STOCHASTIC CLAIM RESERVING BASED ON CRM FOR SOLVENCY II PURPOSES

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Abstract

To model uncertainty of future loss payments has been a primary goal of actuarial literature over last decade. Stochastic methodologies for outstanding claims valuation have indeed been recently developed with the aim to obtain either a variability coefficient or the probability distribution of the reserve, useful for the assessment of the Reserve Risk capital requirement. Nowadays a broad literature is available concerning deterministic and stochastic model used for loss reserving and Bootstrapping and Bayesian methodologies appear now the main frameworks for academics and practitioners in the stochastic field. The International Actuarial Association (IAA)[1] proposed a different way to analyse outstanding claim reserve, focused on a paper by Meyers, Klinker and Lalonde where the overall reserve of a single line of business (LoB) (without any distinction among different accident years) is described as a compound mixed Poisson process, by a similar approach used for aggregate amount of claims when premium risk is to be estimated. Following that approach, we propose to estimate the loss reserve assuming that each single cell of the lower part of the run-off triangle follows a Compound Poisson Process (either Pure or Mixed). In a previous paper[2], it is shown how to obtain, under independence constraint, quite easily the exact moments of the reserve distribution only through the knowledge of the characteristics of the two main random variables (number and claim size of future payments). We extend here this approach introducing a correlation that acts separately between claim count and average cost in the bottom part of the loss triangle. Furthermore, Monte Carlo methods allow to simulate outstanding claims distributions for each accident year, for both the overall reserve until complete run-off and the next calendar year only (in case of a one-year time horizon as prescribed in Solvency II). The model is applied in a Solvency II framework in order to compare the internal model capital requirement to the Standard Formula approaches (both market-wide and undertaking specific). At this regard, the case study concerns different insurers and several lines of business with the purpose to obtain a reliable comparison. Finally in the present paper model's parameters are calibrated by observed data and through the Fisher-Lange (an average cost method not yet widely used) with the aim to estimate the uncertainty related to this deterministic method too.

Keywords: stochastic models for claims reserve, capital requirements, reserve risk, collective risk model, average cost methods, Solvency II.

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[1] See IAA (2004), Appendix B: Non-Life (P&C) Insurance Case Study.

[2] See Savelli N. & Clemente G.P., *A Collective Risk Model for Claims Reserve Distribution,* Proceedings of "Convegno di Teoria del Rischio", 2009, Campobasso and Savelli N. & Clemente G.P., *A Collective Risk Model for Claims Reserve Distribution,* Presented at International Congress of Actuaries, 2010, Cape Town, South Africa.

1. Introduction

The estimation of future claims liabilities has been an increasing key issue in insurance over the time and many deterministic methods have been developed, in the past, in order to obtain an estimated (point) value of provision for outstanding claims. Recently, under the forthcoming Solvency II and new international accounting principles in insurance, this valuation is nowadays requested to involve also a measure of uncertainty around the (point) estimate of claim reserves. At this regard, stochastic models for outstanding claims valuation have been recently carried on with the aim to assess at least a variability coefficient related to the point estimate of the reserve (i.e. central/best estimate). Bootstrapping methodologies and closed formulae (e.g. Mack[3] formula) are two alternative ways to obtain the prediction error of the Chain-Ladder method. Furthermore the probability distribution of claims reserve could be directly derived by the sampling and the following Monte-Carlo procedure, while Mack formula allow to derive that only under some distributional assumptions (e.g. assuming a LogNormal distribution with mean and standard deviation equal to the best estimate of the reserve and the Mack prediction error respectively). Moreover, many actuarial studies are investigating this method in order to propose further developments[4]. Other deterministic method, as Bornhuetter-Ferguson, has been recently implemented with a stochastic structure based on a Bayesian approach[5]. Moreover closed formulae have been obtained also for this method with the target to measure the prediction error of reserve estimate[6]. Finally, the International Actuarial Association (IAA)[7] suggested a different way to analyse outstanding claim reserve with the aim to quantify the capital requirement for Reserve Risk. The methodology, based on a paper of Meyers, Klinker and Lalonde[8] focused on premium risk, describes the overall reserve of a single line of business (LoB) as a compound mixed poisson process under the assumptions that the total number of claims reserved is Negative Binomial distributed and the severity of outstanding claims has mean and variability coefficient known. This approach, as mentioned briefly by other works[9], provides to estimate directly the overall claims reserve without analyzing separately by accident year. We recall here a previous paper[10] where we extended the IAA approach assuming that each single cell of the lower run-off triangle can be described by a Compound Poisson Process (either Pure or Mixed). It has been shown how to obtain, under independence constraint, quite easily the exact moments of the reserve distribution only through the knowledge of the characteristics of the two main random variables (number and claim size of future payments[11]). We try here to overcome the independence assumption introducing through structure variables a correlation that acts separately between claim count and average cost in the bottom part of the loss triangle. Upon some assumptions, it has been derived again the exact mean and variance of the claim reserve. Monte Carlo methods allow to simulate outstanding claims distributions for each accident year, for the overall reserve and for the next calendar year (in case of a one-year time horizon as prescribed in Solvency II). Model's parameters are calibrated from observed data and through a deterministic model (in the paper an average cost method is used, but different algorithmic methods could be assumed) based on the separate estimate of number of claims to be paid and future average costs for each cell of the triangle to be estimated. One of the

