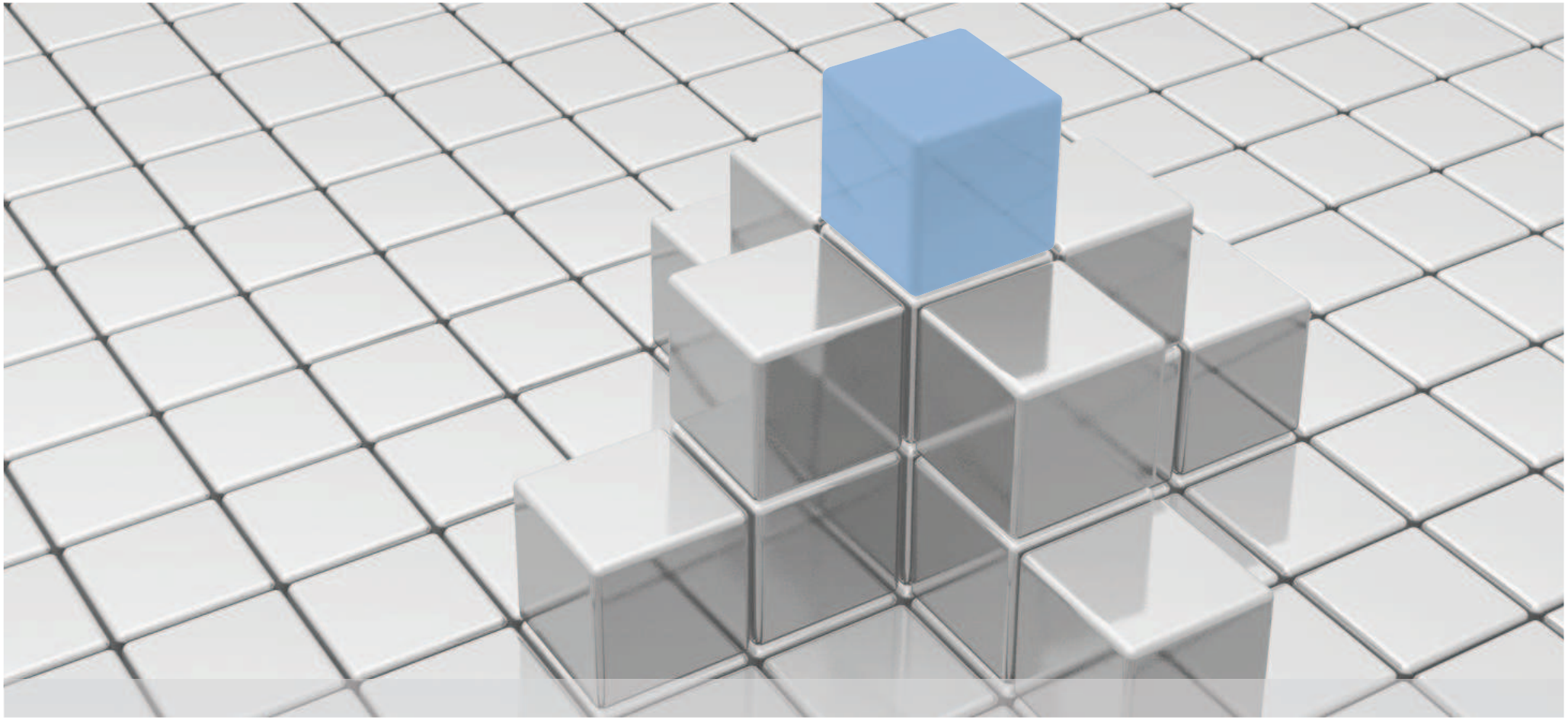

MANAGING EXPOSURE TO REINSURANCE CREDIT RISK EXPOSURE

ASTIN 2011 – Madrid, Spain // *by Yuriy Krvavych (IAG) and Pavel Shevchenko (CSIRO)*

