

DOUBLE CHAIN LADDER AND BORNHUTTER-FERGUSON

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Abstract

In this paper we propose a method close to Double Chain Ladder (DCL) introduced in Martínez-Miranda, Nielsen and Verrall (2011b). Our proposal is motivated by the lack of stability of its precedent. We discuss that the implicit estimation of the underwriting year inflation in the classical Chain Ladder (CCL) method and explicit estimation of it in DCL represents a weak point because the underwriting year inflation might be estimated with significant uncertainty. We show that the underwriting year inflation can be estimated from the less volatile incurred data and then transferred to the into the DCL model. We include an empirical illustration which reveals that the IBNR and RBNS cash flows from DCL are about the double of those from the new method.

Keywords: Bootstrapping; Chain Ladder; Claims Reserves; Reserve Risk

1 Introduction

Double Chain Ladder (DCL) was recently introduced in Martínez-Miranda, Nielsen and Verrall (2011b) and operates on a standard reserving triangle of aggregate paid claims with the addition of the triangle of the numbers of claims. DCL introduces a micro-model of the claims generating process which models and predicts first the reported number of claims. Then, through a delay function and a severity model, it models and predicts future payments. DCL has the attractive feature that when the observed reported claims are replaced by their theoretical expected values, and when one particular estimation procedure is chosen for one parameter, then the resulting prediction is exactly the prediction of the classical Chain Ladder method (CCL). In some senses, one can therefore interpret the DCL method as decomposing classical Chain Ladder into its components. Thus, CCL operates with a theoretical model for the counts data, rather than the observed one. It uses with a delay function and the severity of claims can depend on inflation in the underwriting year direction. In this paper we argue that the implicit estimation of the underwriting year inflation in CCL and explicit estimation of it in DCL represents a weak point because the underwriting year inflation might be estimated with significant uncertainty. We show that the underwriting year inflation can be estimated from the less volatile incurred data and then transferred to the into the DCL model simply by replacing the DCL inflation estimates by those obtained from the incurred data. Because this method replaces the underwriting year parameters in a similar way to Bornhutter-Ferguson, the title of this paper is Double Chain Ladder and Bourhutter-Ferguson.

The rest of the paper is set out as follows. In Section 2 we describe briefly the micro model for DCL and also the new method which we refer by BDCL hereafter. Section 3 summarizes the DCL estimation method, the proposed BDCL and describes the point forecasting expressions. Finally in Section 4 we include an application to personal accident data from a major nonlife insurer. Bootstrap methods close to those proposed by Martínez-Miranda et al. (2011b) provide prediction errors and make possible the inference about IBNR and RBNS claims. The reported results in this Section are appealing by revealing how the last underwriting year inflation is dramatically over-estimated by paid data. This in turn makes the paid data reserve almost double as high as when the unrealistic paid inflation is replaced by the more realistic incurred inflation, following the here proposed BDCL method.

2 The model for aggregated data

We assume the micro model formulated in Verrall, Nielsen and Jessen (2010) and Martínez-Miranda, Nielsen, Nielsen and Verrall (2011a). The micro models allow us to estimate the settlement delay and therefore to predict RBNS and IBNR reserves separately. Far from other approaches which involve also micro models, our proposal becomes simpler and does not require to have individual data to derive the desired forecasts.

The model is constructed by describing three components: the settlement delay, the individual payments and the reported counts. Here we present some notation and the main points of such model (see the papers cited above for a full description).

We assume that two data run-off triangles of dimension m are available: aggregated payments, Δ_m , and incurred counts, \aleph_m . These triangles will be written as follows:

- The aggregated incurred counts triangle: $\aleph_m = \{N_{ij} : (i, j) \in \mathcal{I}\}$, where N_{ij} is the total number of claims of insurance incurred in year i , which have been reported in year $i + j$ i.e. with j periods delay from year i . And $\mathcal{I} = \{(i, j) : i = 1, \dots, m, j = 0, \dots, m - 1; i + j \leq m\}$.
- The aggregated payments triangle: $\Delta_m = \{X_{ij} : (i, j) \in \mathcal{I}\}$, with X_{ij} being the total payments from claims incurred in year i and paid with j periods delay from year i .

