Insurance Risk EC for XL contracts with an inflation stability clause

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Agenda

- Superimposed inflation / stability clause
- Insurance Risk Solutions
- Non-Life Risk / Standard approach
- Non-Life Risk / Stochastic approach (gross of Re)
- Non-Life Reinsurance Risk
- Reinsurance Risk for limited XL contracts
- Extended and bivariate lognormal approximations

Superimposed inflation / stability clause

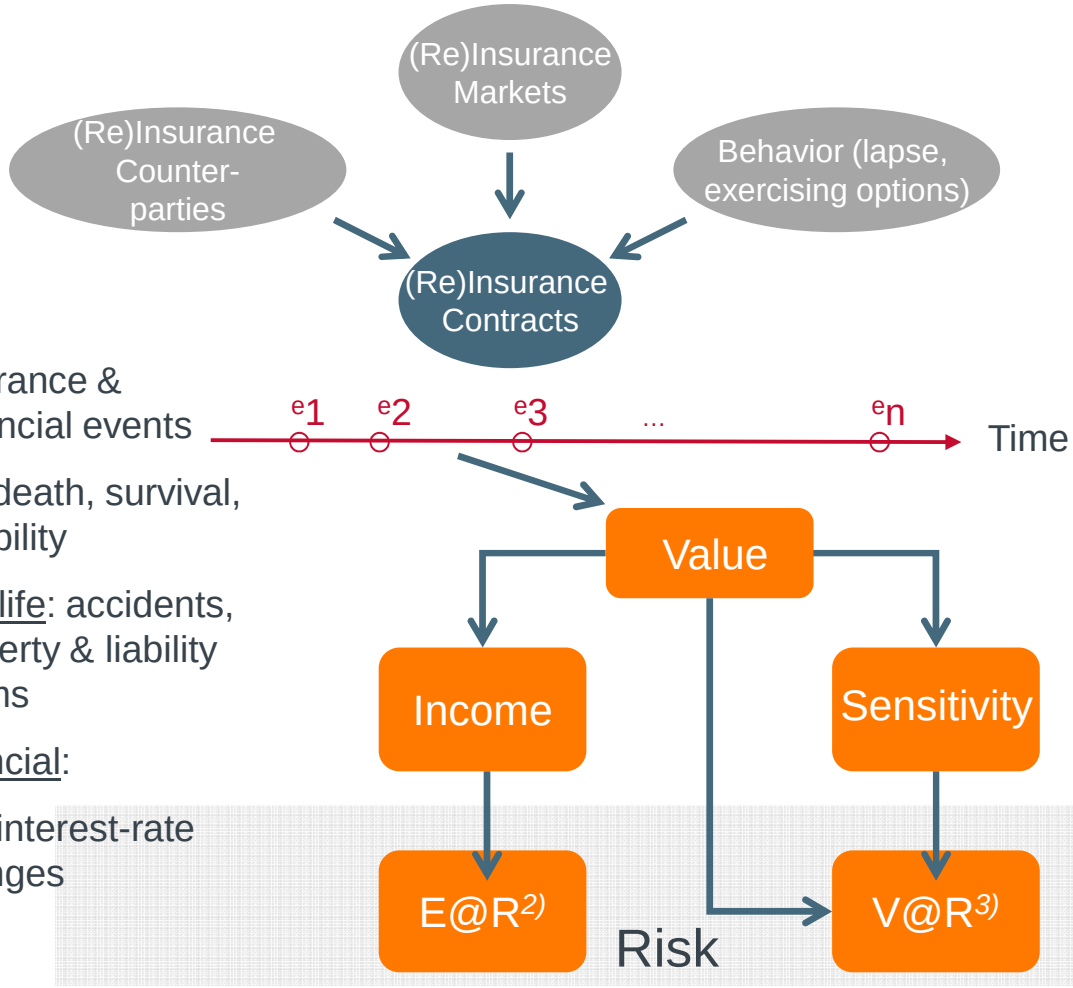
- Higher losses are subject to higher inflation than classical inflation, the so-called superimposed inflation

- Measurement: Masterson Claim Cost Index by Lines of Business

- Reinsurance impact
  - Assume fixed deductible of XL contract over claims development period
  - If a claim exceeds the deductible, then the reinsurer must cover all future increases due to superimposed inflation: inflation moral hazard

