Nonparametric approach to analysing operational risk losses

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Abstract

Nonparametric methods combine most of the advantages of parametric alternatives when assessing risk. We present a new way to address quantile estimation with no distribution assumptions, we discuss the asymptotic properties and refer to the conclusions of an extensive simulation study. In the last part, we show how to implement our method within a Monte Carlo procedure in an internal model for operational risk. We present a case study. We quantify the operational risk linked to external fraud and we evaluate the anti-fraud policy followed by the company, i.e. whether or not the company has a claims auditing system or a special investigation unit. Ultimately we show the impact of on operational risk reduction.

Keywords: Internal Models, Risk measures, Operational Risk, Fraud

1 An introduction to operational risk

The aim of this work is to analyse fraud as an operational risk for the insurance company, to provide an innovative methodological approach and to quantify the effect of failing to detect fraud after investigation in a real case study.

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Operational risk refers to the possibility of unexpected events that occur as a consequence of alterations in regular functioning of companies. In the financial and insurance sector, where the central activity has to do with contracts, investment, economic transactions and customer relationships, the risk of any of these activities not operating correctly exists as in industrial plants. However, we believe that in insurance some events do not have physical consequences, and consequently they are more difficult to recognize than operational failures elsewhere.

The quantification of operational risk meets an additional difficulty due to the lack of data. There are generally not many internal operational risk observations for one given company and there is sometimes a systematic underreporting of occurred losses that are never reported as operational losses. Moreover, large operational events occur even if they are very infrequent and they are significantly relevant, since they represent the tail behaviour of the operational risk distribution.

With the introduction of the Solvency II regulations in the EU, it has become clear to insurance companies that the risk associated with the different operations they carry out needs to be quantified. The primary aim is to quantify the capital needs to enable companies to work in conditions of sufficient solvency. Solvency II implies a radical change and now companies quantify capital needs according to the risks they take on rather than according to the volume of premiums they underwrite. Risk quantification takes on an increasingly fundamental role in insurance companies. Capital requirements have to be calibrated to ensure all quantifiable risks for the insurance or reinsurance company are taken into account. As indicated in Article 101 of the Solvency II Level 1 text ([12]) the necessary capital shall correspond to the Mean-Value-at-Risk of the basic own funds of an insurance or reinsurance undertaking subject to a confidence level of 99.5 % over a one-year period.

One of risks that must be covered is related to operational functions, together with others as market risk, credit risk and so on. According to Article 13(33) of the Solvency II Level 1 text, operational risk is the risk of loss arising from inadequate or failed internal processes, or from personnel and systems, or from external events. The calculation of the capital requirement for operational risk with respect to life insurance contracts shall take account the amount of annual expenses incurred in those insurance obligations. Regarding non-life insurance contracts, the capital requirement for operational risk should be calculated taking into account the earned premiums and technical provisions which are held for those insurance and reinsurance obligations.
The capital requirement for operational risks shall not exceed 30% of the Basic Solvency Capital Requirement relating to those operations (both insurance and reinsurance). For most insurers, operational risk is exceptionally difficult to assess because historical information, either internal or external, is simply inexistent.

A type of operational failure that may be inspected is the presence of undetected fraudulent claims. In other words, the absence or inefficiency of an auditing protocol can lead the firm to severe aggregate losses. Investing in an auditing system scheme reduces the risk of losses in the operational risk category, but the question that is often addressed is only whether its benefits are greater than its costs. Under the new regulation framework imposed by solvency requirements, a new perspective is introduced as operational risk mitigation implies a reduction of capital requirements. Now an auditing system unit establishing systematic inspections of incoming claims provides additional savings to a company when compared to the traditional cost-benefit analysis that has impregnated the literature on fraud deterrence and detection in the past. Our analysis is new because we address the benefit of implementing an anti-fraud policy and we also consider the impact of an auditing technology on operational risk mitigation.

The methods are presented in section 2. In section 3 we provide with an introduction to fraud in insurance and then we describe our case study. In general operational risk takes into account the losses that may come about through errors in the company’s internal processes, in its day-to-day business, due to the actions of its own employees or the influence of external factors. We aim at quantifying the operational risk linked to external fraud, that also means taking into account the anti-fraud policy followed by an insurer, i.e. whether or not the insurance company has a claims auditing system or a special investigation unit. In our application, we discuss the impact of implementing auditing systems in the area of motor insurance and the effect of those systems on diminishing under-reporting of losses. Ultimately, we show the impact of anti-fraud auditing on operational risk reduction. In section 5 we provide a discussion of the empirical analysis and then we conclude.