Both triangles are observed real data, usually available in practice and the only that we assume to be available in practice. Now to model such aggregated data we go to the underlying individual structure. In fact we define a new (unobserved) triangle in between these, which is the triangle of paid claims, $\aleph_m^{paid} = \{N_{ij}^{paid} : (i, j) \in \mathcal{I}\}$. Here N_{ij}^{paid} is the number of payments incurred in year i and settled with j periods delay. Note that the settlement delay (or RBNS delay) is a stochastic component which arises by considering the micro-level unobserved variables, N_{ijl}^{paid} . These variables are the number of the future payments originating from the N_{ij} reported claims, which were finally paid with l periods delay. Let denote by d the assumed maximum periods of delay ($d \leq m - 1$), then

$$N_{ij}^{paid} = \sum_{l=0}^{\min\{j,d\}} N_{i,j-l,l}^{paid}. \quad (1)$$

Finally let denote by $Y_{ij}^{(k)}$ the individual settled payments which arise from N_{ij}^{paid} ($k = 1, \dots, N_{ij}^{paid}$, $(i, j) \in \mathcal{I}$).

With these definitions, the model which we assume in this paper is formulated under the assumptions given below.

- M1. *The RBNS delay.* Given N_{ij} , the distribution of the numbers of paid claims follows a multinomial distribution, so that the random vector $(N_{i,j,0}^{paid}, \dots, N_{i,j,d}^{paid}) \sim \text{Multi}(N_{ij}; p_0, \dots, p_d)$, for each $(i, j) \in \mathcal{I}$. $\mathbf{p} = (p_0, \dots, p_d)$ denotes the delay probabilities such that $\sum_{l=0}^d p_l = 1$ and $0 < p_l < 1, \forall l$.
- M2. *The payments.* The individual payments $Y_{ij}^{(k)}$ are mutually independent with distributions f_i . Let μ_i and σ_i^2 denote the mean and the variance for each $i = 1, \dots, m$. Assume that $\mu_i = \mu\gamma_i$, with μ being a mean factor and γ_i the inflation in the accident years. Also the variances are $\sigma_i^2 = \sigma^2\gamma_i^2$ with σ^2 being a variance factor.
- M3. *The counts:* The counts N_{ij} are independent random variables from a Poisson distribution with multiplicative parametrization $E[N_{ij}] = \alpha_i\beta_j$ and identification (Mack 1991), $\sum_{j=0}^{m-1} \beta_j = 1$.
- M4. *Independence:* We assume also that the variables $Y_{ij}^{(k)}$ are independent of the counts N_{ij} , and also of the RBNS and IBRN delays. Also, it is assumed that the claims are settled with a single payment or maybe as “zero-claims”.

Note that under the above model the observed aggregated payments can be written as

$$X_{ij} = \sum_{k=1}^{N_{ij}^{paid}} Y_{ij}^{(k)}, \quad \text{for each } (i, j) \in \mathcal{I},$$

which have conditional mean given by

$$E[X_{ij}|\mathfrak{N}_m] = E[N_{ij}^{paid}|\mathfrak{N}_m]E[Y_{ij}^{(k)}] = \sum_{l=0}^{\min(j,d)} N_{i,j-l} p_l \mu \gamma_i. \quad (2)$$

Following calculations in Verrall et al. (2010) the conditional variance of X_{ij} is approximately proportional to the mean. Specifically we have that $V[X_{ij}|\mathfrak{N}_m] \approx \varphi_i E[X_{ij}|\mathfrak{N}_m]$, where $\varphi_i = \gamma_i \varphi$ and $\varphi = \frac{\sigma^2 + \mu^2}{\mu}$, and therefore the dispersion parameter depends on the accident year, i . Such approximation described with more detail in Martínez-Miranda et al. (2011b) justified that an over-dispersed Poisson model can be used to estimate the parameters σ^2 and φ (we will go back to this point later).