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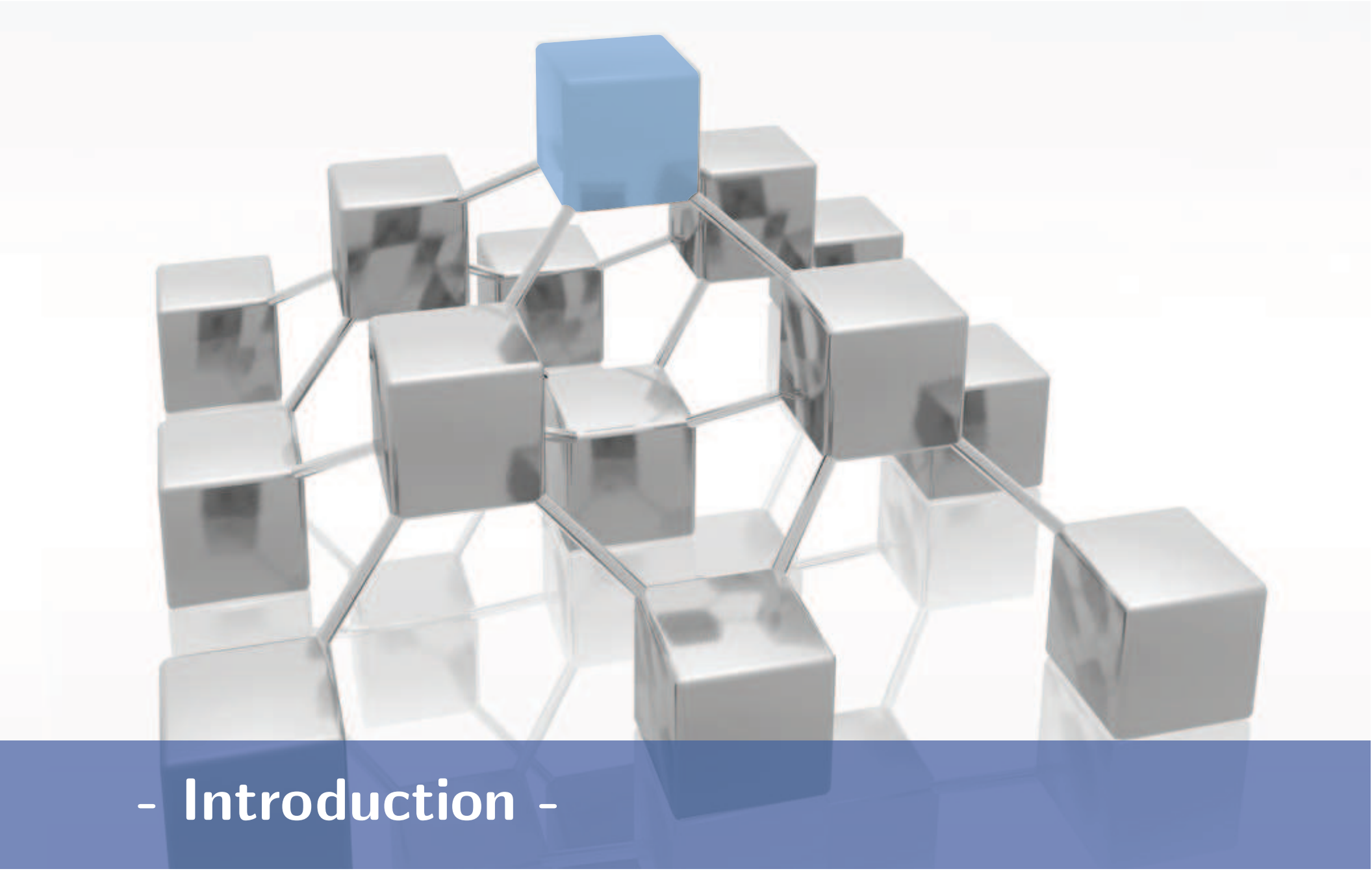
Agenda

