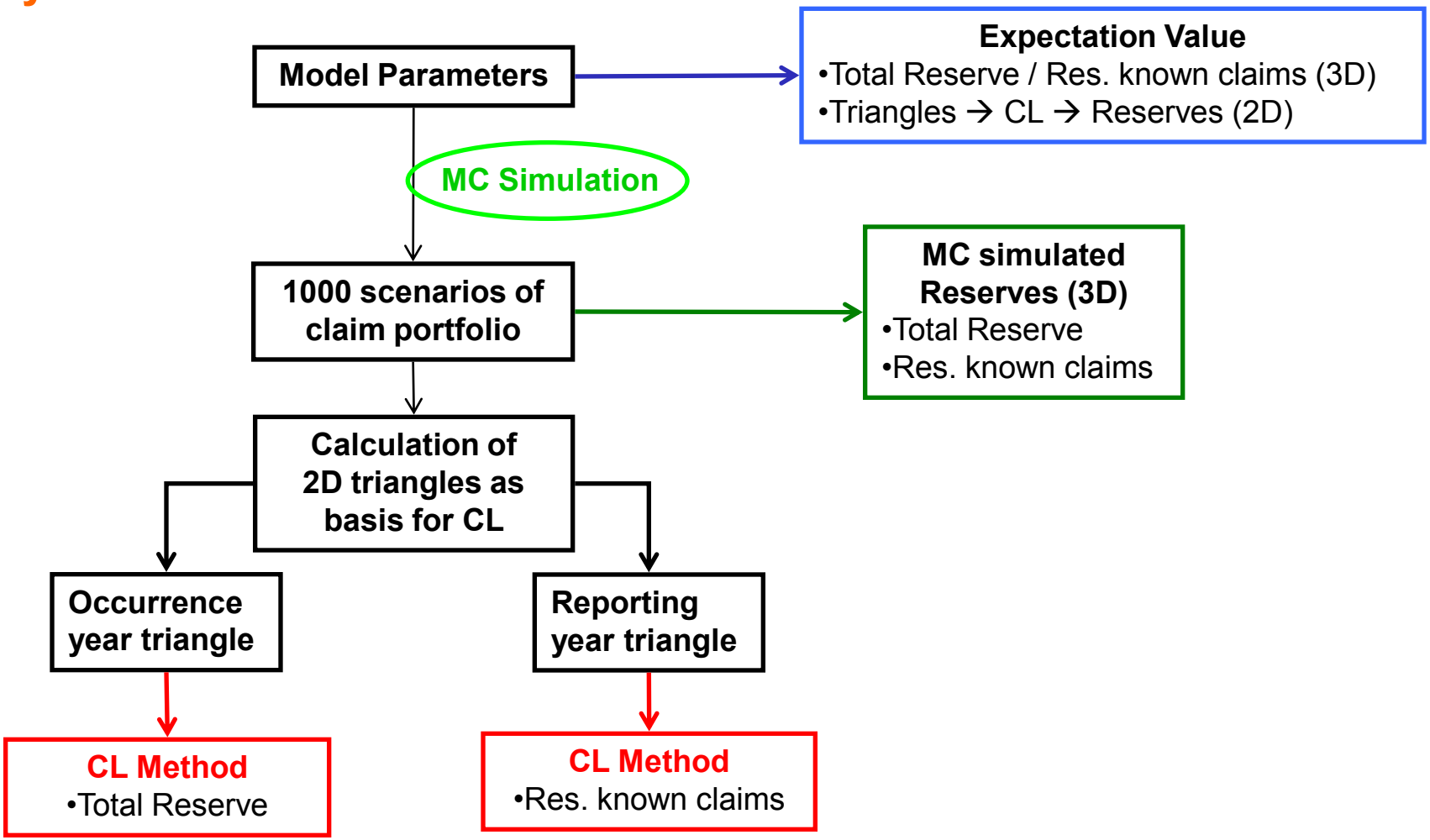
Example 1: $E\left[Z_{ijk}\right] = \alpha_i \beta_j \gamma_k$

Example 2: $E\left[Z_{ijk}\right] = \alpha_i \mu_{jk}$

Example 3: $E\left[Z_{ijk}\right] = \mu_{ij} \gamma_k$

Numerical Examples and MC Simulation

Systematic of numerical calculations



Numerical Examples and MC Simulation – Results

Comparison of estimation and empirical mean values.