[3] See Mack (1993),(1994) and Mack (1999) for an extension with the inclusion of tail factor too.
[4] See Pinheiro et al. (2003), England,Verrall (2006), Taylor (2009).
[5] See England & Verrall (2002) and Verrall (2004), England et al. (2010)
[6] See Mack (2008), Alai et al. (2009), Alai et al. (2010), Saluz et al. (2010)
[7] See IAA (2004).
[8] See Meyers, Klinker, Lalonde (2003).
[9] See Hayne (1989), Hayne (2003), Meyers (2008).
[10] See Savelli, Clemente (2009)
[11] See Havning, Savelli (2005) and Daykin et al. (1994) for exact cumulants of aggregate claims amount of a Compound Poisson Process (pure or mixed).

aims is indeed to build up a stochastic structure for the Fisher-Lange method[12], widely used as deterministic method in the Italian insurance market. Furthermore, we analyze the one-year reserve risk in the perspective adopted by Solvency II[13] applying the "re-reserving"[14] with the target to estimate the variability of claims development result and the percentiles associated to its simulated probability distribution, in order to quantify the capital requirement for the reserve risk. Main results will be compared using different stochastic reserving models, analyzing the effect on both reserve risk capital requirement and risk margin[15]. Finally, CRM and Bootstrap models have been applied to different insurers and to several LoBs in order to analyze the effect on the claims reserve valuation (as Best Estimate plus Risk Margin according to Solvency II QIS5 framework) and on the capital requirement obtained by either Internal Model or Standard Formula (both Market-Wide and Undertaking Specific Approaches).

2. Collective Risk Model

In this section, the Collective Risk Theory is described with the aim to estimate the claims reserve distribution. We can then regard a generic triangle for a single LoB with dimension (N,N^+) where rows $(i=1,\ldots,N)$ represent the claims accident years (AY) and columns (with $j=1,\ldots,N^+$) are development years (DY) for payments. It is to be emphasized that frequently columns are not equal to rows, because of a payments tail in the triangle (in this case $N^+=N+1$). The observations are available in the upper part $\left\{X_{i,j};\, i+j \le N+1 \text{ } and \text{ } 1 \le j \le N+1 \text{ } if \text{ } N = N^+\right\}$ with the cell $(1,N^+)$ known too if the triangle has a tail. $X_{i,j}$ represents the payments related to claims incurred in the generic accident year i and paid in development year j (namely in the calendar year i+j+1).

Furthermore we assume that $n_{i,j}$ is the observed number of paid claims in development period j and for accident year i, and $m_{i,j} = X_{i,j}/n_{i,j}$ is the average cost of claims generated in accident year i and paid after j years. Different deterministic methods, for example Fisher Lange[16], allow to estimate values of $n_{i,j}$ and $m_{i,j}$ for the lower part of the triangle leading to a point reserve estimate. Finally, we could obtain by observed data the variability coefficient c_{Zj} of claim size paid for each development year j.

In order to build up a stochastic model based on a Compound Poisson Process we suppose that in each cell of the bottom part, the incremental paid claim cost will be equal to the aggregate claim amount:

$$\widetilde{X}_{i,j} = \sum_{h=1}^{\widetilde{K}_{i,j}} \widetilde{Z}_{i,j,h}$$

where:

- $\widetilde{K}_{i,j}$ is a random variable that describes the number of claims of accident year i and that will be effectively paid in a future development year j. This variable could be described by a mixed Poisson Process in order to consider the uncertainty linked to the parameter estimation through a

[12] See Fisher-Lange (1973) for the original idea about this deterministic method and Ottaviani (1980) for the formalization of an extended methodology applied by Italian insurers. See Savelli,Clemente (2009) for a brief description of the deterministic method here applied.

[13] See Solvency II Directive (2009)

[14] See Ohlsonn, Lauzenings (2008) and Diers (2009). See AISAM,ACME (2007) for a case study on a One.Year Approach applied to long-tail liabilities.

[15] See De Felice, Moriconi (2003), De Felice, Moriconi (2006) and De Felice et al. (2006) for a YEE approximation to avoid circularity in the risk margin valuation.

[16] See Savelli,Clemente (2009)

multiplicative structure variable $\tilde{q}_{i,j}$. In the next, we will assume that the structure variable is distributed as a Gamma with equal parameters with mean 1 and standard deviation equal to $\sigma_{q_{i,j}}$;

- $\tilde{Z}_{i,j,h}$ is the random amount of the h[th] claim settled after j years and occurred in the accident year i. Despite the distribution of the severity could be fitted on the observed data, we can determine the exact moments of the aggregate claims amount without any assumption on the severity probability distribution. As made for claim count, we introduce for the single claim cost a parameter uncertainty by a multiplicative structure variable having an impact on the variability coefficient c_{Zj}. In particular this uncertainty is described by multiplicative random variables r_j (in the next assumed as well Gamma distributed with the same parameters) having mean 1 and standard deviation σ_{r_j}.