- Managing Reinsurance Credit Risk (RCR) Exposure - Introduction
  - Definition of RCR and how it is different to investment credit risk
  - Active management of RCR Exposure: motivation
  - Hedge vs. Retain & Minimise RCR
- Proposed approach to modelling, measuring and minimising RCR
  - Model setup;
  - Modelling implications;
  - Numerical illustrations;
- Conclusions



- Introduction -

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RCR and incentives to manage it

- Insurers hedge against their risk of having extremely large losses in the reinsurance market. By purchasing reinsurance they smooth insurance results and protect against insolvency. This type of risk hedging automatically gives rise to another type of risk - Reinsurance Credit Risk (RCR).
- RCR - as seen through 'risk managers' glasses'
  - The RCR is the risk of the reinsurance counterparty failing to pay reinsurance recoveries in full to the ceding insurer in a timely manner - *unwillingness* to pay, or even not paying them at all - *inability* to pay;
- Incentives to manage RCR
  - The use of reinsurance is usually well received by shareholders, however the RCR emerging as a by-product of reinsurance purchasing is not expected to bounce back to the ceding insurer.
  - Therefore, the RCR is perceived as a company's specific risk that must be managed to maximise shareholders value.

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RCR vs. Investment Credit Risk

- To some extent they are similar – in both cases the loss can be decomposed into: default *frequency* – *severity* – *recoveries*. However, there are differences between them that arise in relation to default event, risk concentration, and default dependencies:

  - *different default events*: under financial distress reinsurers often go into run-off and enter into a commutation agreement with ceding insurers compared to bond issuers that default as a result of shortfall in interest and/or principal payments;
  - *single name concentration* - the number of reinsurers is small (when compared to the number of bond issuers) and so a typical insurer - however prudent - is likely to have a concentrated exposure to individual names;
  - *industry sector concentration* - by definition reinsurer exposure is specific to one industry sector (insurance) so correlations are likely higher than in a more diversified bond portfolio;
  - *tail dependence* - the ceding insurer is in the same industry - catastrophe events will weaken the balance sheet of the reinsurers at the same time (potentially) as the ceding insurers portfolio is stressed.

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Hedge vs. Retain & Minimise RCR

- *Choice 1: Hedge the risk* - neutralise the RCR via hedging it in the CDS derivative market.
  - *Pros*: natural way of getting rid of the risk as the insurer's main specialty is insurance risk and not making money by retaining RCR. Shareholders perceive the RCR as a company's specific risk and thus do not require reward for holding it. Whenever hedging of RCR is possible the optimal portfolio of reinsurance contracts can be established by optimising the total cost of reinsurance and hedging RCR. This problem was recently studied in **Bodoff 2010 [1]**.
  - *Cons*: the market for RCR hedging through CDS is not always available, and/or in some jurisdictions the use of CDS could be discouraged by regulator, also hedging via CDS will never be a perfect hedge as 'reinsurer's unwillingness to pay' is not covered by CDS and taking CDS gives rise to a residual credit risk of CDS counterparty.
- *Choice 2: Retain the risk* - implies modelling it, holding extra amount of capital for it, and mitigating it through active management of its exposure.

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Hedge vs. Retain & Minimise RCR (2)

- This work is primarily focusing on Choice 2 and, in particular, proposes a quantitative approach to modelling optimal RCR exposure limit per reinsurance counterparty under which the RCR is minimised.
- Here, the exposure limits are optimised such that the insurer's marginal risk attributable to RCR is minimised under constraints of budget cost of reinsurance program. Once it's done the active management then monitors and manages the reinsurance counterparty exposure through:
    - *Replacement of defaulting reinsurer* by another one of the same credit quality;
    - *Commutation/collaterisation* in the case of adverse exposure movements due to increased realised claims, and/or deterioration of reinsurance counterparty credit rating;
    - *Imposing restrictions on credit quality* of reinsurers providing cover, say the minimum acceptable credit rating is BBB+; and
    - *Mitigating the risk of 'unwillingness to pay'* via, for example, using a clientele of reinsurers whom the primary insurer has been maintaining a strong business relationship with.

