Dividend problems in the dual risk model

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Dual Risk Model


\[ U(t) = u - ct + S(t), \quad t \geq 0 \]

- Accumulated reserve up to time \( t \): \( U(t) \)
- \( u \): initial reserve, surplus;
- Individual random gains: \( X_i \sim P(x) \), \( \mathbb{E}[X_i] = p_1 \);
- Accumulated gains up to \( t \): \( S(t) = \sum_{i=0}^{N(t)} X_i \), \( X_0 \equiv 0 \). \( \{X_i\} \), i.i.d.
- \( \{X_i\} \) independent of \( N(t) \);
- \( \{S(t), t \geq 0\} \): compound Poisson process, parameters \( \lambda \) and p.d.f. \( p(x) \);
- \( c \): constant rate of expenses, \( \mu = E[S(1)] - c = \lambda p_1 - c > 0 \), \( \lambda p_1 > c \).
- \( b \geq u \): (Upper) dividend barrier (absorbing);
- Ruin barrier (absorbing).
Dual Risk Model

\[ U(t) = u - ct + S(t), \quad t \geq 0 \]
Problem: Dividend calculation

- We have 2 absorbing barriers: **Dividend barrier** $b$ and **Ruin** level “0”.
- A dividend can be paid if the process is not ruined.
- $b$ is fixed. Once reached a dividend is paid and process re-starts, from $b$.
- If the process is ruined, the investment is **insolvent**.
- Let $\delta (> 0)$ be the force of interest.
- Problem: Calculate the discounted future dividends, e.g., Expected.
- Dividends can be multiple.
- There are results on expected, moments of discounted dividends: Avanzi *et al.* (2007), Cheung & Drekic (2008), Ng (2009, 2010)
- We go further, with a new approach: connection the classical and the dual
Solutions for the dividend calculation

- Probability of ruin, with no dividend barrier: known.
- Setting a dividend barrier, we evaluate:
  - Expected discounted future dividend payments:
    - known, but new approach;
    - higher moments also available;
  - The probability of getting a dividend payment;
  - Distribution for the amount of a dividend payment;
  - Distribution for the number of dividends (infinite time).
Definitions

- Let, process free of the barrier, and continue if it is ruined: Time to ruin, from initial surplus $x$:

$$\tau_x = \inf \{ t > 0 : U(t) = 0 | U(0) = x \} ; \quad (\tau_x = \infty \text{ if } U(t) \geq 0 \ \forall t \geq 0)$$

- Probability of ultimate ruin

$$\psi(x) = \Pr \{ \tau_x < \infty | U(0) = x \} = e^{-Rx}$$

where $R > 0$: $\lambda \left( \int_0^\infty e^{-Rx} \rho(x) dx - 1 \right) = -cR$.

- Time to reach an upper level $b \geq x \geq 0$, from $x$:

$$T_x = \inf \{ t > 0 : U(t) > b | U(0) = x \}$$

- Probability of ruin before reaching $b$: $\xi(u, b) = \Pr (T_x > \tau_x)$.

- Probability of reaching $b$ before ruin: $\chi(u, b) = \Pr (T_x < \tau_x)$

- Single dividend amount: $D_u = \{ U(T_u) - b \wedge T_u < \tau_u \}$. D.f.:

$$G(u, b; x) = \Pr (T_u < \tau_u \text{ and } U(T_u) \leq b + x) | u, b)$$

- $M$: number of dividends to be distributed.
Total discounted dividends

- Total discounted dividends: $D(u, b, \delta)$

$$V(u; b) = \mathbb{E}[D(u, b, \delta)] = \mathbb{E} \left[ \sum_{i=1}^{\infty} e^{-\delta \left( \sum_{j=1}^{i} T(j) \right)} D(i) \right], \quad 0 \leq u \leq b .$$
Expected discounted dividends

- \( V(u; b) = \mathbb{E}[D(u, b, \delta)] \)
- From Avanzi et al. (2007), solutions from solving,

\[
0 = cV'(u; b) + (\lambda + \delta)V(u; b) - \lambda \int_0^{b-u} V(u + y, b)p(y)dy \quad (1)
\]

\[
- \lambda \int_0^\infty V(u - b + y, b)p(y)dy - \lambda V(b, b) [1 - P(b - u)]
\]