We assume here, extending a previous approach[17], only one structure variable $\tilde{q}$ that affects each cell in the lower part of the triangle. Furthermore with the aim to consider the correlations between the claim costs in the same development year, it is considered one r.v. $\tilde{r}_j$ for each development year (i.e. each random value occurred affects only the cells in the same column).
In this context we derive the following exact characteristics of claims reserve. As expected, the average claims reserve is equal to:

(1) $$E(\tilde{R}) = \sum_{i=1}^{N} E(\tilde{R}_i) = \sum_{i=1}^{N} \sum_{j=N-i+2}^{N^+} E(\tilde{X}_{i,j}) = \sum_{i=1}^{N} \sum_{j=N-i+2}^{N^+} m_{i,j} \cdot n_{i,j}$$

This relation shows that the average of the CRM model is exactly coinciding with the Fisher-Lange Best Estimate if the parameters ($m_{i,j}$ and $n_{i,j}$) of the stochastic model are calibrated using this deterministic approach.
The exact standard deviation (ignoring the variables r_j[18], that have a negligible effect on the variability) is described by:

(2) $$\sigma^2(\tilde{R}) = \sum_{i=1}^{N} \sum_{j=N-i+2}^{N^+} n_{i,j} \cdot a_{2,Z_{i,j}} + \left(\sum_{i=1}^{N} \sum_{j=N-i+2}^{N^+} n_{i,j} \cdot a_{1,Z_{i,j}} \right)^2 \cdot \sigma_q^2$$

where the first term is the sum for each DY and AY of variance of single cells of the bottom part of the triangle considering only the Poisson Process (i.e. without the effect of the structure variables).
The second part considers the effect of r.v. q introducing an implicit correlation related to the presence of an only structure variable that affects all the triangle.
It is indeed easy to show that the variance is not lower than the sum of the variance of claims payments in each cell:

(3) $$\sigma^2(\tilde{R}) \geq \sum_{i=1}^{N} \sum_{j=N-i+2}^{N^+} n_{i,j} \cdot a_{2,Z_{i,j}} + \sum_{i=1}^{N} \sum_{j=N-i+2}^{N^+} n_{i,j}^2 \cdot a_{1,Z_{i,j}}^2 \cdot \sigma_q^2 = \sum_{i=1}^{N} \sum_{j=N-i+2}^{N^+} \sigma^2(\tilde{X}_{i,j})$$

The implicit correlation (assumed equal in each cell) could be derived solving previous relations.
Finally the distribution of aggregate claims reserve could be now obtained either following some methodologies based on approximation[19] or using simulation methods. In the next the latter one will be used in order to estimate the distribution.

[17] Savelli, Clemente (2009) shows how to obtain the exact characteristics (mean, variance and skewness) of claims reserve ignoring $\tilde{r}_j$ and assuming that structure variables $\tilde{q}_{i,j}$ are i.i.d. in each cell of the lower part of the triangle.

[18] Some case studies (see Figure 7 in Savelli, Clemente (2009)) prove that the claim size parameter uncertainty contributes in a limited way to the overall variability.

[19] For example Normal Approximations, Recursive Panjer Formula, Fast Fourier Trasform, etc.

3. Case Study (Main Results)

The stochastic model takes into consideration claim experience data on Motor Third Party Liability (MTPL) from two Italian insurance companies, according to accounting years from 1993 to 2004[20]. Clearly proportion of the real data have been modified to save the confidentiality of the data-set. The main information used is regarding number of paid and reserved claims, number of closed and reopened claims, number of reported claims and finally incremental payments and reserved amounts. Next figure shows the historical cost of cumulative paid amounts for the two companies analysed. Data are summed up in a triangle 12x12 with tail reported in Appendix. In particular the triangle SIFA is referred to a small-medium company whereas the triangle AMASES has a company roughly 10 times larger as reference.

Discounted Best Estimate obtained by the Chain-Ladder is indeed equal to roughly € 233 million for SIFA and € 2,566 for AMASES. Data shows how the complete run-off period is longer than DY 12 and a tail must be added. In these triangles the tails are the level of statutory reserve fixed by the companies. As usual in actuarial literature[21], in the following analysis the tail (12^{+}) will be added to the payments of the last development year (12).

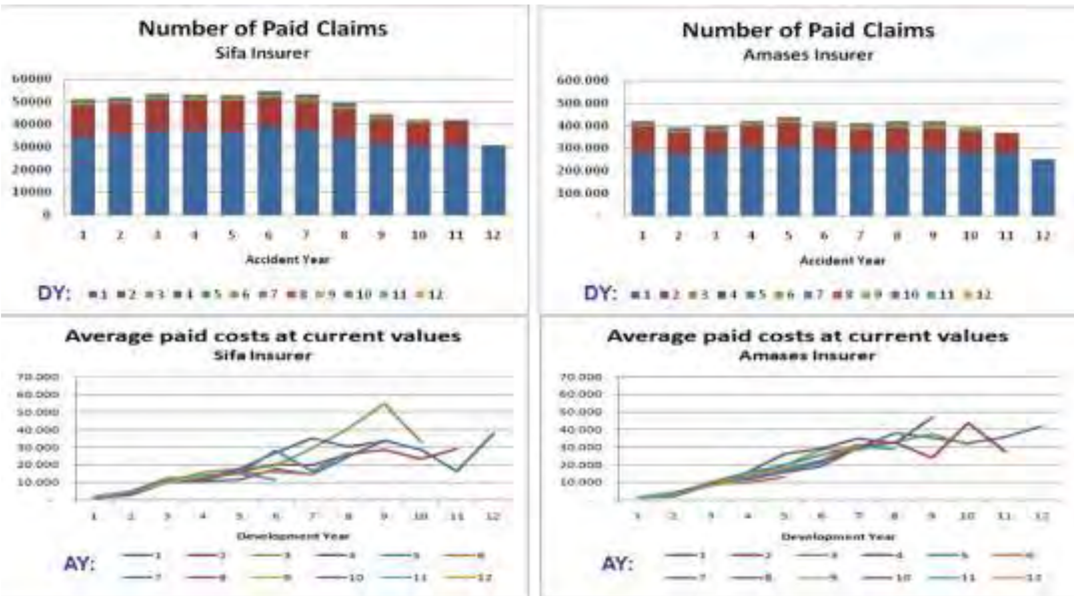

Figure 1 – Number of Paid Claims and Average Paid Costs according to both insurers

Figure 1 summarizes main data used for parameters calibration through the Fisher-Lange method. It is to be emphasized the nature of claims settlement process where usually small claims are firstly paid. Both companies have settled indeed more than 90% of claims (as numbers) within two years from occurrence date while the cumulative amount of payments at DY 2 is almost equal to 60-70% of ultimate cost estimate. Moreover, the percentage of paid claims appears more volatile for small insurer.