- Protection against inflation moral hazard: inflation stability clause
  - Future inflation is shared between the ceding company and the reinsurer
  - Example: "date of payment" stability clause
  - index the deductible and limit of XL contract using the ratio of the sum of actual payments to the sum of inflation adjusted payments

Insurance Risk Solutions



2) Earnings at risk. *3) Value at risk.*

- Any insurance risk analysis and reporting is a combination of:
  - Value
  - Income (=Change in Value)
  - Sensitivity
  - Risk

systematic deviations from biometric life tables, parameter risk, model risk, etc.

non-life premium & reserving risk, process risk, reinsurance risk, catastrophe risk, etc.

Non-Life Risk / Standard approach

- **Standard approach** (only current SCR for combined premium & reserve risk)

V : **Value** of combined volume measure for current year

σ : **Value** of combined standard deviation for current year

V@R measure

$$SCR = \rho_\alpha(\sigma) \cdot V$$

: current year **Solvency Capital Requirement** (**SCR**)

$$\rho_\alpha(\sigma) = \frac{\exp\left\{\Phi^{-1}(\alpha) \cdot \sqrt{\ln\left(1+\sigma^2\right)}\right\}}{\sqrt{1+\sigma^2}} - 1$$

(= value-at-risk of unexpected increase in log-normally distributed combined loss ratio at the confidence level α=99.5%)

Details of specification : QIS5(2010), Sandström(2011). Handbook of Solvency

Further readings : The Solvency II Handbook, Risk Books, Chap. 12 & 13

Non-Life Risk / Stochastic approach (gross of Re) (1)

Input: Line of Business (LoB) characteristics

N : random number of claims (e.g. Poisson distribution)

X_i : random individual claims, $X =_d X_i, \quad i = 1,\ldots,N$ (identical distribution)

$S = \sum_{i=1}^{N} X_i$: random sum of ultimate aggregate paid claims

$t_j = j,\ j = 1,\ldots,n$: claims payment dates ($t_n = n$: final settlement date)

$c(j),\ j = 1,\ldots,n$: claims payment pattern

$f(j),\ j = 0,1,\ldots,n$: inflation index

Cumulative Paid and Incurred Claims

$X_i(j) = c(j)\dfrac{f(j)}{f(0)} X_i,\ j = 1,\ldots,n$: future individual paid claims

$X_i^C(j) = \sum_{k=1}^{j} X_i(k),\ j = 1,\ldots,n,$: future individual cumulative paid claims

$R_i(j) = X_i^C(n) - X_i^C(j),\ j = 1,\ldots,n$: future individual claims reserves

$d(j),\ j = 1,\ldots,n$: future individual reserve deviation pattern

Non-Life Risk / Stochastic approach (gross of Re) (2)

$$X_i^I(j) = X_i^C(j) + d(j)R_i(j),\ j = 1,\dots,n$$: future individual incurred claims (= cumulative paid + outstanding)

Simplifying formulas:

$$X_i^C(j) = a(j)\cdot X_i,\quad X_i^I(j) = b(j)\cdot X_i,\quad j = 1,\dots,n$$ with the inflation adjusted factors

$$a(j) = \sum_{k=1}^{j} c(k)\frac{f(k)}{f(0)},\quad b(j) = a(j) + d(j)\cdot\big(a(n) - a(j)\big),\quad j = 1,\dots,n$$

- **Value of Aggregate Cumulative Paid and Incurred Claims**

$$S^C(j) = \sum_{i=1}^{N} X_i^C(j),\quad j = 1,\dots,n$$: future aggregate cumulative paid claims

$$S^I(j) = \sum_{i=1}^{N} X_i^I(j),\quad j = 1,\dots,n$$: future aggregate incurred claims

- **Income: Incremental Values of Aggr. Cumulative Paid and Incurred Claims**

$$\Delta S^C(1) = S^C(1),\quad \Delta S^C(j) = S^C(j) - S^C(j-1),\quad j = 2,\dots,n$$

$$\Delta S^I(1) = S^I(1),\quad \Delta S^I(j) = S^I(j) - S^I(j-1),\quad j = 2,\dots,n$$