- Also, Laplace transforms from there;
- **Instead**, we propose solving

\[
V(u; b) = \mathbb{E}[D(u, b, \delta)] = \mathbb{E}\left[ \sum_{i=1}^\infty e^{-\delta(\sum_{j=1}^i T(j))} D(i) \right]
\]

- \( \left\{ \left( T(i), D(i) \right) \right\}_{i=2}^\infty \), sequence of i.i.d. random pairs.
- \( \left( T(i), D(i) \right) \overset{d}{=} (T_b, D_b) \)
- Independent of \( \left( T(1), D(1) \right) \). Let \( \left( T(1), D(1) \right) \overset{d}{=} (T_u, D_u) \).
Expected discounted dividends

Independence gives

\[ V(u; b) = \mathbb{E} \left[ \sum_{i=1}^{\infty} e^{-\delta (\sum_{j=1}^{i} T(j))} D(i) \right] = \sum_{i=1}^{\infty} \mathbb{E} \left[ e^{-\delta (\sum_{j=1}^{i} T(j))} D(i) \right] \]

\[ = \mathbb{E} \left( e^{-\delta T(1)} D(1) \right) + \mathbb{E} \left( e^{-\delta T(2)} \right) \mathbb{E} \left( e^{-\delta T(1)} D(2) \right) + \mathbb{E} \left( e^{-\delta T(2)} \right) \mathbb{E} \left( e^{-\delta T(3)} D(3) \right) + \ldots \]

Now, \((T_b, D_b) \overset{d}{=} (T(i), D(i)), i = 2, 3, \ldots; (T_u, D_u) \overset{d}{=} (T(1), D(1)),\)

\[ V(u; b) = \mathbb{E} \left( e^{-\delta T_u} D_u \right) + \mathbb{E} \left( e^{-\delta T_u} \right) \mathbb{E} \left( e^{-\delta T_b} D_b \right) \times \left[ 1 + \mathbb{E} \left( e^{-\delta T_b} \right) + \mathbb{E} \left( e^{-\delta T_b} \right)^2 + \ldots \right] \]

\[ = \mathbb{E} \left( e^{-\delta T_u} D_u \right) + \mathbb{E} \left( e^{-\delta T_u} \right) \frac{\mathbb{E} \left( e^{-\delta T_b} D_b \right)}{1 - \mathbb{E} \left( e^{-\delta T_b} \right)}. \]
Similarly,

\[ V_2(u; b) = \mathbb{E}[D(u, b, \delta)^2] = \sum_{i=1}^{\infty} \mathbb{E}[Z_i^2] + 2 \sum_{i=1}^{\infty} \sum_{j=i+1}^{\infty} \mathbb{E}[Z_iZ_j] \]

\[ Z_i = e^{-\delta(\sum_{j=1}^{i} T_{(j)})} D(i) \]

\[ \sum_{i=1}^{\infty} \mathbb{E}[Z_i^2] = \mathbb{E}\left(e^{-2\delta T_u} D_u^2\right) + \frac{\mathbb{E}\left(e^{-2\delta T_u}\right) \mathbb{E}\left(e^{-2\delta T_b} D_b^2\right)}{1 - \mathbb{E}\left(e^{-2\delta T_b}\right)} \]

\[ \sum_{i=2}^{\infty} \mathbb{E}[Z_iZ_j] = \frac{\mathbb{E}\left(e^{-2\delta T_u}\right) \mathbb{E}\left(e^{-2\delta T_b} D_b\right) \mathbb{E}\left(e^{-\delta T_b} D_b\right)}{[1 - \mathbb{E}\left(e^{-\delta T_b}\right)] [1 - \mathbb{E}\left(e^{-2\delta T_b}\right)]} \]
Discounted Dividend Expectations