Lower figures (Figure 1) represent the average paid cost (at current value) pattern according to different development years. It is confirmed an increasing average cost moving towards the right part of the triangle because of a greater proportion of claims for bodily injuries. Furthermore, as observed in market data too, the volatility increases for higher durations. Finally the smaller insurer highlights a greater variability of average cost.

[20] The effect of direct reimbursement (CARD), introduced in 2007 in the Italian Insurance Market, is here not considered.

[21] Main approaches (as Bootstrap, Merz and Wuthrich formula, etc.) are usually applied to triangles without tail.

The CRM Model has been applied by MonteCarlo simulations (50,000 simulations) to both companies. In particular, we suppose that the number of claims (in each single cell of the bottom triangle) is distributed as a Negative Binomial, obtained from the mixture of a Poisson with mean $n_{i,j}$ (estimated by the Fisher-Lange assuming as settlement speed the average computed on the last three diagonals) multiplied by an only structure variable $\tilde{q}$ described by a Gamma having mean 1 and standard deviation equal to 8% for SIFA and 3% for AMASES. Standard deviations have been estimated by the exact moments assuming the same correlations (roughly 0.1, equal to that one implicitly derived by Mack prediction error formula) between claim counts for the two insurers.
Moreover, the costs of the single claims are distributed as a Gamma[22] with mean equal to the average cost of the cell $m_{i,j}$ (estimated by Fisher-Lange as the mean of all average costs available in the upper part of the triangle expressed at current values and projected by the future claim inflation structure, here used as deterministic).
The standard deviation is based on the next variability coefficients c_{zj} different for each development year and depending on both the analysed line of business and the company's portfolio characteristics:

Development Years		2	3	4	5	6	7	8	9	10	11	12
c_{Zj}		0.50	0.75	1.00	1.25	1.50	2.00	2.50	3.00	4.00	5.00	7.00

Table 1 – Variability coefficient of average paid costs according to both insurers

We regard estimation of variability coefficients c_{zj} (obtainable from the Claim DataBase of the company and different for each development year) as a key issue. As well known these data are available mainly as an internal actuary, while it is rather difficult to get them for an external actuary (e.g. involved in a take-over evaluation). This might produce minor diffusion of the approach compared to other methods based on aggregate run-off triangles mainly. Furthermore, in order to consider parameter uncertainty, we assume that c_{zj} are affected by a structure variable r_j (one for each development year) distributed according to a Gamma having mean 1 and std equal to 3% for both companies.

Table 2 shows a comparison between the two companies (SIFA and AMASES) of variability coefficient and skewness using not only the CRM, with parameters based on Fisher-Lange assumption, but also the well-known Bootstrapping model, based on Chain-Ladder method and developed using adjusted Pearson residuals with a constant scale parameter ϕ[23] for different development years and through LogNormal simulations. As expected, the CRM model leads for both Insurers to a best estimate, equal to the Fisher-Lange deterministic method claims reserve estimate. It is obtained for SIFA a variability coefficient equal to 8.69% and a not negligible skewness (+0.15). It could be point out a lower variability and skewness when considering the bigger Insurer. We recall indeed that both cumulants of incremental payments are decreasing according to a greater number of claims.
Furthermore it is to be pointed out that stochastic models lead to different averages (on both discounted and undiscounted bases), clearly due to the different assumptions implicitly included in

[22] The choice of the claim-size distribution may have relevant impacts on the reserve distribution and then on the capital requirement. Skewness of claim reserve is an important element respect to Bootstrap when the simulation is carried on through different claim-size distribution family. The distribution family has, obviously, a negligible impact on variability but a large effect on skewness (see at this regard Savelli, Clemente (2009), section 5)

[23] See England & Verrall (1999) and England (2002) for Bootstrap model. See Pinheiro et al. (2003) and England & Verrall (2006) for different types of residuals and for scale parameters ϕ_j differentiated for each development year.

the two underlying deterministic methods (Fisher-Lange and Chain-Ladder) both applied here without any "professional judgement" (instead present, as obvious, in the real valuations).
Secondly, it is to be emphasized how the different dimensions and development patterns of two triangles are affecting on the different variability provided by the stochastic methods leading to a lower variability for the bigger Insurer. It should also be noted that there is not a predominant model. AMASES gets a variability equal to 3.3% by CRM (lower than 3.7% obtained by Bootstrap), while the sampling with replacement methodology shows lower variability (4.6%) than CRM (8.7%) for SIFA Company. Finally, the skewness is not affected so much when the Bootstrap method is used in case of either different triangle dimensions and different probabilistic assumptions (LogNormal, ODP, Normal, Negative Binomial, etc.). Results of 50,000 simulations (not reported here) based on different probabilistic assumptions (LogNormal, ODP, Normal and Negative Binomial) applied to Bootstrap samples show a skewness from 0.05 to 0.08 for SIFA Insurer and from 0.078 to 0.085 for AMASES. Instead CRM returns a different skewness between insurers, with a lower value for the bigger company because of the larger number of claims paid.