3 Bornhuetter-Ferguson and Double Chain Ladder

3.1 Model estimation: the DCL method

The Double Chain Ladder method proposed by Martínez-Miranda et al. (2011b) consider the simple chain-ladder algorithm applied to the triangles of paid claims, Δ_m , and incurred counts, \aleph_m , to estimate all the parameters in the models. Therefore as implied by the name Double Chain Ladder, the classical technique CCL is applied twice and from this everything needed to estimate the outstanding claims is available. Also these authors discussed the estimation procedure gives exactly results as the CCL for paid data when the observed counts are replaced by their fitted values.

The DCL estimation method uses the estimates of the chain ladder parameters from the triangle of counts and the triangle of payments. Let denote such estimates by $(\widehat{\alpha}_i, \widehat{\beta}_j)$ and $(\widetilde{\alpha}_i, \widetilde{\beta}_j)$, respectively, for $i = 1, \dots, m, j = 0, \dots, m - 1$. To derive such estimates is straightforward using the estimates of the development factors provided by the chain ladder algorithm (Verrall, 1991). Let consider the counts triangle (similarly for the parameters of the paid triangle) and let denote by $\widehat{\lambda}_j, j = 1, 2, \dots, m - 1$, the corresponding estimated development factors. Then the estimates of β_j for $j = 0, \dots, m - 1$ can be calculated by

$$\widehat{\beta}_0 = \frac{1}{\prod_{l=1}^{m-1} \widehat{\lambda}_l} \tag{3}$$

and

$$\widehat{\beta}_j = \frac{\widehat{\lambda}_j - 1}{\prod_{l=j}^{m-1} \widehat{\lambda}_l} \tag{4}$$

for $j = 1, \dots, m - 1$. The estimates of the parameters for the accident years can be derived from the latest cumulative entry in each row through the formula:

$$\widehat{\alpha}_i = \sum_{j=0}^{m-i} N_{ij} \prod_{j=m-i+1}^{m-1} \widehat{\lambda}_j. \tag{5}$$

Now the DCL method estimates the rest of the parameters in the model (formulated along M1-M4) basically from the above estimates $(\widehat{\alpha}_i, \widehat{\beta}_j)$ and $(\widetilde{\alpha}_i, \widetilde{\beta}_j)$ ($i = 1, \dots, m, j = 0, \dots, m - 1$). Specifically the reporting delay probabilities $\{p_0, \dots, p_d\}$ can be estimated by solving the following linear system:

$$\begin{pmatrix} \tilde{\beta}_0 \\ \vdots \\ \tilde{\beta}_{m-1} \end{pmatrix} = \begin{pmatrix} \beta_0 & 0 & \cdots & 0 \\ \beta_1 & \beta_0 & \ddots & 0 \\ \vdots & \ddots & \ddots & 0 \\ \beta_{m-1} & \cdots & \beta_1 & \beta_0 \end{pmatrix} \begin{pmatrix} \pi_0 \\ \vdots \\ \pi_{m-1} \end{pmatrix}. \quad (6)$$

Once the solution $\{\hat{\pi}_0, \dots, \hat{\pi}_{m-1}\}$ is obtained, these preliminary delay parameters are adjusted to have the desired real probability vector, $(\hat{p}_0, \dots, \hat{p}_d)$ which verifies that $0 < \hat{p}_l < 1$ and $\sum_{l=0}^d \hat{p}_l = 1$, with d being the maximum estimated delay period.

Now the mean and variance of the distribution of individual payments are estimated. The method proposes to estimate the inflation parameters, $\gamma = \{\gamma_i : i = 1, \dots, m\}$, and the mean factor, μ through the expression:

$$\hat{\gamma}_i = \frac{\hat{\alpha}_i}{\hat{\alpha}_i \hat{\mu}} \quad i = 1, \dots, m. \quad (7)$$

To ensure identifiability it simply sets $\gamma_1 = 1$, and then we can estimate μ by

$$\hat{\mu} = \frac{\hat{\alpha}_1}{\hat{\alpha}_1}, \quad (8)$$

and the estimated inflation parameters, $\hat{\gamma}_i$, are estimated by substituting $\hat{\mu}$ in the equation (7).