- We need the expectations \( \mathbb{E}(e^{-\delta T_u} D_u), \mathbb{E}(e^{-\delta T_b} D_b), \mathbb{E}(e^{-\delta T_u}) \) and \( \mathbb{E}(e^{-\delta T_u}) \) ...
- Adapt from the classical severity, Dickson & Waters (2004),

\[
\phi_n(u^*, b, \delta) = \mathbb{E}[e^{-\delta T_u} D_u^n], \ n = 0, 1, 2, ..., \quad u^* = b - u
\]

\[
\frac{d}{du^*} \phi_n(u^*, b, \delta) = \frac{\lambda + \delta}{c} \phi_n(u^*, b, \delta) - \frac{\lambda}{c} \int_0^{u^*} \phi_n(u^* - y, b, \delta) p(y) dy
\]

\[-\frac{\lambda}{c} \int_{u^*}^{\infty} (y - u^*)^n p(y) dy.
\]

- Laplace transform of \( \phi_n(u^*, b, \delta) \) w.r.t. \( u^* \) ( \( \geq 0 \), extended )

\[
\bar{\phi}_n(s, b, \delta) = \frac{c \phi_n(0, b, \delta) - \lambda p_n \frac{1 - \bar{p}_n(s)}{s}}{cs - (\lambda + \delta) + \lambda \bar{p}(s)}.
\] (2)

with boundary condition \( (\phi_n(b, b, \delta) = 0) \) we find \( \phi_n(0, b, \delta) \).
Probability of a single dividend payment

**Definition**

\[ \chi(u, b) = \Pr \{ T_u < \tau_u \} \text{ and } \xi(u, b) = \Pr \{ T_u > \tau_u \}, \quad u \leq b, \]

Solutions through Integro-differential equations or Laplace transforms:

- Standard approach, \((t_0 : u - ct_0 = 0), \quad 0 < u < b.\)

\[ \xi(u, b) = e^{-\lambda t_0} + \int_0^{t_0} \lambda e^{-\lambda t} \int_0^{b-(u-ct)} p(x) \xi(u - ct + x, b) \, dx \, dt, \]

differentiating and rearranging (Exact solutions in some cases)

\[ \lambda \xi(u, b) + c \frac{d}{du} \xi(u, b) = \lambda \int_0^{b-u} p(x) \xi(u + x, b) \, dx \]  \hspace{1cm} (3)

\[ \frac{d}{du} \log \xi(u, b) = \frac{\lambda}{c} \left( \int_0^{b-u} p(x) \xi(x, b-u) \, dx - 1 \right) \]

Boundary condition: \( \xi(0, b) = 1. \)
Laplace transform:
Change of variable: $z = b - u$ and $\mathcal{E}(z, b) = \bar{\xi}(b - z, b) = \bar{\xi}(u, b)$

$$\lambda \mathcal{E}(z, b) - c \frac{\partial}{\partial z} \mathcal{E}(z, b) - \lambda \int_0^z p(z - y)\mathcal{E}(y, b) \, dy = 0.$$ 

In $\mathcal{E}(z, b)$ extend the range of $z$ from $0 \leq z \leq b$ to $0 \leq z \leq \infty$, resulting function by $\epsilon(z)$

Compute its Laplace transform, $\bar{\epsilon}(s)$, [$\bar{p}(s)$ is transform of $p(x)$]

$$\bar{\epsilon}(s) = \frac{c \epsilon(0)}{cs - \lambda + \lambda \bar{p}(s)}$$

$\epsilon(0) = \bar{\xi}(b, b), \ [\epsilon(b) = \mathcal{E}(b, b) = \bar{\xi}(0, b) = 1].$
Single dividend amount distribution

Definition

\[ G(u, b; x) = \Pr(T_u < \tau_u \text{ and } U(T_u) \leq b + x) | u, b) \]

- Ruin probability \( \psi(x) = e^{-Rx} \) can be written as

\[
\psi(u) = \xi(u, b) + \int_{0}^{\infty} g(u, b; x)\psi(b + x)\,dx \\
= \xi(u, b) + \psi(b) \int_{0}^{\infty} g(u, b; x)\psi(x)\,dx
\]

\[
\xi(u, b) = e^{-Ru} - e^{-Rb}\bar{g}(u, b; R)
\]

