Provisions for Loss Adjustment Expenses

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Abstract
In this paper two different approaches to the estimation of provisions for loss adjustment expenses (LAE) are presented. We explain a heuristic version of the New York Method (the 50-50 rule), see for example Johnson (1989) or Strain (1984), and present an easily applicable calibration method. Along the lines of Buchwalder et al. (2006) we attempt to formalize the latter idea in a stochastic framework. From the stochastic framework cash-flows for the provisions for the LAE can be estimated and we propose a way of calculating the run-off result.

Keywords: Loss Adjustment Expenses, New York Method (the 50-50 rule), Overdispersed Poisson Model, Chain Ladder, Run-Off Result and Cash-Flow.

1 Introduction

Provisions for loss adjustment expenses (LAE provisions) are a mandatory number in every annual report for non-life insurance companies. Typically the LAE expenses are split in two parts: The allocated and the unallocated LAE. This paper deals exclusively with the unallocated part. As described in Buchwalder et al. (2006) the allocated LAE can be considered as a part of the claim payments and hence they are often included in the regular claims provisions. The provision for the allocated LAE can thus be included in the IBNR provisions.
The claims provisions consist of two parts: one part represents the claims which are Reported But Not Settled (RBNS), these provisions are sometimes called the case provisions. The other part represents the claims which are Incurred But Not Reported (IBNR).
There is no clear consensus or industry standard regarding the calculation of LAE provisions. However, a very common solution is to use a certain percentage of claims provisions. Usually no arguments are given as to how to estimate this percentage and the reason why this approach is meaningful. This paper attempts to describe this problem mathematically and put forward a simple and easily applicable solution.

The paper is organized as follows: In Section 2 the New York Method is described and a new way of calibrating the method is presented. In Section 3 we propose a way of formalizing the idea of LAE provisions in a mathematical framework. A data example is considered in Section 4.

2 The New York Method

The New York Method is a simple way of calculating the LAE provisions (related to the unallocated LAE). It is based on the following rationale: for some line of business it is assumed that the LAE are proportional to the total claim sum (paid-to-paid principle). The total LAE hence equals

$$LAE \;=\; \varepsilon\,(Paid + RBNS + IBNR) \quad , \tag{1}$$

where $\varepsilon > 0$ is a constant that determines the loss adjustment expense ratio. The LAE ratio obviously depends on the line of business in focus.
One can split the total LAE into a part that has already been settled and a part needed to settle all claims fully. The part of the LAE corresponding to the paid claims, $\varepsilon Paid$, has already been used, whereas the part corresponding to the IBNR claims, $\varepsilon IBNR$, has to be set aside as a provision. However LAE corresponding to the RBNS claims, $\varepsilon RBNS$, are split, because some of the LAE have been used for opening of the claim files and some will be used for settling open claims. Therefore it is assumed that a constant $\omega \in [0, 1]$ denotes the used part of the LAE, and $(1 - \omega)$ is the part of the LAE which is used to run-off the RBNS provisions.
We hence conclude that the used part of the LAE is given by

$$LAE_{Used} = \varepsilon \left(Paid + \omega RBNS \right) \quad ,$$

whereas the LAE provisions are given by

$$LAE_{Prov} = \varepsilon \left(IBNR + (1 - \omega) RBNS \right) \quad . \tag{2}$$

The constant ω should be decided in cooperation with the claims handling department.
In the standard New York Method it is assumed that $\omega = 50\%$ which means that half of the LAE are used to open the claim files and setup the RBNS provisions and the other half is used to run-off the RBNS provisions (the 50-50 rule). From here onwards ω is considered a fixed and known number.