	CV		Skewness	
	CRM (FL)	Bootstrap (CHL)	CRM (FL)	Bootstrap (CHL)
SIFA	8.7%	4.6%	0.15	0.08
AMASES	3.3%	3.7%	0.06	0.08

Table 2 - Comparison between CRM and Bootstrap LogNormal for both insurers (50,000 simulations – discounted values)

Both stochastic models (CRM and Bootstrap) are implemented in order to quantify the technical reserve amount and the capital requirement for reserve risk. The capital requirement has been obtained under the One-Year approach[24] provided in Solvency II framework. The "re-reserving" method has been applied in order to derive the distribution of the insurer obligations at year end of next accounting year. Moreover the discounting rates, used in this case study, are determined by the term structure of risk-free interest rates, given in QIS5 Technical Specifications and equal for non-life obligations to the risk-free rate as at 31 December 2009 obtained by swap rates and including a 50% illiquidity premium[25].

The application of the One-Year approach shows (see Table 3) how the coefficient concerning the variability of year 1 (CV_{OY}) is located between 70% and 80% of the CV_{Tot} related to total Run-Off variability (roughly 79-80% for SIFA and 72-74% for AMASES). This result is close to that obtained applying the closed formula proposed by Wüthrich and Merz[26], while it has to be emphasized how different patterns are in force according to accident years. Moreover CRM provides a major increase in the skewness of the "One-Year" reserve distribution.

	CV		Skewness	
	CRM (FL)	Bootstrap (CHL)	CRM (FL)	Bootstrap (CHL)
SIFA	7.0%	3.7%	1.27	0.18
AMASES	2.4%	2.7%	0.46	0.16

Table 3 –CV and skewness (One-Year approach) obtained by CRM and Bootstrap LogNormal for both insurers

[24] The IM capital requirement $^{\alpha}$ is here obtained as the difference between the percentile of the distribution of the insurer obligations at year end of next accounting year (R_1^-), opportunely discounted at time zero with the risk-free discount factor v_1, and the best estimate at time 0 (R_0). Both values are posted without considering risk margin.
[25] QIS5 provides discount factors equal to a risk-free rate plus a percentage of a illiquidity premium. The percentage is different according to the nature and the risk profile of the liabilities and it moves from 50% to 100%.
[26] See Wüthrich and Merz (2008)

(50,000 simulations – discounted values)

Table 4 shows the Ratios between SCR for reserve risk and the Best Estimate of Loss Reserve, obtained by Bootstrap and CRM for both insurers according to three different confidence levels and regarding the 99.5%, adopted by Solvency II Directive, as our benchmark.
As expected, the smaller insurer has a higher SCR ratio due to the more variable and skewed claim reserve distribution. Moreover differences between models are more significant for SIFA Insurer and for higher confidence levels because of the skewness effect.

	SCR_0/BE_0					
	CRM(FL)			Bootstrap(CHL)		
	99.00%	99.50%	99.97%	99.0%	99.5%	99.97%
SIFA	22.4%	26.4%	44.4%	9.1%	10.1%	13.6%
AMASES	25.8%	6.6%	10.0%	6.7%	7.5%	10.3%

Table 4 – SCR ratio (equal to capital requirement for reserve risk divided by Best Estimate) obtained by CRM and Bootstrap LogNormal for both insurers (50,000 simulations – discounted values)

Focusing on the benchmark level of 99.5%, it is interesting to verify if an Internal Model allows to save some capital compared to the standard formula. Figure 2 shows the SCR ratio (derived by the capital requirement at 99.5% for reserve risk divided by the initial Best Estimate) obtained by several approaches. In particular, besides the internal models (Bootstrapping and CRM), capital is quantified by QIS5 standard formula through either market-wide or one of several undertaking specific approach. The capital requirement could indeed be estimated as a function of a fixed volatility factor (defined market-wide approach (MW)) or of a credibility structure (defined undertaking-specific approach (USP)) equal to a weighted average of market-wide volatility factor and a specific estimate of σ. Three undertaking-specific methodologies are allowed in QIS5 for reserve risk, two of them are based on Wüthrich and Merz formula. Here it is considered the approach number 3 that evaluates σ as the ratio between the square root of mean square error prediction (MSEP) and the best estimate of provision for claims outstanding derived by Chain-Ladder (CLPCO).

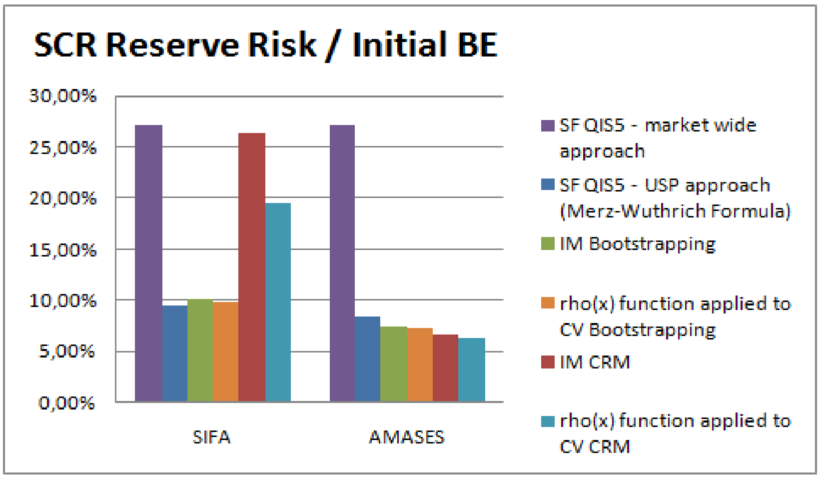

Figure 2 – SCR ratio derived by different methodologies

As expected both insurers have the same capital ratios obtained by the market-wide formula[27], while it is noteworthy that USP approach gives a significant saving of capital for both insurers respect to the market-wide formula. It is interesting to observe that this approach is not so far from