The estimation of the variances, σ_i^2 ($i = 1, \dots, m$) comes from estimating first the overdispersion parameter φ (defined in Section 2) by

$$\hat{\varphi} = \frac{1}{n - (d + 1)} \sum_{i,j \in \mathcal{I}} \frac{(X_{ij} - \hat{X}_{ij}^{DCL})^2}{\hat{X}_{ij}^{DCL} \hat{\gamma}_i}, \quad (9)$$

where $n = m(m + 1)/2$ and $\hat{X}_{ij}^{DCL} = \sum_{l=0}^{\min(j,d)} N_{i,j-l} \hat{p}_l \hat{\mu} \hat{\gamma}_i$. Then the variance factor of individual payment can be estimated by

$$\hat{\sigma}_i^2 = \hat{\sigma}^2 \hat{\gamma}_i^2 \quad (10)$$

for each $i = 1, \dots, m$, where $\hat{\sigma}^2 = \hat{\mu} \hat{\varphi} - \hat{\mu}^2$.

3.2 The BDCL method

The BDCL method follows identical steps as DCL but the inflation parameters are adjusted using some deterministic extra information.

Here we perform such adjustment using the inflation parameters which result from running DCL with reported counts and incurred instead of aggregated payments. Specifically we introduce the BDCL method through the following two-step procedure:

- *Step 1: Parameter estimation.* First estimate the model parameters from DCL as proposed Martínez-Miranda et al. (2011b) from the triangles \aleph_m and Δ_m . Let denote such estimates as $\hat{\mathbf{p}}, \hat{\mu}, \hat{\sigma}^2$ and $\{\hat{\gamma}_{i,0} : i = 1, \dots, m\}$. Then estimate again the model using DCL but replacing the paid triangle by $\check{\Delta}_m = \{\check{X}_{ij} : (i, j) \in \mathcal{I}\}$, with \check{X}_{ij} being the incurred data corresponding to the accident year i and development period j . Keep only the resulting estimated inflation parameters, let denote them by $\{\hat{\gamma}_{i,1} : i = 1, \dots, m\}$.
- *Step 2: BF adjustment.* Replace the inflation parameters $\{\hat{\gamma}_{i,0} : i = 1, \dots, m\}$ from the paid data by some more realistic using available extra information. Specifically we propose to use the estimates from the incurred triangle, $\{\hat{\gamma}_{i,1} : i = 1, \dots, m\}$. Let denote them hereafter by $\{\hat{\gamma}_i : i = 1, \dots, m\}$.

From Steps 1-2 the final parameters estimate will be $\hat{\theta} = \{\hat{p}_l, \hat{\mu}, \hat{\sigma}^2, \hat{\gamma}_i, l = 0, \dots, d, i = 1, \dots, m\}$. In general Step 2 could be defined for an arbitrary percentage of BF adjustment, however in the empirical illustration provided in the paper, for simplicity, we have carried the two-step procedure by replacing all the inflation parameters by the derived from incurred data.

3.3 Justification of BDCL

The CCL and Bornhuetter-Ferguson (BF) methods have become the easiest claim reserving methods, due to their simplicity CCL and BF are the most commonly used techniques in practice (see for example Alai, Merz and Wüthrich, 2009, 2010; Mack, 2008; Schmidt and Zocher, 2008, Verrall 2004, for a recent discussion about the method). The BF method introduced by Bornhuetter and Ferguson (1972) aims to solve the well known weakness of CCL against outliers. In this aim the BF method incorporates prior knowledge from experts and therefore is rather robust than the CCL method which relies completely on the data contained in the run-off triangle.