2.1 Calibration of the New York Method

In this section a way of estimating ε is presented. Consider a period of time, for example one year. The claims provisions at the beginning and end of the year are denoted by $RBNS_1$, $IBNR_1$ and $RBNS_2$, $IBNR_2$, respectively. $IBNR_2$ and $RBNS_2$ are split in two; the part regarding the newest underwriting year, $IBNR_2^{new}$, $RBNS_2^{new}$, and the part regarding the old underwriting years, $RBNS_2^{old}$, $IBNR_2^{old}$, such that $RBNS_2^{new} + RBNS_2^{old} = RBNS_2$ and $IBNR_2^{new} + IBNR_2^{old} = IBNR_2$. Denote by ΔLAE_{Used} the LAE which have been used during the year under consideration and let $\Delta Paid^{new}$ be the amount of payments related to the newest underwriting

year. It now holds that a zero-run-off of the LAE provisions is equivalent to

$$\begin{aligned}
&\varepsilon\left(IBNR_1+(1-\omega)RBNS_1\right)+\varepsilon\left(IBNR_2^{new}+RBNS_2^{new}+\Delta Paid^{new}\right)\\
&=\\
&\varepsilon\left(IBNR_2+(1-\omega)RBNS_2\right)+\Delta LAE_{Used} \quad .
\end{aligned}$$

By reordering the terms we obtain the following

$$\begin{aligned}
\tfrac{1}{\varepsilon}\Delta LAE_{Used} &= \left(IBNR_1+(1-\omega)RBNS_1\right)\\
&\quad+\left(IBNR_2^{new}+RBNS_2^{new}+\Delta Paid^{new}\right)\\
&\quad-\left(IBNR_2+(1-\omega)RBNS_2\right) \quad .
\end{aligned} \tag{3}$$

The total payments which have been made during the year under consideration are given by

$$\Delta Paid=\Delta Paid^{old}+\Delta Paid^{new} \quad ,$$

where $\Delta Paid^{old}$ is related to the old underwriting years.
The run-off result for the claims provisions is moreover defined by

$$RO_2=\left(IBNR_1+RBNS_1\right)-\left(IBNR_2^{old}+RBNS_2^{old}\right)-\Delta Paid^{old} \quad .$$

Combining the run-off result with equation (3) and rearranging the terms yields that

$$\begin{aligned}
\tfrac{1}{\varepsilon}\Delta LAE_{Used} &= \left(IBNR_1+RBNS_1\right)-\left(IBNR_2^{old}+RBNS_2^{old}\right)-\Delta Paid^{old}\\
&\quad-\omega RBNS_1+\omega RBNS_2+\Delta Paid^{old}+\Delta Paid^{new}\\
&= RO_2+\Delta Paid+\omega\left(RBNS_2-RBNS_1\right)
\end{aligned}$$

or equivalently

$$\widehat{\varepsilon} = \frac{\Delta LAE_{Used}}{RO_2+\Delta Paid+\omega\left(RBNS_2-RBNS_1\right)} \quad . \tag{4}$$

However, this calculation will be affected by some random noise. Therefore, in the estimating procedure of ε one should also take some actuarial judgement into account or/and data from more than one accounting period.

If the portfolio is roughly constant over time and $(RBNS_2-RBNS_1)$ and RO_2 are small compared to $\Delta Paid$ then ε is roughly proportional to $\Delta Paid$. One can hence approximate (4) by

$$\widehat{\varepsilon}\approx\frac{\Delta LAE_{Used}}{\Delta Paid} \quad . \tag{5}$$

The LAE provisions are obtained by inserting $\hat{\varepsilon}$ into equation (2).

3 The Overdispersed Poisson Model

In this section we consider an Overdispersed Poisson Model in the spirit of Venter (2007) and England and Verrall (1999). The idea is to calculate the LAE provision in the same framework as the one used to calculate claims provisions.
Denote by X_{ij} the entries in a paid incremental run-off triangle with dimension n and let Y_{ij} be the entries in the corresponding incurred incremental run-off triangle. It is assumed that the vectors $(X_{ij}, Y_{ij}), 1 \leq i, j \leq n$, are mutually independent and that

$$\begin{aligned} EX_{ij} &= T_i\beta_j \qquad \text{and} \qquad VX_{ij} = \phi EX_{ij} \\ EY_{ij} &= T_i'\beta_j' \qquad \text{and} \qquad VY_{ij} = \phi' EY_{ij} \quad . \end{aligned}$$