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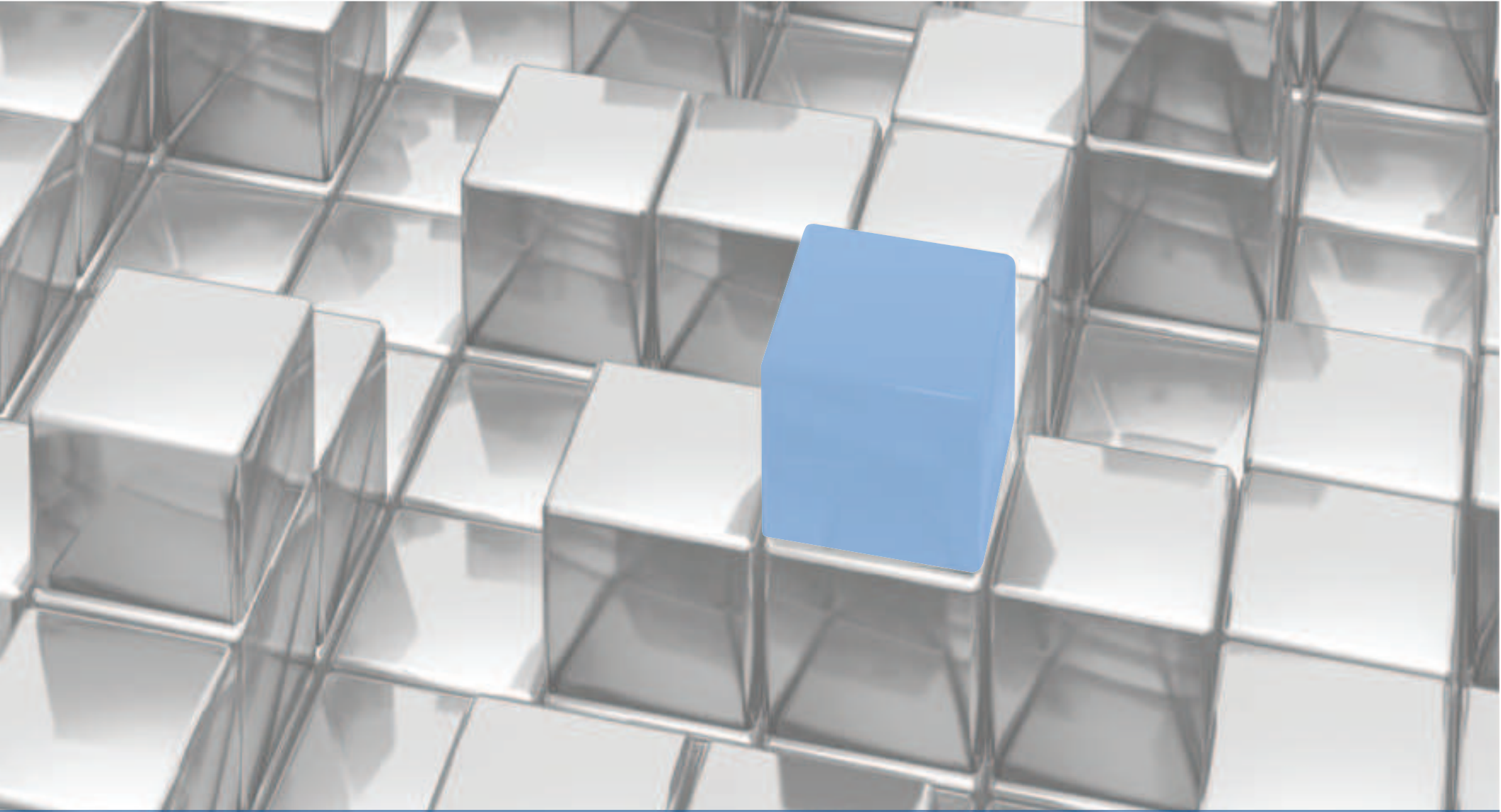

- Minimisation of retained RCR -

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RCR modelling - foundations

- *Key assumption:* the reinsurance default event is defined as an asset impairment event, whereas the credit markets usually consider a bond issuer default as having occurred when there is a shortfall in interest and/or principal payments on its obligation.
- The insolvency of a reinsurer will lead to costs on the part of the insurer:
  - any amounts owing as a result of claims settled with the reinsurer but not yet paid will be impaired;
  - amounts potentially owing in respect of claims incurred but not yet advised to the ceding insurer are potentially not recoverable;
  - any potential recoveries in respect of policies in force are unlikely to be met in full;
  - the ceding company may be required to purchase replacement cover to remain protected against events from the time of reinsurance default to the time the contract would have expired.