Specifically CCL method estimates of outstanding claims for accident year $i > 1$ by

$$\hat{R}_i^{CCL} = C_{i,m-i} \left(\prod_{k=m-i+1}^{m-1} \tilde{\lambda}_k - 1 \right)$$

where $C_{i,m-i}$ are the observed cumulative claims and $\tilde{\lambda}_1, \dots, \tilde{\lambda}_{m-1}$ are the development factors. Therefore the CCL reserve strongly depends on the current amount $C_{i,m-i}$. And then nonsense predictions can be derived mainly for the last years, where the triangle contains less and more volatile information. What the BF method does is to avoid this dependence and replace the latest cumulative claims by an external (prior) estimate. Such estimate is obtained from an estimate of ultimate claims, U_i^{prior} . So BF replaces $C_{i,m-i}$ by $U_i^{prior} / (\prod_{k=m-i+1}^{m-1} \tilde{\lambda}_k)$ and the BF estimate of outstanding claims is

$$\widehat{R}_i^{BF} = \frac{U_i^{prior}}{\prod_{k=m-i+1}^{m-1} \tilde{\lambda}_k} \left(\prod_{k=m-i+1}^{m-1} \tilde{\lambda}_k - 1 \right).$$

Assuming the Poisson model which formulates a multiplicative structure, $E[X_{ij}] = \tilde{\alpha}_i \tilde{\beta}_j$, the relationship between CCL and BF reserve is showed through the following expressions:

$$\widehat{R}_i^{CCL} = \widehat{\alpha}_i \sum_{k=0}^{m-1} \widehat{\beta}_k \left(\frac{\sum_{k=m-i+1}^{m-1} \widehat{\beta}_k}{\sum_{k=0}^{m-1} \widehat{\beta}_k} \right) = \widehat{U}_i^{CCL} \left(\frac{\sum_{k=m-i+1}^{m-1} \widehat{\beta}_k}{\sum_{k=0}^{m-1} \widehat{\beta}_k} \right)$$

and

$$\widehat{R}_i^{BF} = U_i^{prior} \left(\frac{\sum_{k=m-i+1}^{m-1} \widehat{\beta}_k}{\sum_{k=0}^{m-1} \widehat{\beta}_k} \right) \quad (11)$$

with \widehat{U}_i^{CCL} and $\widehat{\alpha}_i$ and $\widehat{\beta}_k$ being the CCL estimation of the ultimate claims and the parameters in the model, respectively.

Assuming the Mack's identification scheme, $\sum_{k=0}^{m-1} \beta_k = 1$, $\widehat{U}_i^{CCL} = \widehat{\alpha}_i$, and therefore what BF actually replaces are the estimated row parameters in the Poisson model.

The proposed BDCL method follows the same spirit of BF+CCL in equation (11). In fact the aim is to stabilize the row parameters with extra information, in this case what comes from the incurred data. If we look at the assumed structure of the parameters in the DCL model, the row parameters are

$$\tilde{\alpha}_i = \alpha_i \mu \gamma_i$$

for the paid data, and similarly for the incurred data,

$$\check{\alpha}_i = \alpha_i \check{\mu} \check{\gamma}_i,$$

with α_i being the row parameters in the model for the reported counts. Therefore when BDCL uses the estimated inflation parameters from the incurred, what actually the method is doing is replacing the more volatile parameter into $\tilde{\alpha}_i$ i.e. the inflation by accident year, γ_i , by the estimate derived from the triangles $(\aleph_m, \check{\Delta}_m)$. With such replacement the predictions become more stable and realistic. Indeed, as we describe with the following empirical illustration, the BDCL reserve becomes quite smaller than those derived from simple DCL and CCL methods.

3.4 Forecasting RBNS and IBNR reserve

The estimated parameters, $\hat{\theta} = \{\hat{p}_l, \hat{\mu}, \hat{\sigma}^2, \hat{\gamma}_i, l = 0, \dots, d, i = 1, \dots, m\}$, derived from Steps 1-2 above can be used to calculate a point forecast of the RBNS and IBNR components of the reserve. Using the notation of Verrall et al. (2010) and Martínez-Miranda et al. (2011a,b), we consider predictions and extend the model assumptions presented in Section 2 over the following triangles:

$$\begin{aligned} \mathcal{J}_1 &= \{i = 2, \dots, m; j = 0, \dots, m-1 \text{ so } i+j = m+1, \dots, m+d\} \\ \mathcal{J}_2 &= \{i = 1, \dots, m; j = m, \dots, m+d \text{ so } i+j = m+1, \dots, m+d\} \\ \mathcal{J}_3 &= \{i = 2, \dots, m; j = m, \dots, m+d \text{ so } i+j = m+d+1, \dots, 2m+d-1\}. \end{aligned}$$

As these authors pointed out the CCL method would produce forecasts over only \mathcal{J}_1 . In contrast, DCL and consequently BDCL provide also the tail over $\mathcal{J}_2 \cup \mathcal{J}_3$.