Integro-differential equation for \( G(u, b; x) \), using usual procedure,

\[
G(u, b; x) = \int_{0}^{t} \lambda e^{-\lambda t} \left[ \int_{0}^{b-(u-ct)} p(y) G(u - ct + y, b; x) \,dy \\
+ \int_{b-(u-ct)+x}^{b-(u-ct)+x} p(y)\,dy \right] \,dt.
\]
Single dividend amount distribution

- Rearranging and differentiating.

\[
\lambda G(u, b; x) + c \frac{\partial}{\partial u} G(u, b; x) = \lambda \int_u^b p(y - u) G(y, b; x) \, dy
\]
\[
+ \lambda \left[ P(b - u + x) - P(b - u) \right].
\]

- Boundary condition \( G(0, b; x) = 0 \). Similarly,

\[
\lambda g(u, b; x) + c \frac{\partial}{\partial u} g(u, b; x) = \lambda \int_0^{b-u} p(y) g(u + y, b; x) \, dy + \lambda p(b - u + x).
\]

- Let \( \mathcal{G}(z, b; x) = G(b - z, b; x) \). Then \( \mathcal{G}(b, b; x) = G(0, b; x) = 0 \)
Single dividend amount distribution

Use of **Laplace transforms**

\[
\lambda G(z, b; x) - c \frac{\partial}{\partial z} G(z, b; x) - \lambda \int_0^z p(z - y) G(y, b; x) \, dy \\
- \lambda (P(z + x) - P(z)) = 0
\]

Let \( \rho(z, x) \) be the function resulting from extending the range of \( z \).

Taking Laplace transforms,

\[
\lambda \bar{\rho}(s, x) - c \left[ s \bar{\rho}(s, x) - \rho(0, x) \right] - \lambda \bar{\rho}(s, x) \bar{p}(s) \\
+ \lambda \left[ \frac{\bar{p}(s)}{s} - \hat{p}(s, x) \right] = 0,
\]

\[
\bar{\rho}(s, x) = \frac{c \rho(0, x) + \lambda \left[ \bar{p}(s) / s - \hat{p}(s, x) \right]}{cs - \lambda + \lambda \bar{p}(s)} \tag{6}
\]

where

\[
\hat{p}(s, x) = \int_0^\infty e^{-sz} P(z + x) \, dz = e^{sx} \int_x^\infty e^{-sy} P(y) \, dy.
\]
Using the Classical Model

Consider the process continuing even if ruin occurs. The process can cross the upper dividend level before or after ruin.

**Definition**

(proper) Distribution of the amount by which the process first upcrosses $b$, 

$$H(u, b; x) = \Pr [U(T_u) \leq b + x]$$

$$H(u, b; x) = H(u, b; x| T_u < \tau_u)\chi(u, b) + H(u, b; x| T_u > \tau_u)\zeta(u, b)$$

$$= G(u, b; x) + \zeta(u, b)H(0, b; x).$$

- Equation above simply means that $H(u, b; x)$ equals the probability of a dividend claim less or equal than $x$ plus the probability of a similar amount but that cannot be a dividend.
Using the Classical Model

- Compute now $H(u, b; x)$, through expressions of the severity of ruin from the classical model

\[
G^*(b - u; x) = G(u, b; x) + \xi(u, b)G^*(b; x)
\]

\[
G(u, b; x) = G^*(b - u; x) - \xi(u, b)G^*(b; x)
\]

- For $u = b$,

\[
G(b, b; x) = G^*(0; x) - \xi(b, b)G^*(b; x)
\]

\[
g(b, b; x) = g^*(0; x) - \xi(b, b)g^*(b; x)
\]

- Calculate Laplace transforms at $R$ and (5)

\[
\xi(b, b) = \frac{1 - \overline{g^*}(0; R)}{1 - \overline{g^*}(b; R)e^{-Rb}} e^{-Rb}
\]

with $g^*(0; x) = p_1^{-1} [1 - P(x)]$ and

\[
\overline{g^*}(0; R) = \frac{1}{R\lambda p_1} (1 - \int_0^\infty e^{-Rx} p(x)dx) = c / \lambda p_1.
\]
Number of dividend payments

**Definition**

Let $M$ : number of dividends to be claimed, or the number of times the process upcrosses the upper level $b$. $M$ follows a zero-modified geometric distribution.