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RCR modelling - foundations (2)

- In **Britt and Krvavych 2009 [2]** authors considered quantification of RCR under the following modelling assumptions:
  - *Exposure* - a small number of representative 'proxy reinsurers' is created to capture the company's exposure to reinsurers default. The company's exposure to a proxy reinsurer varies by exposure type, e.g. by catastrophe (cat) and non-cat, small and large cat events.
  - *Dependent default events* - the default of any proxy is assumed to occur at the beginning of any projection time period (quarter), and is modelled as a binary event using Bernoulli random variable with the default rate dependent on the state ('normal' or 'stressed') of the global reinsurance market;
  - *Tail dependency* between large natural peril losses sustained by a primary insurer and the global reinsurance market transiting to 'stressed' state;
  - *Loss Given Default* calculated by applying a recovery rate to the exposure to proxy reinsurers (i.e. reinsurance recoveries) at the end of previous period and the replacement cost of unexpired reinsurance cover.

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RCR exposure limits

- The set of m plausible credit ratings is defined by the primary insurer. If say lowest plausible credit quality is BBB+, then $m = 8$.
- For each plausible credit rating a single representative *proxy reinsurer* is used to capture the company's exposure to reinsurer default.
- The proxy reinsurers are chosen so as to capture the credit exposure and concentration levels typical of company's reinsurer exposure. These representative exposures will remain static over the life of the projection.
- Two sets of exposure limits are maintained: one for catastrophe exposure in *main catastrophe tower*(Upper Layers) and the one for both catastrophe exposure below main cat retention and long-tail (non-catastrophe) exposure (Lower Layers).
- Exposure limits for upper(lower) layers are defined as positive weights $w_i^{U(L)} \in [0, 1], i = 1, ..., m$ - a maximum share of exposure-at-default (i.e. potential reinsurance recoverables in the event of default) the primary insurer is willing to assign to a single proxy reinsurer of credit quality i. For each set of exposure limits those weights will add up to one, i.e. $\sum_{i=1}^{m} w_i^{U(L)} = 1$.

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Modelling dependent default events

The following formula formalises the default event of reinsurance proxy i in a quarter period j of year one:

$$\{\mathcal{D}_{ij}(Z)|Z=z\} \sim Be\left[q_n^{I(z,j)} q_s^{1-I(z,j)}\right], \tag{1}$$

where q_n and q_s are quarterly normal and stressed default rates respectively,

$$I(z,j) = \begin{cases} 1, & z > j; \\ 0, & z \leq j \end{cases} \tag{2}$$

and $Z \sim TruncGeom[p]$ is the Truncated Geometric random variable over the period of four quarters with the quarterly transition rate p and distribution

$$\mathbb{P}[Z=z] = \begin{cases} (1-p)^{z-1}p, & z = 1, ..., 4; \\ (1-p)^4, & z = 5 \end{cases} \tag{3}$$

where $z = 1, ..., 4$ is the ordering number of quarter when for the first time the market transits into 'stressed' state, $z = 5$ indicates that the market remains in normal state during the first four quarters.