For the RBNS reserve we follow the original suggestion of Verrall et al. (2010) and use the expression of the conditional mean in equation (2) i.e.

$$\hat{X}_{ij}^{rbns} = \sum_{l=i-m+j}^j N_{i,j-l} \hat{\pi}_l \hat{\mu} \hat{\gamma}_i, \quad (12)$$

with $(i, j) \in \mathcal{J}_1 \cup \mathcal{J}_2$.

Similarly is proposed for the IBNR forecast reserve but involving chain ladder predictions of future numbers of reported claims, $\hat{N}_{i,j}$, i.e.

$$\hat{X}_{ij}^{ibnr} = \sum_{l=0}^{i-m+j-1} \hat{N}_{i,j-l} \hat{\pi}_l \hat{\mu} \hat{\gamma}_i, \quad (13)$$

with $(i, j) \in \mathcal{J}_1 \cup \mathcal{J}_2 \cup \mathcal{J}_3$.

We also derive the bootstrap predictive distribution as Martínez-Miranda et al. (2011a,b) proposed, using the data in the observed triangles \aleph_m and Δ_m , and the model with estimated parameter $\hat{\theta}$.

4 An empirical illustration

In this paper we consider a personal accident data set from a major insurer. The data available consists of three incremental run-off triangles of dimension $m = 19$, the reported counts, the aggregated paid data and the incurred. The periods considered in the triangles are years.

We illustrate the method proposed in this paper which combines Double Chain Ladder and Bourhutter-Ferguson (BDCL) and compare it with the simple DCL method. Also as a benchmark for comparison purposes, we have calculated the predicted reserve from the CCL method.

Table 1 gives the estimates of the parameters for the model from BDCL and from DCL. Also Figure 1 shows the estimated inflation by DCL from the paid data and also from the incurred data.

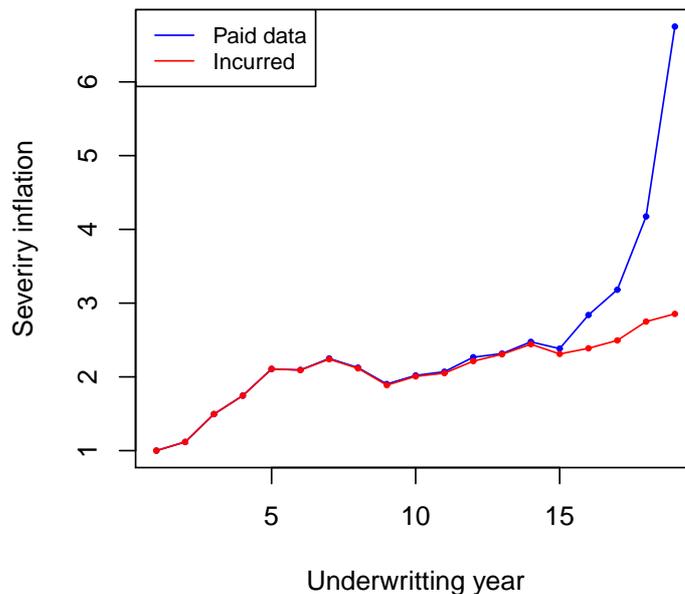


Figure 1: Estimated inflation by DCL from the paid data (blue curve) and the incurred data (red curve).