[27] We recall indeed that market-wide approach does not take into account the size factor.

the results obtained by Bootstrapping method. CRM model leads instead to a SCR ratio similar to market-wide standard formula for SIFA and to a capital lower than USP and bootstrapping for AMASES.
Finally always in Figure 2 we aim to test the assumption of Lognormal distribution of claims reserve assumed in the standard formula. At this regard we quantify the capital requirement applying the ρ(x) function to the variability coefficient obtained respectively by Bootstrap and CRM model. As in premium risk valuation, it is confirmed that the LogNormal (with only two parameters)[28] can underestimate the capital requirement when the underlying random variable has a significant positive skewness. It is shown that the high positive skewness derived by CRM (1.27 for SIFA and 0.46 for AMASES, see Table 3) leads to ratios lower than Internal Model results.
LogNormal assumption appears instead reliable in Bootstrap models, because the skewness is usually not affected so much when this method is used.

Figure 3 compares the technical provisions of both companies estimated under the current regulatory environment (Solvency I) to the same reserve evaluated in Solvency II – QIS5 framework. The former one quantifies claims reserve as the ultimate cost estimated to be paid (*not discounted*), while, as described briefly in the first section, Solvency II identifies the current exit value methodology for the valuation. Non-hedgeable liabilities, as provision for claims outstanding, are then obtained as the sum of a best estimate on discounted basis and risk margin. Risk Margin is, here, determined by the cost of capital approach according to QIS5 methodologies including SCR for reserve risk (by Internal Model) and for operational risk (by Standard Formula as a percentage of the technical provisions[29]). No default risk is here considered (no reinsurance is indeed assumed) and no unavoidable market risk (usually supposed equal to zero in Non-Life business)[30]. Finally we assume no diversification effects with other LoBs, we use the proportional method on BE, proposed as simplification by QIS5, to obtain future SCRs and the risk-free rate without illiquidity premium for discounting (as provided by CEIOPS too). At this regard, it is noteworthy that the proportional approach is not always a good proxy to estimate the risk of following accounting years. For example Wüthrich (2010)[31] shows how this method could underestimate future SCR.

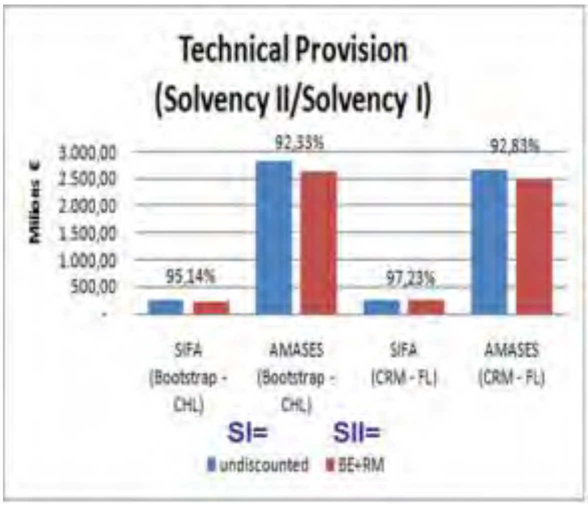

Figure 3 – Technical provision obtained under local legislation and Solvency II framework

[28] We remind that for a LogNormal distribution it could be proved the next relation between variability coefficient and skewness: $\gamma(X) = CV(X) \cdot (3 + CV(X)^2)$
[29] See QIS5 TS
[30] See QIS5 TS
[31] Applying an alternative approach, developed in Salzmann and Wüthrich (2010) according to Chain-Ladder assumptions, to several triangles from European and US insurers, a different pattern for both long-tail and short-tail Lobs is derived.

It could be observed how the ratio between Solvency II and Solvency I Loss Reserve is always lower than 100%. The reduction is given by the discounting effect (more relevant for AMASES because of the lower settlement speed) not considered in current legislation. Differences between insurers and models derive mainly by the risk margin effect. Risk Margin appears very low (roughly 2%) because of a lower SCR. Only the higher Capital Requirement, derived by CRM, leads to a significant Risk Margin (for SIFA only).

The same methodologies have been applied to General Third-Party Liabilities (GTPL) and Motor Damages (MOD) LoBs too in order to analyze the effect on different triangles. Next figures summarize main characteristics of number of paid claims and costs according to both Insurers.

Similarly to MPTL, parameters have been calibrated using Fisher-Langer method for these LoBs too. Moreover, we assume a greater variability coefficient of claim size for GTPL and a lower for MOD (see Table 5). Number of paid claims of GTPL is affected by an only one r.v. q with σ_q equal respectively to 19% and 8.5% for SIFA and AMASES. These values have been calibrated assuming a correlation equal to roughly 0.08, that is implicitly obtained by Mack prediction error formula.
At the same time, MOD provides an only one r.v. q too with σ_q equal to 7.5% and 1.5% for SIFA and AMASES respectively (calibrated as for previous LoBs by the implicit correlation in the Mack formula equal to roughly 0.04).
Finally both lines assume, as in MTPL, the presence of r.v. r_j for each development year with σ_r equal to 3% for both Insurers.