For the model to be identifiable we assume that

$$\sum_{j=1}^{n} \beta_j = \sum_{j=1}^{n} \beta_j' = 1 \quad . \tag{6}$$

We start by considering a simplified model where each triangle (X_{ij}) and (Y_{ij}) are considered separately. McCullagh and Nelder (1989) suggest that a Poisson quasi-likelihood equivalent to the likelihood considered in Mack (1991) can be maximized to obtain the parameter estimators.
To predict the unknown entries the classic triangle methods, chain ladder, Bornhuetter-Ferguson and Benkthander, can be obtained, by estimating T_i respectively, by

$$\begin{aligned} \hat{T}_i &= \frac{\sum_{j=1}^{n+1-i} X_{ij}}{\sum_{j=1}^{n+1-i} \beta_j} \\ \hat{T}_i &= T_i \\ \hat{T}_i &= \sum_{j=1}^{n+1-i} X_{ij} + T_i \left(1 - \sum_{j=1}^{n+1-i} \beta_j\right) \quad . \end{aligned}$$

By applying the Langranges method under the identification condition (6) the estimates of β_j are connected to the chain ladder factors in the following sense

$$\hat{\beta}_k = \frac{1}{\prod_{j=k}^{n-1} \hat{f}_j} \left(1 - \frac{1}{\hat{f}_{k-1}}\right) \quad , 1 \leq k \leq n,$$

where $\hat{f}_k$ denote the chain ladder factors defined by

$$\hat{f}_k = \frac{\sum_{i=1}^{n-k} \sum_{j=1}^{k+1} X_{ij}}{\sum_{i=1}^{n-k} \sum_{j=1}^{k} X_{ij}} \quad , k = 1, \ldots, n-1.$$

Notice that X_{ij} and Y_{ij} are dependent and the estimation of β_j and β'_j should therefore be carried out simultaneously. The suggested estimation method is hence not optimal. The simultaneous quasi-maximum-likelihood equations for (X_{ij}, Y_{ij}) do however not have explicitly given formulas for the solutions. The problem can of course be solved numerically, but this process is considered to be outside the scope of this paper.

If one or more of the chain ladder factors for the incurred triangle, $\hat{f}'_k$, are less than 1, then the variance assumption in the Overdispersed Poisson Model is not fulfilled. In practice one can often solve this problem by defining the relevant chain ladder factors to be 1. Especially if the LAE provisions are the main point of interest, we would recommend this pragmatic approach.

To obtain a model that has the same properties as the New York method (50-50 rule), it is assumed that the payment pattern of the LAE have the following structure

$$\tilde{\beta}_j = \left(\omega\beta_j + (1-\omega)\beta'_j\right)\varepsilon \quad ,$$

where $\omega \in [0,1]$ and $\varepsilon > 0$ have exactly the same interpretation as in section 2.
By noticing that the last diagonal of a run-off triangle corresponds to the the latest calendar year and by using the notation from section 2, it can be assumed that

$$\Delta LAE_{Used} = \sum_{i=1}^{n} \tilde{T}_i \tilde{\beta}_{n+1-i} \quad ,$$

such that

$$\hat{\varepsilon} = \frac{\Delta LAE_{Used}}{\sum_{i=1}^{n} \tilde{T}_i(\omega\beta_{n+1-i} + (1-\omega)\beta'_{n+1-i})} \quad . \tag{7}$$

Data studies indicate that estimation of ε using (7) is a more robust approach compared to the method given in section 2.

The LAE provision for the k'th underwriting year can be written as

$$LAE_{Prov}^{(k)} = \sum_{j=n+2-k}^{n} \tilde{T}_k \tilde{\beta}_j \quad 2 \leq k \leq n. \tag{8}$$

This is similar to the calculation of the claims provisions where one sums over the same indices but depending on the reserving model chooses different T_k's and β_j's.