	Only Model parameter dependent	MC simulated	Only Model parameter dependent	MC simulated
	3D Expectation	3D Mean	CL Expectation	CL Mean
Example 1 $E[Z_{ijk}] = \alpha_i \beta_j \gamma_k$ Total Reserve	338366	338103	338366	341029
Example 1 Reserve of known claims	189427	190158	189427	190390
Example 2 $E[Z_{ijk}] = \alpha_i \mu_{jk}$ Total Reserve	587676	589595	587676	591713
Example 2 Reserve of known claims	294221	295583	297428	298571
Example 3 $E[Z_{ijk}] = \mu_{ij} \gamma_k$ Total Reserve	576124	577304	589941	596173
Example 3 Reserve of known claims	268032	268004	272400	272347

Numerical Examples and MC Simulation – Results

Comparison of estimation and empirical mean values.

Result for mean values: Applicability of CL depends on data structure

	Only Model parameter dependent	MC simulated	Only Model parameter dependent	MC simulated
	3D Expectation	3D Mean	CL Expectation	CL Mean
Example 1 $E[Z_{ijk}] = \alpha_i \beta_j \gamma_k$ Total Reserve	338366	338103	338366	341029
Example 1 Reserve of known claims	189427	190158	189427	190390
Example 2 $E[Z_{ijk}] = \alpha_i \mu_{jk}$ Total Reserve	587676	589595	587676	591713
Example 2 Reserve of known claims	294221	295583	297428	298571
Example 3 $E[Z_{ijk}] = \mu_{ij} \gamma_k$ Total Reserve	576124	577304	589941	596173
Example 3 Reserve of known claims	268032	268004	272400	272347

Numerical Examples and MC Simulation – Results

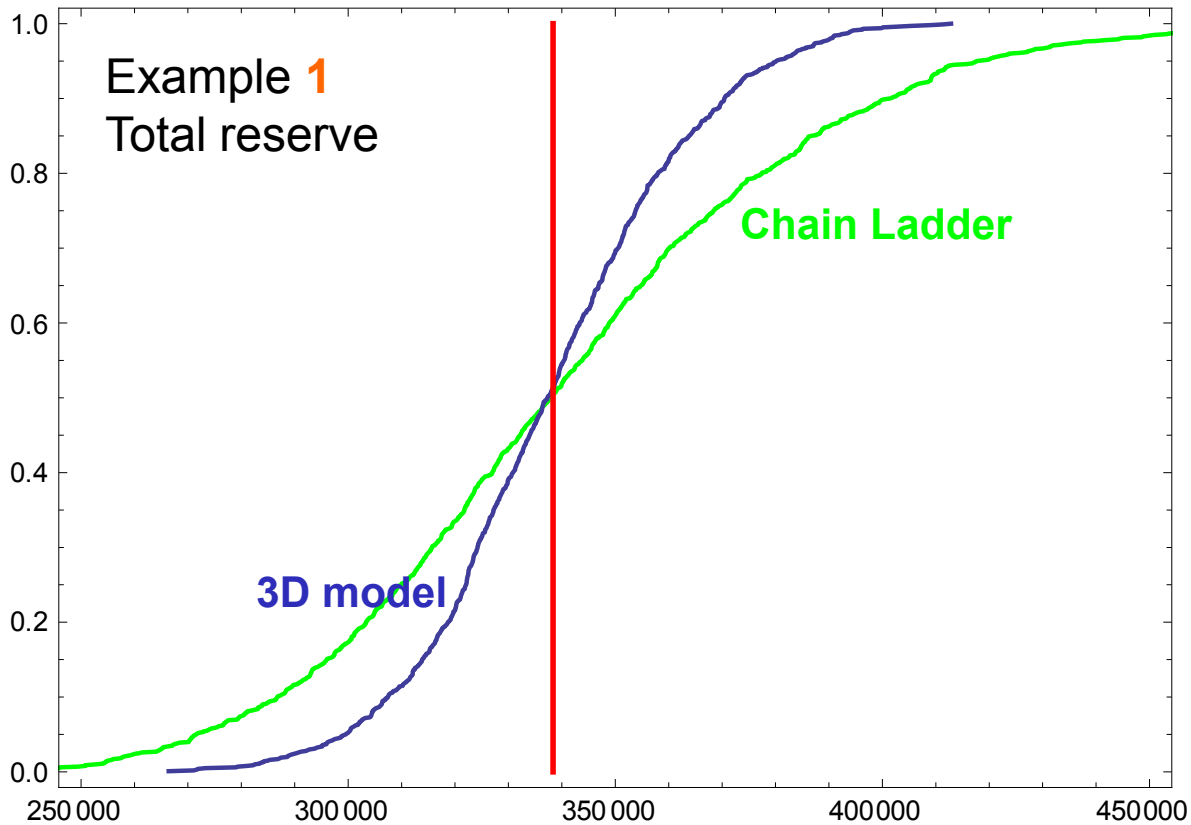
Comparison of estimation and empirical mean values.