Modelling dependent default events (2)

- The use of a 'stress' scenario that affects all reinsurers is a useful method by which co-dependency between reinsurer defaults can be allowed for.
- It is assumed that the global reinsurance market transits into 'stressed' state in a particular period j if the quarterly gross natural peril loss X_j exceeds its $(1-p)$-th percentile, i.e. when $X_j > \mathrm{VaR}_{1-p}(X_j)$, and once it has transited into stressed state, it remains stressed for some period of time. The reasonable duration of 'stress' state is considered to be between 1 to 4 years
- It is also assumed that the quarterly transition of the market is independent of the cumulative catastrophe losses incurred in the periods prior to market transition, as it is believed that 'unstressed' reinsurers sustaining losses in a quarter would quickly re-capitalise during the same period. Those additional assumptions allow for tail dependencies between the primary insurer and reinsurers providing cover to it.

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Loss given default assumptions

The ultimate default cost on i-th reinsurer proxy providing cover on both upper and lower layers, and defaulting in quarter j is defined as

$$(1-\rho_i) \times \left[w_i^U \times R_U(X_j) + w_i^L \times (R_L(X_j) + R_{LT}(j))\right] \tag{4}$$
$$+\frac{4-j}{4}\left[w_i^U \times \pi_U(i) + w_i^L \times \pi_L(i)\right],$$

where

- $R_U(X_j)$ and $R_L(X_j) + R_{LT}(j)$ is the gross exposure-at-default for upper and lower layers respectively, with $R_{U(L)}(X_j)$ being the total recoveries from upper(lower) layers of the reinsurance program on catastrophe losses X_j incurred in period j , and $R_{LT}(j)$ being the total recoverables on long-tail (liability) losses at the end of period j.
- $\frac{4-j}{4}\left(w_i^U \times \pi_U(i) + w_i^L \times \pi_L(i)\right)$ is a portion of Deferred Reinsurance Expense of reinsurance program shared by reinsurer proxy i, which, in the event of default, indicates (an 'optimistic' estimate of) the total cost of replacing residual cover lost due to default of the proxy. Here, $\pi_{U(L)}(i)$ is the cost of the part of company's reinsurance program providing cover in upper(lower) layers assuming the cover is 100% placed with reinsurance proxy i.

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Ultimate default cost – 'carrier' of RCR

The ultimate total default cost specific to upper(lower) layers of the reinsurance program is defined as a risk function (carrier) linear in weights (i.e. exposure limits) $\mathbf{w}^U = \left(w_1^U, ..., w_m^U\right)$ and $\mathbf{w}^L = \left(w_1^L, ..., w_m^L\right)$

$$C_L\left(\mathbf{w}^L\right) = \sum_{i=1}^{m}\sum_{j=1}^{4}\Big\{(1-\rho_i)\times w_i^L \times (R_L(X_j) + R_{LT}(j)) + \frac{4-j}{4} w_i^L \times \pi_L(i)\Big\} \times \mathcal{D}_{ij}(Z), \tag{5}$$

and

$$C_U\left(\mathbf{w}^U\right) = \sum_{i=1}^{m}\sum_{j=1}^{4}\Big\{(1-\rho_i)\times w_i^U \times R_U(X_j) + \frac{4-j}{4} w_i^U \times \pi_U(i)\Big\} \times \mathcal{D}_{ij}(Z),$$

Then the total ultimate cost default across the reinsurance program is

$$C\left(\mathbf{w}^L, \mathbf{w}^U\right) = C_L\left(\mathbf{w}^L\right) + C_U\left(\mathbf{w}^U\right).$$

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Measuring RCR

The following risk measures are used to assess the impact of retained RCR on the benefits anticipated from underlying reinsurance program:

- $\mathbf{RM}_L$ - (preferably) a TVaR risk measure averaging risk carrier above its central estimate, or a standard deviation. The risk measure $\mathbf{RM}_L$ is used to measure the reduction in the benefits of the reinsurance program in lower layers through the difference

$$\begin{aligned}&\mathbf{RM}_L\left[(X-R)+(Y-R_{LT})+C_L\left(\mathbf{w}^L\right)\right]\\&-\mathbf{RM}_L\left[(X-R)+(Y-R_{LT})\right],\end{aligned}$$

  where $X=\sum_{j=1}^{4}X_j$ is the total catastrophe losses incurred over the period of one year, $R=\sum_{j=1}^{4}R\left(X_j\right)$ is the total cat recoveries, Y and R_{LT} - is the total attritional claims (i.e. small working plus large liability claims) incurred over the period of one year and their non-cat reinsurance recoveries respectively, and $C_L\left(\mathbf{w}^L\right)$ is the total ultimate cost of reinsurance default over one year specific to lower layers of reinsurance program; and