Non-Life Risk / Stochastic approach (gross of Re) (3)

- **Aggregate Loss Reserves and Incremental Aggregate Loss Reserves**

$$R(j) = S^I(j) - S^C(j), \quad j = 1,\ldots,n$$

$$\Delta R(1) = S^I(1) - S^C(1), \quad \Delta R(j) = \Delta S^I(j) - \Delta S^C(j), \quad j = 2,\ldots,n$$

- **SCR and RM gross of Reinsurance**

Model assumption: deterministic pattern of future earned premiums

Then, the future unexpected increase of losses are given by

$$L(j) - E[L(j)] = \Delta S^C(j) - E[\Delta S^C(j)] + \Delta R(j) - E[\Delta R(j)] = \Delta S^I(j) - E[\Delta S^I(j)], \quad j = 1,\ldots,n$$

Solvency Capital Requirements (**SCR's**) gross of Reinsurance (**V@R** resp. **E@R**):

$$SCR_0^{VaR} = VaR_\alpha[S^I(1)] - E[S^I(1)], \quad SCR_k^{VaR} = VaR_\alpha[\Delta S^I(k+1)] - E[\Delta S^I(k+1)], \quad k = 1,\ldots,n-1$$

Risk margin (**RM**) gross of Reinsurance (restricted to premium & reserve risk):

$$RM^{VaR} = i_{CoC} \cdot \sum_{k=1}^{n-1} v_f^k \cdot SCR_k^{VaR}$$ (i_{CoC} : cost-of-capital rate; v_f : risk-free discount rate)

Non-Life Reinsurance Risk (1)

- **Input: splitting of LoB characteristics between Cedent and Reinsurer**

$X_i = X_{i,c} + X_{i,r}$: splitting of individual claims

$S = S_c + S_r$: splitting of ultimate aggregate paid claims $S_c = \sum_{i=1}^{N} X_{i,c} \quad S_r = \sum_{i=1}^{N} X_{i,r}$

- **Splitting of Incremental Aggregate Incurred Claims**

$$\Delta S^I(1) = \Delta S_c^I(1) + \Delta S_r^I(1) = S_c^I(1) + S_r^I(1), \quad \Delta S^I(j) = \Delta S_c^I(j) + \Delta S_r^I(j), \quad j = 2,\ldots,n$$

- **SCR and RM of Reinsurance**

Solvency Capital Requirements (**SCR's**) of Reinsurance (**V@R** resp. **E@R**):

$$SCR_{r,k}^{VaR} = VaR_\alpha\left[\Delta S_r^I(k+1)\right] - E\left[\Delta S_r^I(k+1)\right], \quad k = 0,\ldots,n-1$$

Risk margin (**RM**) of Reinsurance (restricted to premium & reserve risk):

$$RM_r^{VaR} = i_{CoC} \cdot \sum_{k=1}^{n-1} v_f^k \cdot SCR_{r,k}^{VaR}$$

Non-Life Reinsurance Risk (2)

- **Examples:** splitting (individual/aggregate) claims between Cedent and Reinsurer

Proportional Reinsurance

Example 1: quota-share $X_{i,c} = (1-q)\cdot X_i, \quad X_{i,r} = q\cdot X_i$

Example 2: surplus $X_{i,c} = \min\left(1, \frac{R}{SI_{h(i)}}\right)\cdot X_i, \quad X_{i,r} = \left(1-\frac{R}{SI_{h(i)}}\right)_+ \cdot X_i$

R line = maximal amount insurer is willing to pay for each policy

$SI_{h(i)}$ sum insured of the policy hit by the i-th individual claim

Non-Proportional Reinsurance

Example 3: excess-of-loss $X_{i,c} = \min(d, X_i), \quad X_{i,r} = \max(X_i - d, 0)$

Example 4: stop-loss $S_c = \min(d, S), \quad S_r = \max(S-d, 0)$

(splitting of ultimate aggregate paid claims)

Reinsurance Risk for limited XL contract (1)

Limited XL contract with an inflation stability clause

$$S_r = \sum_{i=1}^{N} X_{i,r},$$

$$X_{i,r} = \min\left(m, \left(X_i - \ell\right)_+\right)$$

Given are the following input characteristics:

$t_j = j,\ j = 1,\ldots,n$: claims payment dates

$c(j),\ j = 1,\ldots,n$: claims payment pattern

$f(j),\ j = 0,1,\ldots,n$: inflation index

$s(j),\ j = 0,1,\ldots,n$: superimposed inflation index

Then, the cumulative paid and incurred loss sequences are given by

$$X_{i,r}^{C}(j) = \min\left(r(j)m, \left(a(j)\cdot X_i - r(j)\ell\right)_+\right),$$

$$X_{i,r}^{I}(j) = \min\left(r(j)m, \left(b(j)\cdot X_i - r(j)\ell\right)_+\right), \quad j = 1,\ldots,n$$

with

$$r(j) = \frac{a(j)}{\sum_{k=1}^{j} c(k)\frac{s(k)}{s(0)}\frac{f(0)}{f(k)}}, \qquad a(j) = \sum_{k=1}^{j} c(k)\frac{s(k)}{s(0)},$$

$$b(j) = a(j) + d(j)\cdot\left(a(n) - a(j)\right), \quad j = 1,\ldots,n$$

Reinsurance Risk for limited XL contract (2)

Example: compound Poisson Pareto reinsurance model

$X =_d X_i, \quad i = 1,\ldots,N$: Pareto claims with distribution $F_X(x) = 1 - \left(\frac{x}{OP}\right)^{-\gamma}$,

$N \sim Poisson(\lambda)$: number of claims > OP

$x \geq OP > 0, \quad \gamma > 1$

Recall that $VaR_\alpha\left[\Delta S_r^I(j)\right]$ depends upon the incremental incurred losses

$$\Delta S_r^I(1) = S_r^I(1), \quad \Delta S_r^I(j) = S_r^I(j) - S_r^I(j-1), \quad j = 2,\ldots,n$$

Need to know the bivariate distributions of the random loss vectors

$$\left(S_r^I(j-1), S_r^I(j)\right), \; j = 2,\ldots,n$$

Pragmatic approach: bivariate lognormal approximations

Based on formulas for the mean, coefficient of variation of $S_r^I(j)$ and correlation coefficients between $S_r^I(j-1)$ and $S_r^I(j)$ (details in paper)

Reinsurance Risk for limited XL contract (3)

1) <u>Extended Solvency II standard approach</u>

Simple univariate lognormal approximation based on the mean and standard deviation of the spread $\Delta S_r^I(j) = S_r^I(j) - S_r^I(j-1)$

<u>Difficulty 1</u>: negative loss spreads (overstated loss reserves)

Apply a lognormal approximation to the profit spread:

<u>Proposition A.2</u> Let Z be a loss with non-zero finite mean and variance

<u>Case 1</u>: $\mu_+ = E[Z] > 0$, $k_+ = \sqrt{Var[Z]}/\mu_+$

$$SCR_\alpha^{VaR}[Z] = \left(\frac{\exp\left\{\Phi^{-1}(\alpha)\cdot\sqrt{\ln(1+k_+^2)}\right\}}{\sqrt{1+k_+^2}} - 1 \right)\cdot\mu_+, \quad SCR_\alpha^{CVaR}[Z] = \left(\frac{1-\Phi\left\{\Phi^{-1}(\alpha)-\sqrt{\ln(1+k_+^2)}\right\}}{\varepsilon} - 1 \right)\cdot\mu_+$$

<u>Case 2</u>: $\mu_- = E[-Z] > 0$, $k_- = \sqrt{Var[Z]}/\mu_-$

$$SCR_\alpha^{VaR}[Z] = \left(1 - \frac{\exp\left\{\Phi^{-1}(\varepsilon)\cdot\sqrt{\ln(1+k_-^2)}\right\}}{\sqrt{1+k_-^2}} \right)\cdot\mu_-, \quad SCR_\alpha^{CVaR}[Z] = \left(1 - \frac{\Phi\left\{\Phi^{-1}(\varepsilon)-\sqrt{\ln(1+k_-^2)}\right\}}{\varepsilon} \right)\cdot\mu_-$$

<u>Difficulty 2</u>: underestimation of the VaR and CVaR risk measures

Reinsurance Risk for limited XL contract (4)

2) <u>Bivariate lognormal approximation</u> (close to Solvency II standard model)

Assume $\left(S_r^I(j-1), S_r^I(j)\right),\ j = 2,\ldots,n$ follows a bivariate lognormal distribution

<u>Distribution and stop-loss transform of the bivariate lognormal spread</u>

Assume (S_1, S_2) bivariate lognormal with parameters $(\mu_1, \nu_1, \mu_2, \nu_2, \rho)$, i.e.