As was defined in the previous section the DCL method estimates the parameters in the model from the pair of triangles consisting of the reported counts and the paid data. Note that the inflation estimates from the paid data are quite different from the obtained from the incurred data. The

distance between them is actually big for the two last years. As expected the predicted reserve from from DCL and also from CCL becomes bigger than the BDCL reserve. And this is exactly that we can observe when we get point forecasts from such methods, indeed the reserve seems to double in DCL and CCL. Table 2 shows the predicted RBNS and IBNR reserve and also the total (RBNS + IBNR) reserve for BDCL and DCL methods. The comparison with the predicted reserve from CCL is given in the last column. From these forecasts it is appealing the fact that the total reserve from BDCL is about the 58% of the DCL and CCL reserve. Looking at the RBNS and IBNR reserve separately the same problem can be observed, in fact the RBNS reserve predicted by BDCL is about the 60.6% of that from DCL. Also the IBNR reserve predicted by DCL is 40.6% bigger than the one derived from BDCL. In this sense both DCL and CCL methods seem to exaggerate the underwriting year inflation dramatically the last years, and therefore the predicted reserve increases notably.

The predictive distribution of the RBNS and IBNR reserve is calculated using the bootstrap methods proposed by Martínez-Miranda et al. (2011b). In this aim the model is estimated from both BDCL and DCL methods and the corresponding cash-flows are simulated. Also we calculate the predictive distribution by CCL using the bootstrap method introduced by England and Verrall (1999), which does not provide the split off between RBNS and IBNR reserve. The summary statistics for the RBNS, IBNR and total (RBNS + IBNR) cash-flows are shown in Table 3, these results provide a numerical comparison among the cash-flows derived from the BDCL, DCL and CCL methods.

Again it is remarkable how DCL and CCL provide upper quantiles for the total reserve which are about the double of those by the BDCL method. Figure 2 shows box plots of the predictive distribution of the total reserve in the future from BDCL, DCL and CCL. Also histograms of the overall total reserve for the next years represent the predictive distribution of the three compared methods. Note that the BDCL cash-flows move around smaller values than those by the DCL method. Also the CCL cash-flows seem to fail dramatically describing the distribution of the reserve, as was discussed previously by Martínez-Miranda et al. (2011a,b).

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\hat{p}_l	$\hat{\gamma}_{i,BDCL}$	$\hat{\gamma}_{i,DCL}$
0.0592	1.00	1.00
0.3097	1.12	1.12
0.2032	1.50	1.49
0.1996	1.74	1.75
0.1388	2.11	2.11
0.0440	2.09	2.09
0.0227	2.24	2.25
0.0095	2.12	2.13
0.0017	1.89	1.90
0.0029	2.01	2.02
0.0002	2.05	2.07
0.0026	2.21	2.27
0.0019	2.31	2.32
0.0031	2.44	2.47
0.0006	2.31	2.38
0.0000	2.39	2.84
0.0000	2.49	3.18
0.0000	2.75	4.17
0.0000	2.85	6.75
$\hat{\mu} = 2579.002$		
$\hat{\sigma}_{BDCL}^2 = 350504716$		
$\hat{\sigma}_{DCL}^2 = 286809586$		

Table 1: Estimated parameters: the delay probabilities \hat{p}_l ($l = 0, \dots, d = 18$), the inflation parameters $\hat{\gamma}_i$ ($i = 1, \dots, k = 19$) and the mean and variance factors, μ and σ^2 , from BDCL and DCL methods.

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Future	BDCL			DCL			CCL
	RBNS	IBNR	Total	RBNS	IBNR	Total	
1	37812	615	38427	59844	1387	61230	61091
2	25878	3294	29171	41446	7406	48852	48061
3	17804	2537	20340	31015	5611	36626	36266
4	9485	2495	11980	17542	5501	23043	22990
5	3699	1867	5566	6443	4069	10512	10439
6	1839	821	2660	3192	1720	4912	4914
7	905	462	1366	1446	945	2390	2380
8	512	246	758	675	487	1162	1174
9	457	113	571	642	210	853	848
10	329	87	416	424	169	592	600
11	337	40	377	536	72	608	594
12	242	49	292	404	99	504	496
13	163	37	200	335	74	409	397
14	28	46	73	60	97	157	136
15	0	18	18	0	37	37	109
16	0	7	7	0	12	12	0
17	0	4	4	0	7	7	0
18	0	2	2	0	4	4	0
19	0	1	1	0	2	2	
20	0	1	1	0	1	1	
21	0	0	0	0	1	1	
22	0	0	0	0	0	0	
Total	99490	12741	112231	164003	27910	191913	190496

Table 2: Point forecasts by calendar year. Columns 2-4 show the predictions from BDCL. Columns 5-7 show the predictions by DCL, and column 8 the classical Chain Ladder predictions (CCL). The quantities are given in thousands.