Result: 3D empirical mean fits better than CL estimation // CL positive bias

	Only Model parameter dependent	MC simulated	Only Model parameter dependent	MC simulated
	3D Expectation	3D Mean	CL Expectation	CL Mean
Example 1 $E[Z_{ijk}] = \alpha_i \beta_j \gamma_k$ Total Reserve	338366	338103	338366	341029
Example 1 Reserve of known claims	189427	190158	189427	190390
Example 2 $E[Z_{ijk}] = \alpha_i \mu_{jk}$ Total Reserve	587676	589595	587676	591713
Example 2 Reserve of known claims	294221	295583	297428	298571
Example 3 $E[Z_{ijk}] = \mu_{ij} \gamma_k$ Total Reserve	576124	577304	589941	596173
Example 3 Reserve of known claims	268032	268004	272400	272347

Numerical Examples and MC Simulation – Results

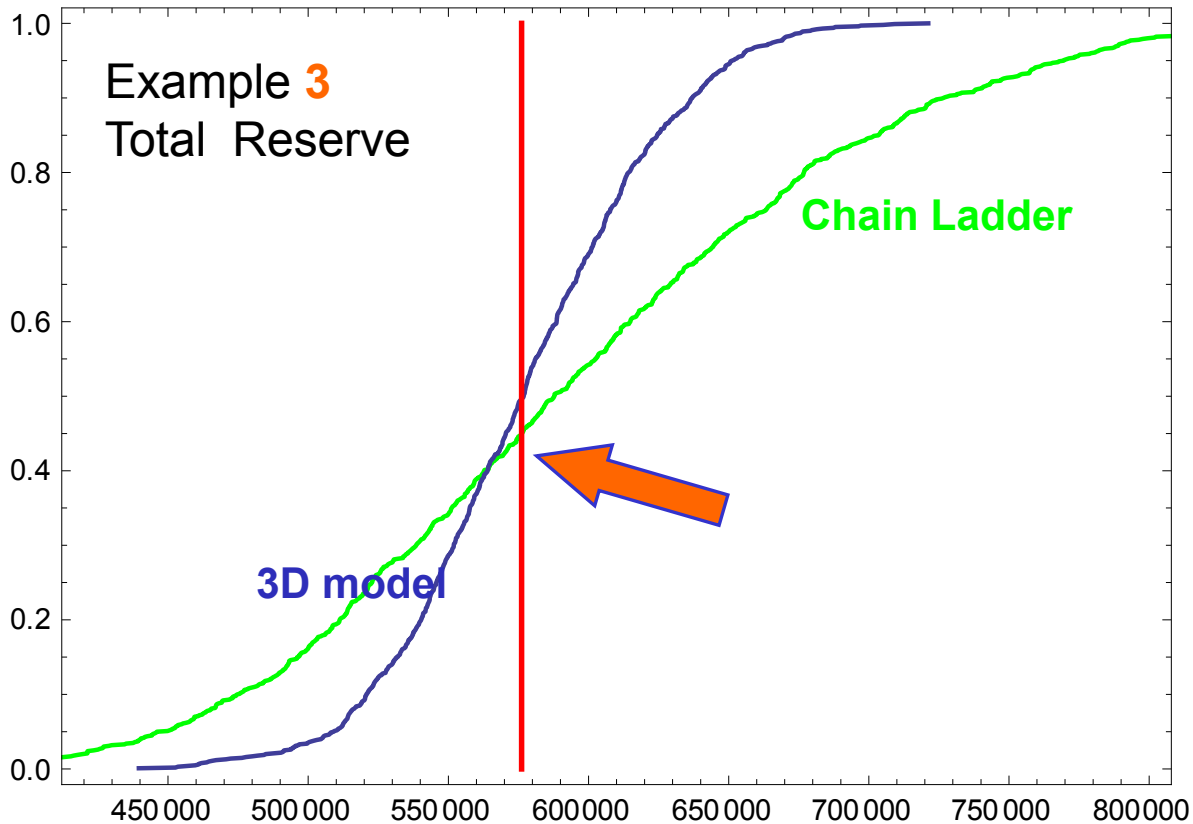
MC reserve distributions



Result: 3D model distribution is narrow → efficient capital use