Development Years		2	3	4	5	6	7	8	9	10	11	12
c_{Zj} (GTPL)		0.50	1.13	1.50	1.88	2.25	3.00	3.75	4.50	6.00	7.50	10.50
c_{Zj} (MOD)		0.25	0.38	0.50	0.63	0.75	1.00	1.25	1.50	2.00	2.50	3.50

Table 5 – variability coefficient of claim size according to GTPL and MOD lines

For sake of simplicity, we summarize here main results obtained in a One-Year view. As expected all approaches (simulations and closed formulae) show a greater variability for GTPL line. It has to be emphasized that Bootstrapping CVs are very far from that derived by Wüthrich and Merz formula despite both methods are based on Chain-Ladder assumptions. It seems indeed that CV differences between the methods increase according to triangle with a higher variability.
Considering MOD we get for AMASES a CV larger than SIFA under the Chain-Ladder technique (due also to link ratios variability for DY4) whereas the CRM provides a very low CV (1%), not completely explained by the different size (in MOD the size factor is roughly 2 to 1 only). CRM model confirms a more skewed distribution and a greater dimensional effect respect to Bootstrapping. Focusing on capital ratios obtained by Internal Model and Standard Formula for all lines analyzed, it could be observed that GTPL provides higher SCR ratio than MTPL because of more significant CV. Moreover, it is noteworthy that standard formula (GTPL market-wide volatility factor is equal to 11%) leads to lower capital requirement than Internal Model (either Bootstrapping or CRM) for SIFA. Only USP approach guarantees a saving of capital because of the very low CV. The bigger company has instead lower requirements by both IM and USP with a greater reduction using methodologies based on Chain-Ladder assumptions. Finally, it is confirmed an underestimated capital for small insurer by the $\rho(x)$ LogNormal-based transformation.
As regard to MOD, models based on Chain-Ladder assumptions (as Bootstrapping and Wüthrich-Merz Formula) provide higher SCR ratios for AMASES. On the contrary, CRM reflects the dimensional effect returning a negligible capital for AMASES and a ratio higher than MW approach for SIFA. Standard formula, based on a 10% volatility factor, gives lower requirements (see CRM for SIFA and Bootstrap for AMASES).

Finally, despite of a higher discounting effect than MTPL, SIFA GTPL claims reserves are close to the value obtained under the current legislation because of a significant risk margin (roughly 15% of Best Estimate). The lower capital requirement leads to a reduced risk margin providing a saving of liabilities for the bigger Insurer (the ratio Solvency II divided by Solvency I moves indeed from 92% with Bootstrap to 94% with CRM). Analyzing MOD, we can observe both a lower discounting effect because of the short tail business and a not negligible risk margin (2-3% for SIFA and 1-3% for AMASES). It is noteworthy a ratio between liabilities always close to 1 with lower values only for SIFA (with bootstrap model) and AMASES (with CRM).

Conclusion

According to us the CRM model seems to represent a valid stochastic model to be added to models suggested at the moment by the broad actuarial literature. Parameters of the models may be easily estimated using claims DataBase and the deterministic model "Fisher-Lange", based on the estimate of separate number of claims to be paid and future average costs. Moreover, the same model could be used also with deterministic methods other than Fisher-Lange, providing separate estimates for either number of future payments and average claim costs (i.e. using other incremental average cost methods).

Some case studies show that CRM has a different behaviour respect to Bootstrapping according to the line of business analyzed. In general, variability coefficients and skewness of claims reserve distribution are more sensitive to the insurer size when CRM model is applied. As regard to capital assessment, market-wide formula is usually higher than IM results. Only GTPL provides greater capital ratios than that obtained by using CEIOPS volatility factors. USP approach, based on Wüthrich and Merz formula, ensures a saving of capital and the lowest aggregated ratio. Moreover the discounting effect leads to technical provisions, evaluated under Solvency II framework, lower than undiscounted reserve, while very high Risk Margin reverses the result in some cases (e.g. for GTPL line).

Nevertheless some sensitivity analyses put in evidence the relevance of a reliable parameter calibration and the strict link between parameter uncertainty and the variability of the overall claim reserve. In particular it needs to pay attention to the estimation of the standard deviation of the structure variable q in order to properly capture the correlation inside the triangle. Variability coefficient of total run-off claims reserve distribution is significantly affected by that value. We regard estimation of variability coefficients c_{zj} (obtainable from the Claim DataBase of the company and different for each development year) as a key issue. As well known these data are available mainly as an internal actuary, while it is rather difficult to get them for an external actuary (e.g. involved in a take-over evaluation). This aspect might produce minor diffusion of the approach compared to other methods based on aggregate run-off triangles mainly.

The One-Year approach is representing a key approach to be fully investigated in order to perform evaluations consistent with Solvency II. Nevertheless, according to us, this approach used for reserve risk might guarantee a major consistency with the premium risk modelling, when applicable. Furthermore, it should be remarked both the need of an international Data Base on run-off triangles and that the results given for the 3 LoBs must be taken with great care and numerous analyses should be carried on in practice for consistent estimation of main parameters. Further research improvements will regard methods to introduce stochastic inflation. Secondly it appears a key issue to capture the variability inside the triangle provided by the new Direct Indemnity System in force since 2007 for MTPL (CARD) as prescribed by Italian legislation. Finally the analyses should be carried on in order to evaluate the effect of tail factor on both One-Year approach and Total Run-Off.