3.1 Cash-flow

A major advantage of this method compared to the New York Method, is the ability to estimate a cash-flow for the LAE provisions. To obtain the expected cash-flow for the LAE in the future periods CF_k^{LAE}, $k > n$, we sum along the diagonals:

$$CF_{LAE}^{(k)} = \sum_{i=k+1-n}^{n} \tilde{T}_i \tilde{\beta}_{k+1-i} \quad , k > n. \tag{9}$$

3.2 Run-off Result

It may be relevant to calculate run-off result for the LAE provisions. This calculation can be carried out by using the following ad hoc method. Split ΔLAE_{Used} by

$$\Delta LAE_{Used}^{(k)} = \frac{\tilde{T}_k \tilde{\beta}_{n+1-k}}{\sum_{i=1}^{n} \tilde{T}_i \tilde{\beta}_{n+1-i}} \Delta LAE_{Used} \quad , 1 \leq k \leq n. \tag{10}$$

This gives an allocation of the used LAE to the individual underwriting years. The run-off result for the k'th underwriting year can thus be defined as

$$RO_{LAE}^{(k)} = LAE_{Prov}^{(k)}(Old) - \left(\Delta LAE_{Used}^{(k)} + \sum_{j=n+2-k}^{n} \tilde{T}_k \tilde{\beta}_j \right) \quad , 1 \leq k \leq n-1, \tag{11}$$

where $LAE^{(k)}_{Prov}(Old)$ denotes the loss adjustment expense provision for the k'th underwriting year evaluated at the beginning of the year. The results derived in section 3.1 and 3.2 make it possible to treat the LAE provision just like an ordinary claims provision with respect to discounting and run-off calculations.

4 Data study

In this section a data set from the LB Group is considered. Data contains 13 years of run-off for a portfolio of personal accident insurances. Paid and Incurred run-off triangles are given below in Table 1 and Table 2.

$i\backslash j$	1	2	3	4	5	6	7	8	9	10	11	12	13
1	675	2685	1894	608	487	135	87	171	94	27	115	12	14
2	504	2783	2165	497	196	146	145	73	26	44	18	18	
3	450	2635	1949	675	269	120	37	68	61	31	136		
4	479	3442	1592	951	270	108	212	38	7	23			
5	561	3708	2346	649	428	192	47	33	17				
6	569	4052	2647	890	335	233	51	53					
7	875	5009	2962	768	213	294	4						
8	688	5394	3187	928	322	316							
9	707	6223	3501	799	609								
10	1232	6652	4154	1060									
11	1102	6711	4734										
12	1099	7054											
13	970												

Table 1: Paid incremental run-off triangle.

$i\backslash j$	1	2	3	4	5	6	7	8	9	10	11	12	13
1	8676	−1636	−510	−141	252	87	117	162	51	120	222	−32	33
2	8649	−694	−1284	−128	3	66	81	58	0	29	8	2	
3	8262	−1094	−674	25	−120	−40	32	74	41	−3	214		
4	8186	78	−1546	174	221	45	189	23	6	−50			
5	9500	−461	−1094	51	150	61	23	21	5				
6	10227	−398	−636	−274	84	58	10	18					
7	12266	−84	−1512	−277	185	11	−64						
8	13823	−1196	−1847	122	60	176							
9	13492	456	−1124	−569	92								
10	15189	1546	−1727	−947									
11	15522	2534	−3636										
12	15816	−152											
13	17817												

Table 2: Incurred incremental run-off triangle.