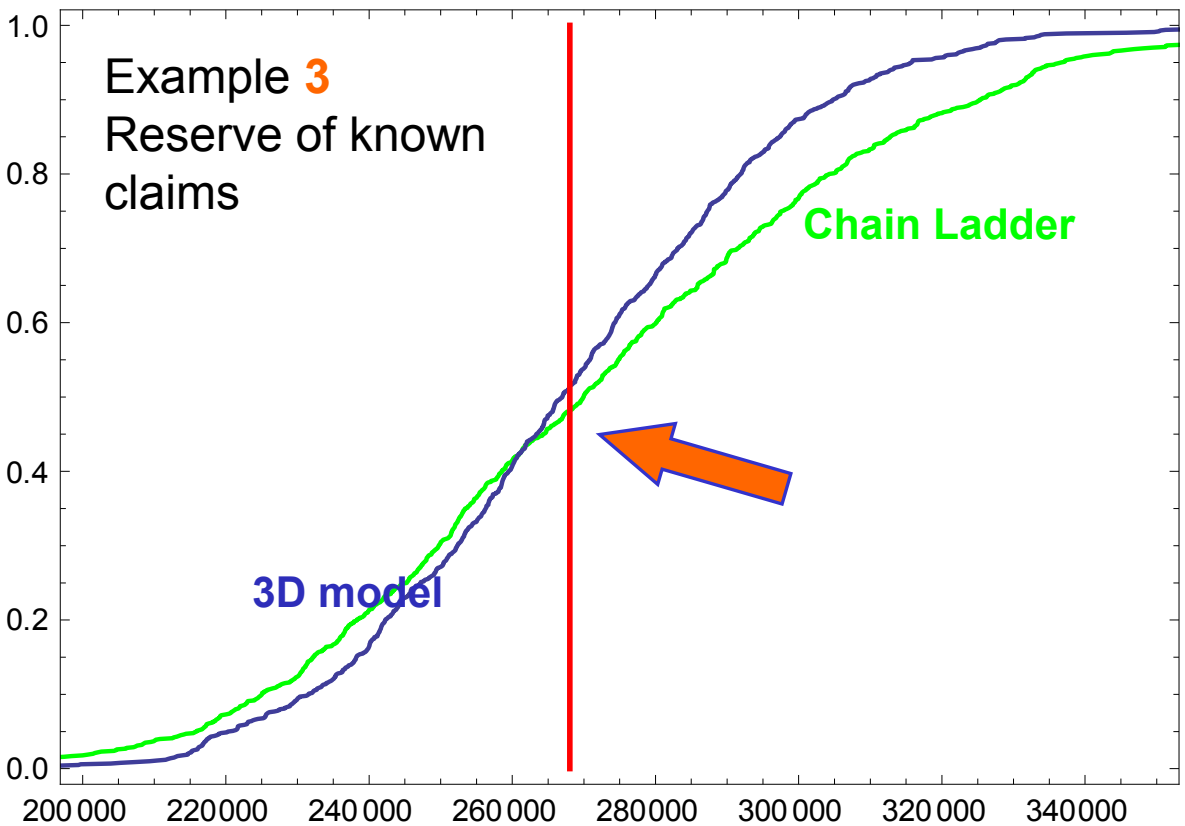
Numerical Examples and MC Simulation – Results

MC reserve distributions



Numerical Examples and MC Simulation – Results

MC reserve distributions



Thank you for your attention !

I look forward to your questions and our discussion!

Back up

ASTIN 2011 Magda Schiegl

Structure and Applicability

What does this mean for the 3D Model?

1) Occurrence – run – off matrix:

$$\frac{S^{(1)}_{in+1}}{S^{(1)}_{in}} = 1 + \frac{\sum_{\{j,k \mid j+k=n+1\}} Z_{ijk}}{\sum_{y=1}^{n} \sum_{\{j,k \mid j+k=y\}} Z_{ijk}}$$

Independence on i
⇒CL is appropriate forecast for total reserve

2) Reporting – run – off matrix:

$$\frac{S^{(2)}_{xn+1}}{S^{(2)}_{xn}} = 1 + \frac{\sum_{\{i,j \mid i+j=x\}} Z_{ijn+1}}{\sum_{k=1}^{n} \sum_{\{i,j \mid i+j=x\}} Z_{ijk}}$$

Independence on x = i+j
⇒CL is appropriate forecast for reported claims„ reserve