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Appendix. Run-Off Triangles

AY/DY	1	2	3	4	5	6	7	8	9	10	11	12	12+
1	28,446	29,251	12,464	5,144	2,727	2,359	1,334	1,238	941	860	282	727	1,068
2	31,963	36,106	13,441	5,868	2,882	2,422	918	1,076	734	458	456		
3	37,775	40,125	12,951	6,034	3,010	1,264	1,250	1,135	904	559			
4	40,418	44,499	15,370	5,594	2,616	1,984	2,137	1,184	873				
5	44,116	45,490	15,339	5,478	2,541	2,906	1,294	1,124					
6	50,294	48,040	17,843	7,035	3,934	2,726	2,267						
7	49,620	49,991	19,570	10,047	5,750	3,313							
8	46,410	49,694	20,881	8,202	4,714								
9	48,295	49,354	18,304	8,833									
10	52,590	50,606	18,604										
11	58,599	53,743											
12	60,361												

Figure 4 - Triangle SIFA – LoB MTPL
(Historical costs of incremental paid amounts, thousands of Euro)

AY/DY	1	2	3	4	5	6	7	8	9	10	11	12	12+
1	193,474	172,618	87,200	45,798	29,768	19,795	19,782	17,315	13,372	12,552	8,831	8,053	19,889
2	199,854	168,966	80,543	40,656	29,053	21,121	19,964	14,249	10,720	13,684	6,008		
3	225,578	186,764	93,349	47,609	30,971	26,291	17,621	18,410	14,662	7,591			
4	256,398	236,678	105,616	51,172	37,338	24,085	20,754	12,082	14,137				
5	282,956	263,196	120,383	63,689	37,220	29,239	23,120	15,509					
6	292,428	284,401	141,400	56,390	40,195	27,955	29,987						
7	312,350	285,506	131,687	75,252	46,549	38,731							
8	327,673	307,992	161,516	77,965	52,696								
9	339,899	326,280	185,911	101,273									
10	371,275	385,847	193,006										
11	388,025	390,737											
12	398,686												

Figure 5 - Triangle AMASES – LoB MTPL
(Historical costs of incremental paid amounts, thousands of Euro)

AY/DY	1	2	3	4	5	6	7	8	9	10	11	12	12+
1	976	1,508	674	416	515	904	1,026	822	434	329	242	368	2,929
2	1,254	2,602	743	438	1,432	1,355	907	672	466	559	238		
3	1,777	2,458	812	763	1,010	849	1,131	1,875	636	1,364			
4	1,256	1,917	1,020	552	777	1,440	2,978	1,979	729				
5	998	1,976	957	445	599	793	1,110	1,144					
6	992	1,651	612	379	452	579	657						
7	1,025	2,026	545	304	358	1,103							
8	1,045	1,963	641	436	902								
9	890	1,906	3,482	882									
10	640	1,422	690										
11	785	1,663											
12	894												

Figure 6 - Triangle SIFA – LoB GTPL
(Historical costs of incremental paid amounts, thousands of Euro)

AY/DY	1	2	3	4	5	6	7	8	9	10	11	12	12+
1	13,861	22,303	14,025	9,469	8,627	10,122	9,176	7,663	7,908	6,052	5,091	3,147	16,601
2	15,602	20,247	14,735	10,855	10,320	14,837	9,462	6,843	6,028	6,088	4,358		
3	18,115	25,441	15,099	13,133	12,671	10,299	11,776	7,467	4,901	6,233			
4	16,988	26,186	20,205	20,399	13,756	12,483	12,410	11,218	7,269				
5	18,166	32,737	24,608	17,171	15,913	13,783	11,467	11,492					
6	16,601	31,350	21,314	17,272	14,651	10,314	12,011						
7	16,862	30,628	34,888	16,066	10,703	21,147							
8	16,088	28,624	19,634	15,402	10,086								
9	15,764	22,721	14,961	9,999									
10	16,592	28,925	17,748										
11	17,942	28,771											
12	17,128												

Figure 7 - Triangle AMASES - LoB GTPL
(Historical costs of incremental paid amounts, thousands of Euro)

AY/DY	1	2	3	4	5	6	7	8	9	10	11	12	12+
1	14,364	5,755	77	3	32	10	28	13	4	17	6	4	130
2	16,551	6,512	278	109	28	41	118	51	7	7	23		
3	16,100	6,677	281	115	17	40	66	31	8	25			
4	15,543	6,352	145	12	20	40	6	1	3				
5	15,372	7,162	402	17	16	10	40	2					
6	17,581	7,196	259	150	159	14	28						
7	18,641	6,702	212	121	28	59							
8	21,062	7,196	499	164	75								
9	25,591	8,682	302	74									
10	25,921	8,686	581										
11	28,350	9,142											
12	41,858												

Figure 8- Triangle SIFA - LoB MOD
(Historical costs of incremental paid amounts, thousands of Euro)

AY/DY	1	2	3	4	5	6	7	8	9	10	11	12	12+
1	44,497	18,158	239	16	7	6	117	25	9	9	7	7	54
2	47,531	20,245	344	9	66	9	11	12	32	68	20		
3	54,502	22,691	412	174	64	168	37	49	41	17			
4	52,831	21,478	623	234	139	70	9	31	52				
5	46,943	24,072	1,426	292	65	52	21	33					
6	55,724	24,508	1,159	1,968	146	30	113						
7	63,507	27,141	1,783	149	33	37							
8	68,291	29,937	995	167	101								
9	73,209	28,736	1,367	343									
10	74,537	31,020	1,567										
11	74,210	27,706											
12	72,418												

Figure 9 - Triangle AMASES - LoB MOD
(Historical costs of incremental paid amounts, thousands of Euro)