The amount of LAE used for the personal accident portfolio during the period which corresponds to the last diagonal of the triangles equals

$$\Delta LAE_{Used} = 780 \quad .$$

When it comes to determining the parameter $\omega \in [0, 1]$, we decided to use the standard

$$\omega = 50\% \quad .$$

4.1 Data study: New York Method

To apply the New York Method, the run-off result for the claims provisions is needed. We have therefore made a simple approach using a paid chain ladder method to obtain the IBNR provision, ignoring tail factors and other types of adjustments.
By applying the chain ladder method to the paid triangle in Table 1 we obtain

$$RBNS_2^{Old} + IBNR_2^{Old} = 14,592 \quad ,$$

which is the claims provisions regarding underwriting years $1, \ldots, 12$ evaluated at the end of the 13th period.

To obtain the comparable provisions evaluated at the end of the 12th period we have removed the last diagonal from Table 1 and applied the chain ladder method to this reduced table,

$$RBNS_1 + IBNR_1 = 27,437 \quad ,$$

which is the claims provisions regarding the underwriting years $1, \ldots, 12$ evaluated at the end of the 12th period.
We are now able to calculate the run-off result which is

$$RO_2 = 27,437 - 14,592 - 14,036 = -1,191 \quad .$$

By using formula (4), the LAE ratio is estimated

$$\hat{\varepsilon} = \frac{780}{-1,191 + 15,007 + 50\% \, (30,006 - 31,505)} = 6.0\% \quad .$$

The LAE provisions are calculated using formula (2),

$$LAE_{Prov} = 6.0\% \, (-2,800 + 50\% \cdot 30,006) = 728 \quad .$$

On the other hand if one chooses the approximated LAE ratio, in formula (5),

$$\hat{\varepsilon} \approx \frac{780}{15,007} = 5.2\% \quad ,$$

then by using formula (2) the corresponding LAE provisions are

$$LAE_{Prov} \approx 5.2\% \, (-2,800 + 50\% \cdot 30,006) = 634 \quad .$$

4.2 Data study: The Overdispersed Poisson Model

In this section we will apply the model from Section 3 to the data in Table 1 and 2. We start out by estimating the row parameters, $(\tilde{T}_i)$, the three sets of column parameters, $(\tilde{\beta}_j, \beta_j, \beta'_j)$, and let $\omega = 50\%$, see Table 3.
As in the previous section, we rely our IBNR prediction on the chain ladder method applied to the paid data in Table 1 and we therefore set the row

parameters for the LAE prediction equal to the ones from the paid data:

j	$\tilde{T}_j$	$\tilde{\beta}_j$	$\hat{\beta}_j$	$\hat{\beta}'_j$
1	7004	58.9%	7.1%	110.6%
2	6626	22.1%	45.0%	−0.9%
3	6458	7.2%	28.4%	−13.9%
4	7247	3.2%	8.3%	−2.0%
5	8162	2.5%	3.9%	1.1%
6	9082	1.5%	2.3%	0.7%
7	10522	0.9%	1.1%	0.7%
8	11384	0.9%	1.0%	0.8%
9	12748	0.4%	0.6%	0.3%
10	14732	0.4%	0.5%	0.3%
11	15570	1.7%	1.3%	2.1%
12	15630	0.0%	0.2%	−0.2%
13	13584	0.3%	0.2%	0.4%

Table 3: Estimated parameters for the LAE projection, $(\tilde{T}_j, \tilde{\beta}_j)$, and estimated column parameters from Table 1 and Table 2.

The LAE ratio is estimated by using formula (7),

$$\hat{\varepsilon} = 5.6\% \quad .$$

By using formula (8) and (9) the LAE provisions and the corresponding cash-flow are estimated. See Table 4 below

k	14	15	16	17	18	19	20	21	22	23	24	25	LAE_{Prov}
$CF_{LAE}^{(k)}$	314	143	88	65	46	36	30	24	21	16	3	2	786

Table 4: LAE cash flow and LAE provision.