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Bootstrap predictive distribution							
	BDCL			DCL			CCL
	RBNS	IBNR	Total	RBNS	IBNR	Total	
mean	97900	12509	110409	163534	28246	191780	172992
pe	18671	6121	23160	37387	13876	48439	62343
1%	61032	3144	68131	100702	7857	112348	-32993
5%	71695	4960	78153	114031	11233	128032	96811
50%	96706	11512	108451	158880	25662	184786	171333
95%	130606	23887	149298	232537	53395	280328	257696
99%	155128	32733	185638	276334	73146	343335	373510

Table 3: Predictive distribution of RBNS, IBNR and total (RBNS + IBNR) reserve. The six first columns give the summary of the distribution from the bootstrap method for BDCL. The following three columns show the results from DCL. The last column shows the England and Verrall (1999) distribution. The quantities are given in thousands.

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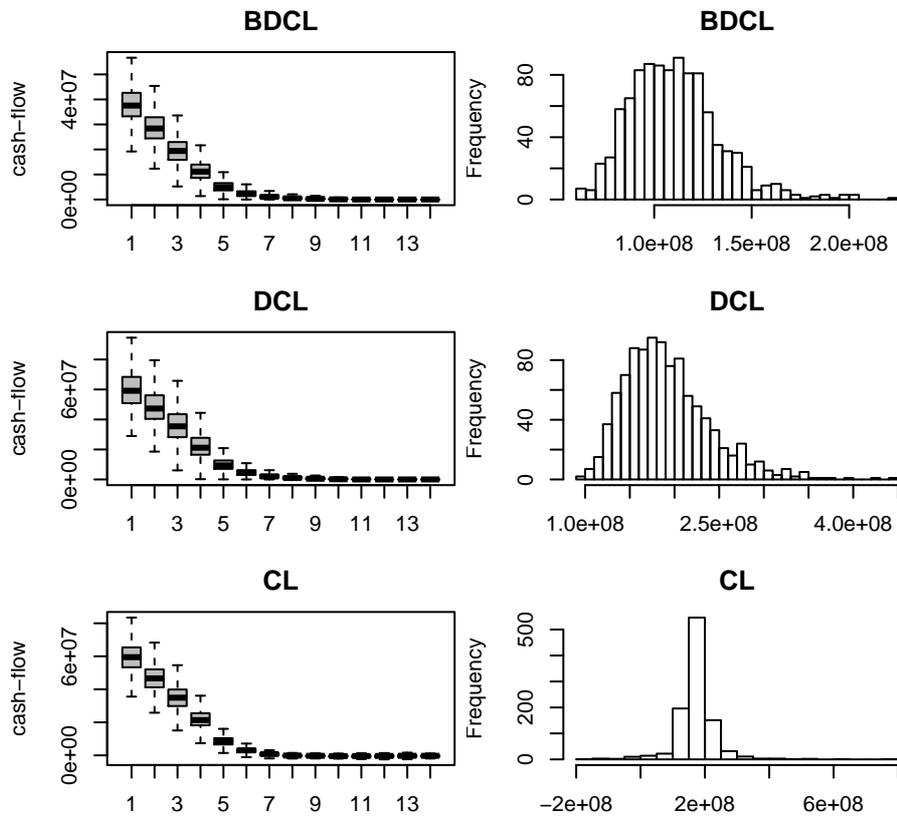


Figure 2: Box plots representing the predictive distribution of the total (RBNS+IBNR) reserve in the future from BDCL, DCL and CCL (rows 1,2 and 3 respectively). Right panels show the histograms of the total reserve (the overall total for the next years) by the three methods.