- $\mathbf{RM}_U$ - a TVaR risk measure averaging risk carrier in the tail of its distribution above a certain percentile. The total cost of reinsurance default, $C\left(\mathbf{w}^U,\mathbf{w}^L\right)$, plays the role of risk carrier here. This risk measure is used to measure capital consumptions due to retained RCR.

Convex optimisation of exposure limits

We consider the following problem of minimising retained reinsurance credit risk

$$\begin{cases} \min\limits_{\mathbf{w}^L, \mathbf{w}^U} & \left\{a \times \mathbf{RM}_L \left[Z + C_L\left(\mathbf{w}^L\right)\right] + \mathbf{RM}_U \left[C\left(\mathbf{w}^U, \mathbf{w}^L\right)\right]\right\} \\ \text{s.t.} & \sum_{i=1}^m w_i^L = 1; \ \ \sum_{i=1}^m w_i^U = 1; \\ & w_i^L \geq 0; \ \ w_i^U \geq 0; \\ & \sum_{i=1}^m \left(w_i^L \times \pi_L(i) + w_i^U \times \pi_U(i)\right) = c > 0 \end{cases} \tag{6}$$

where $Z = (X - R) + (Y - R_{LT})$ is the total annual net(of reinsurance) claims expense excluding cost of default, and the last constraint equality is budget constraint of insurer's annual reinsurance expense.

- The target function of the minimisation problem (6) is the mixture of two risk measures $\mathbf{RM}_L$ and $\mathbf{RM}_U$ measuring impact of retained reinsurance credit risk on effectiveness of underlying reinsurance program in lower and upper layers respectively.
- The coefficient a is the positive weight assigned to $\mathbf{RM}_L$ to indicate the level of significance of this risk measure relative to $\mathbf{RM}_U$ in the minimisation of target function.

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Convex optimisation of exposure limits (2)

- The optimisation problem (6) can be reduced to convex programming and then solved numerically via simulation. The convexity of (6) is mainly due to convexity of chosen risk measures – standard deviation is convex, and TVaR is coherent which implies it is convex too.
- Direct use of TVaR in its canonical form would require calculation of VaR, which could be problematic when applying it to non-continuous random variables (e.g. random mixture of Bernouili random variables). This is overcome by replacing TVaR with the limiting function from its 'representation form'. In general, for a TVaR of a random variable V its corresponding representation form is:

$$\begin{aligned} \mathrm{TVaR}_\alpha[V] &= \inf_{x\in\mathbb{R}} \Upsilon_\alpha[V,x] \\ &= \inf_{x\in\mathbb{R}} \left\{ x + \frac{1}{1-\alpha}\mathbb{E}\left[(V-x)_+\right] \right\}. \end{aligned} \tag{7}$$

Convex optimisation of exposure limits (3)

- The TVaR representation form is induced by the following regulator's problem of finding an optimal solvency capital requirements $r[V]$ (from the set of risk measures) that minimises the total of policyholder deficit $\mathbb{E}\left[(V - r[V])_+\right]$ and the cost of capital $\varepsilon \times r[V]$ with $\varepsilon = 1 - \alpha,\ \varepsilon \in (0, 1)$:

$$\min_{r[V]} \left\{\varepsilon \times r[V] + \mathbb{E}\left[(V - r[V])_+\right]\right\}, \tag{8}$$

  which gives $\mathrm{VaR}_{1-\varepsilon}[V]$ as the optimal solution (please refer to **Denuit et al. 2005 [3]** for an elegant geometric proof of this result).

- Furthermore, when the random variable V is the linear combination of different random variables with coefficients $\mathbf{w}$ the function $\Upsilon_\alpha[V(\mathbf{w}), x]$ is convex in $(\mathbf{w}, x)$, and, as it was shown in **Rockafeler and Uryasev 2002 [4]**, the convexity holds for general random variables admitting discreteness.