4.3 Run-off Results for the LAE Provisions

In this section we will calculate the run-off result for the LAE provisions by using formula (11).
To obtain the LAE provisions evaluated at the end of the 12th period, we

have kept ω and ε fixed and used the reduced triangles to reestimate the row and column parameters, by using formula (8) on the old data we have obtained $\left(LAE^{(k)}_{Prov}(Old)\right)$.
By using formula (10) and (11), we are able to calculate the run-off result for the LAE provisions and the results are shown in Table 5 below

i	$LAE^{(i)}_{Prov}(Old)$	$\Delta LAE^{(i)}_{Used}$	$LAE^{(i)}_{Prov}$	$RO^{(i)}_{LAE}$
1	0	1	0	-1
2	0	0	1	-1
3	4	6	1	-3
4	7	2	8	-3
5	11	2	11	-2
6	17	4	15	-2
7	26	5	22	-1
8	37	10	29	-2
9	59	18	44	-3
10	100	26	71	3
11	166	63	103	0
12	360	193	167	0
13	NA	193	167	NA
$Total$	786	780	786	-15

Table 5: Run-off result for the LAE provisions.

From the results in Table 5 one can see that the relative aggregated run-off result equals -1.9%, which is a very satisfying result.

4.4 Uncertainty on the choice of ω

A concern about all three methods presented in this paper is that a central parameter, ω, is not estimated from data.
Therefore we need to examine how much the prediction of the LAE provisions depends on the choice of ω and how volatile the provisions are to changes in ω. In Table 6 we have shown the LAE provision level for different choices of

ω.

ω	N. Y. Method	Approx. N. Y. Method	ODP Method
10%	11	10	206
20%	185	166	357
30%	362	322	504
40%	543	478	647
50%	728	634	786
60%	918	790	922
70%	1,112	946	1,054
80%	1,310	1,102	1,183
90%	1,514	1,257	1,309

Table 6: The LAE provisions dependence on ω.

The results in Table 6 show that the Overdispersed Poisson Model is slightly more stable when ω changes.
By construction the LAE ratio of the Approximated New York Method does not depend on ω, therefore ω only has an influence on the LAE provision level.
In the other methods both the estimation of the LAE ratio and the LAE provision level depend on ω. The two methods have a reverse dependence on ω. In the New York Method the dependence on ω increases if ω increases, whereas in the Overdispersed Poisson Model the dependence on ω decreases if ω increases.

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6 Bibliography

[1] Benktander, G. (1976): An approach to credibility in calculating IBNR for casualty excess reinsurance. Actuar. Rev. 3, no. 2, 7.
[2] Bornhuetter, R. L. and Ferguson, R. E. (1972): The Actuary and IBNR. Proceedings of the Casualty Actuarial Society, vol. LIX, 181-195.
[3] Buchwalder, M. Bühlmann, H., Merz, M. and Wüthrich, M. V. (2006): Estimation of unallocated loss adjustment expenses. Bulletin Swiss Association of Actuaries, no. 1, 43-53.
[4] England, P. D. and Verrall, R. J. (1999): Analytic and bootstrap estimates of prediction errors in claims reserving. Insurance: Mathematics and Economics, vol. 25, 281-293.
[5] Hachemeister, C. A. and Stanard, J. N. (1975): IBNR claims counts estimation with static lag function. 12th ASTIN Colloquium, IAA, Portimao, Portugal.
[6] Johnson, W. (1989): Determination of outstanding liabilities for unallocated los adjustment expenses. Proceedings CAS V76, 111-125.
[7] Mack, T. (1991): A simple parametric model for rating automobile insurance or estimating IBNR claims reserves. Astin Bulletin, vol. 21, no. 1, 93-108.
[8] McCullagh, P. and Nelder, J. A. (1989): Generalized Linear Models, second edition. Chapman & Hall, London.
[9] Strain, R. W. (1984): Property-Liability Insurance Accounting, New York: The College of Insurance. 1984, 82 & 189.
[10] Venter, G. G. (2007): Refining reserve runoff Ranges. CAS E-forum: http://casact.org/pubs/forum/07sforum/07s-venter2.pdf