$(Z_1, Z_2) = \left(\dfrac{\ln S_1 - \mu_1}{\nu_1}, \dfrac{\ln S_2 - \mu_2}{\nu_2}\right)$ standard bivariate normal with correlation ρ

Let $\bar{F}(z) = P(S_1 - S_2 \geq z),\quad \pi(z) = E\left[(S_1 - S_2 - z)_+\right]$ denote the survival function and the stop-loss transform of the spread

<u>Proposition B.1</u> (Integral representations)

$$\bar{F}(z) = -\pi'(z) = \int_{-\infty}^{\infty} \Phi\left(A(y,z)\right) \cdot \varphi(y)dy,$$

$$\pi(z) = \exp\left(\mu_1 + \tfrac{1}{2}\nu_1^2\right) \cdot I_1(z) - \exp\left(\mu_2 + \tfrac{1}{2}\nu_2^2\right) \cdot I_2(z) - z \cdot \bar{F}(z),$$

$$I_1(z) = \int_{-\infty}^{\infty} \Phi\left(A(y + \rho\nu_1, z) + \sqrt{1-\rho^2} \cdot \nu_1\right) \cdot \varphi(y)dy, \quad I_2(z) = \int_{-\infty}^{\infty} \Phi\left(A(y + \nu_2, z)\right) \cdot \varphi(y)dy,$$

with $$A(y,z) = \frac{\mu_1 + \rho\nu_1 y - \ln\left(z + \exp(\mu_2 + \nu_2 y)\right)}{\nu_1 \cdot \sqrt{1-\rho^2}}$$

Reinsurance Risk for limited XL contract (5)

Special case: Margrabe's formula

$$\pi(0) = \exp\left(\mu_1 + \tfrac{1}{2}\nu_1^2\right)\cdot\Phi\left(\frac{\mu_1 - \mu_2 + \nu_1^2 - \rho\nu_1\nu_2}{\sqrt{\nu_1^2 + \nu_2^2 - 2\rho\nu_1\nu_2}}\right) - \exp\left(\mu_2 + \tfrac{1}{2}\nu_2^2\right)\cdot\Phi\left(\frac{\mu_1 - \mu_2 - \nu_2^2 + \rho\nu_1\nu_2}{\sqrt{\nu_1^2 + \nu_2^2 - 2\rho\nu_1\nu_2}}\right)$$

Analytical approximations (Deng, Li and Zhou(2008)). Idea: quadratic expansion of the auxiliary function $A(y+c,z)$ around $y = y_0 - c$ with $y_0 = 0$

Proposition B.2 (2nd order closed-form approx.) (details in paper)

Corollary B.3 (1st order closed-form approx., generalized Margrabe formula)

$$\bar{F}(z) \cong \Phi\left(\frac{a(z)}{\sqrt{1+b(z)^2}}\right), \quad \pi(z) \cong \exp\left(\mu_1 + \tfrac{1}{2}\nu_1^2\right)\cdot\Phi\left(\frac{a(z)+b(z)\cdot\rho\nu_1 + \sqrt{1-\rho^2}\cdot\nu_1}{\sqrt{1+b(z)^2}}\right)$$

$$-\exp\left(\mu_2 + \tfrac{1}{2}\nu_2^2\right)\cdot\Phi\left(\frac{a(z)+b(z)\cdot\nu_2}{\sqrt{1+b(z)^2}}\right) - z\cdot\Phi\left(\frac{a(z)}{\sqrt{1+b(z)^2}}\right).$$

Property 1: $\pi(0)$ is Margrabe's formula

Property 2: $\bar{F}(z) \to 0, \pi(z) \to 0$ as $z \to \infty$ (necessary VaR/CVaR conditions)