Convex optimisation of exposure limits (4)

- By replacing the TVaR risk measure with the limiting function Υ we restate the target function in (6) as:

$$f\left(\mathbf{w}^U, \mathbf{w}^L, x, y\right) = \Upsilon_\alpha\left[C\left(\mathbf{w}^U, \mathbf{w}^L\right), x\right] + \begin{cases} a \times \Upsilon_{0.5}\left[Z + C_L\left(\mathbf{w}^L\right), y\right], & \text{(when using TVaR)} \\ a \times \mathbf{RM}_L\left[Z + C_L\left(\mathbf{w}^L\right)\right], & \text{(when using variance-based)} \end{cases}$$

which is convex in $\left(\mathbf{w}^U, \mathbf{w}^L, x, y\right)$, and apply numerical convex programming to minimise $f\left(\mathbf{w}^U, \mathbf{w}^L, x, y\right)$ with respect to $\left(\mathbf{w}^U, \mathbf{w}^L, x, y\right)$.

- Numerical example -

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Numerical example

Consider a general insurer that writes insurance business with the following profile of loss exposure:

- aggregate annual attritional loss Y that follows a log-normal distribution with mean 2.5Bn, CoV of 20%, and no reinsurance cover for large liability losses, i.e. $R_{LT} = 0$;
- aggregate quarterly catastrophe loss X_j following a Pareto distribution with mean 70M and CoV of 300%;
- the insurer buys an annual reinsurance cat XOL cover of 100M xs 1,900M for the main cat tower (upper layers), and a combined (XOL and Stop-Loss) cover for losses between 25M and 100M (lower layers), and all the covers can be purchased from reinsurers of credit quality not worse than BBB+;
- aggregate quarterly recoveries on cat losses from lower and upper layers are respectively
    - $R_L(X_j)$ following a Pareto distribution with mean 50M and CoV of 125%; and
    - $R_U(X_j)$ following a Pareto distribution with mean 70M and CoV of 400%.

Numerical example (2)

Cost of reinsurance program for both lower and upper layers varies by credit rating of reinsurer as follows (premiums in M) and the total reinsurance expense is budgeted at 260M:

	AAA	AA+	AA	AA-	A+	A	A-	BBB+
π_U	160	153	147	140	135	130	124	118
π_L	130	120	115	110	107	105	102	100

Both risk measures specific to upper(lower) layers of the reinsurance program are defined by a TVaR with 50-th percentile used for the risk measure specific to lower layers and 90-th percentile for the one specific to upper layers.

Numerical example (3)

The conditional default rates were calibrated in **Britt and Krvavych 2009 [2]** using reinsurance default modelling assumptions to recover unconditional default rates from the AM Best research study titled "Securitisation of Reinsurance Recoverables" published in August 2007:

Default rates - annual forward rates in 'normal' and 'stressed' states

Calibrated conditional 'normal' annual forward rates of default (in %)

YEAR	AAA	AA+	AA	AA-	A+	A	A-	BBB+
1	6.9×10^{-5}	0.038	0.089	0.148	0.218	0.277	0.350	0.744

Calibrated conditional 'stressed' annual forward rates of default (in %)

YEAR	AAA	AA+	AA	AA-	A+	A	A-	BBB+
1	0.987	2.466	3.688	4.839	6.016	6.926	7.951	12.589

It is assumed that the global reinsurance market enters 'stressed' state with the transition rate of 10% p.a., and that the duration of 'stress' state equals two years.

Numerical example (4)

Average loss rates given default modelled:

AAA	AA+	AA	AA-	A+	A	A-	BBB+
0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6

Optimal RCR exposure limits (in %):

	AAA	AA+	AA	AA-	A+	A	A-	BBB+
w^U	19.7	17.0	16.4	15.7	13.1	9.8	6.6	1.6
w^L	17.2	16.7	16.4	16.2	13.9	10.6	7.2	1.8

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