$$
\Pr[M = 0] = \zeta(u, b)
$$

$$
\Pr[M = k] = \chi(u, b) \chi(b, b)^{k-1} \zeta(b, b), \quad k = 1, 2, \ldots
$$

Hence, the total amount of dividend payments (not discounted) follows a zero modified compound distribution.
Illustration

Example

\[ p(x) = \alpha e^{-\alpha x}, \ x > 0, \ (\alpha > 0) \]

\[
E \left( e^{-\delta T_u} D_u^n \right) = \frac{n! \lambda}{\alpha^n} \left( \frac{\alpha}{c} \right) \frac{e^{-r_2 u} - e^{-r_1 u}}{(r_1 + \alpha) e^{-r_2 b} - (r_2 + \alpha) e^{-r_1 b}}
\]

\[
V(u, b, \delta) = \frac{\lambda}{\alpha} \frac{e^{-r_2 u} - e^{-r_1 u}}{(\delta - cr_2) e^{-r_2 b} - (\delta - cr_1) e^{-r_1 b}},
\]

where \( r_1 < 0 \) and \( r_2 > 0 \) are solutions of the equation (7)

\[
s^2 + \left( \alpha - \frac{\lambda + \delta}{c} \right) s - \frac{\alpha \delta}{c} = 0. \quad (7)
\]
Illustration (cont’d)

\[ \chi(u, b) = \frac{\lambda - \lambda e^{-Ru}}{\lambda - \alpha ce^{-Rb}}; \quad \xi(u, b) = \frac{\lambda e^{-Ru} - \alpha ce^{-Rb}}{\lambda - \alpha ce^{-Rb}}. \]
Illustration (cont’d)

\[ \chi(u, b) = \frac{\lambda - \lambda e^{-Ru}}{\lambda - \alpha ce^{-Rb}}; \quad \bar{\zeta}(u, b) = \frac{\lambda e^{-Ru} - \alpha ce^{-Rb}}{\lambda - \alpha ce^{-Rb}}. \]

\[ G(u, b; x) = (1 - e^{-\alpha x}) \frac{\lambda - \lambda e^{-Ru}}{\lambda - \alpha ce^{-Rb}}. \]

\[ \frac{G(u, b; x)}{\chi(u, b)} = 1 - e^{-\alpha x}. \]
Illustration (cont’d)

\[ \chi(u, b) = \frac{\lambda - \lambda e^{-Ru}}{\lambda - \alpha ce^{-Rb}}; \quad \xi(u, b) = \frac{\lambda e^{-Ru} - \alpha ce^{-Rb}}{\lambda - \alpha ce^{-Rb}}. \]

\[ G(u, b; x) = (1 - e^{-\alpha x}) \frac{\lambda - \lambda e^{-Ru}}{\lambda - \alpha ce^{-Rb}}. \]

\[ \frac{G(u, b; x)}{\chi(u, b)} = 1 - e^{-\alpha x}. \]

\[ \mathbb{E} \left( e^{-\delta T_u} D_u^n \mid T_u < \tau_u \right) = \mathbb{E} \left( D_u^n \mid T_u < \tau_u \right) \mathbb{E} \left( e^{-\delta T_u} \mid T_u < \tau_u \right) \]
Illustration (cont’d)

Let $\lambda = \alpha = 1$, $c = 0.75$ and $\delta = 0.02$

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Table: Values for $V(u, b, 0.02)$ for $u = 1, 3, 5, 10, 15, 20; b = 2, 3, 6, 10, 30, 40$
Let $\lambda = \alpha = 1$, $c = 0.75$ and $\delta = 0.02$

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Table: Values for $\mathbb{E} \left( e^{-\delta T_u D_u} \right)$ for $u = 1, 3, 5, 10, 15, 20$; $b = 2, 3, 6, 10, 30, 40$
Let $\lambda = \alpha = 1$, $c = 0.75$ and $\delta = 0.02$

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</table>

**Table:** Values for $\chi(u, b)$ for $u = 1, 3, 5, 10, 15, 20$; $b = 2, 3, 6, 10, 30, 40$