2 Nonparametric method to calculate operational risk quantiles

Quantifying loss risk can be carried out by applying alternative methodologies (see [14], for an extensive review of loss models). We chose value-at risk ($VaR$) as the basic risk measure in this
study, then we propose a non-parametric estimation method for the risk measure, which is then used to analyze the detection or failure to detect fraudulent claims (see [4], [3], [5] and [6]).

We assume a single operational loss is a random variable which we may want to study in internal models for economic capital requirement. Risk measurement in that area has generally been based on three possible statistical approaches: i) the empirical statistical distribution of the loss, ii) the normal or Student t distribution and iii) some alternatives parametric approximations. This means that one should either rely on the data and the empirical distribution function, for which a minimum sample size is required, or one should go for the normal approximation, meaning that a straightforward expression is available for the most popular risk measures, although the loss may be quite far from having a normal shape, or, finally, one should find a suitable parametric density to which the loss data should fit reasonably well.

Nonparametric methods to obtain a risk assessment can combine most of the advantages of the previous methods. On the one hand, the nonparametric approach is easy to implement (such as the normal or Student t approximation), and on the other hand, it is very flexible so it is comparable to the empirical distribution approach. One of the advantages of our approach is that it can smooth down data and deal with tail behavior, something that is inherently difficult because extreme data are infrequent. We present the method, the asymptotic properties and summarize an extensive simulation study shown in [4]. We also discuss how to implement the method within a Monte Carlo procedure in an internal model for operational risk.

Function $F_X$, which is the cumulative distribution function (cdf) of a single operational risk loss random variable $X$, is fit by means of a non-parametric method, based on a double transformed kernel estimation, proposed by [5] and [6]. These authors show that this is a good approximation to fitting the probability distribution function (pdf) $f_X$ of $X$, specially if this random variable has strong right skewness. Later, this method was generalized by [4] to fit the cdf $F_X$ and its quantiles.

Classical kernel estimator does not provide with good fits of a cdf when the analysed variable has much positive skewness. In the right tail, where the data are scarce, the classical kernel estimator has a shape similar to then empirical estimator. Therefore, the classical kernel estimator does not extrapolate well the value of cdf associated to large losses. An alternative is the transformed kernel estimation. In general, the transformed kernel estimation consists of a transformation of the original variable to a new transformed variable with symmetric distribution,
which we can fit easily with the classical kernel estimator (see [20], [7] and [9]).

Let $X_i, i = 1 \ldots n$ be a random sample of $n$ observed losses $X$ with cdf $F_X$. The classical kernel estimation of $F_X$ is obtained by integration of the classical kernel estimation of its pdf $f_X$. By means of a change of variable, the usual expression for the kernel estimator of a cdf is obtained:

$$
\hat{F}_X(x) = \int_{-\infty}^{x} \hat{f}_X(u) du = \int_{-\infty}^{x} \frac{1}{nb} \sum_{i=1}^{n} k \left( \frac{u-X_i}{b} \right) du
$$

where $k(\cdot)$ is a fixed pdf, which is known as the kernel function. It is usually a symmetric pdf, but this does not imply that the final estimate of $F_X$ is symmetric. Some examples of very common kernel functions are the Epanenechnikov and the Gaussian kernel (see [18]). Function $K(\cdot)$ is the cdf of $k(\cdot)$. Parameter $b$ is known as the bandwidth. It controls the smoothness of the distribution estimate. The larger $b$ is, the smoother the resulting cdf.

Properties of kernel cdf estimator were analysed by [2] and [17]. Both pointed out that when $n \to \infty$, the mean squared error (MSE) of $\hat{F}_X(x)$ can be approximated by:

$$
E \left\{ \left( \hat{F}_X(x) - F_X(x) \right)^2 \right\} \sim \frac{F_X(x)[1-F_X(x)]}{n} - \frac{1}{n} \sum_{i=1}^{n} k \left( \frac{1}{b} \right) dt + b^4 \left( \frac{1}{2} f_X(x) \int_{-1}^{1} k(t) dt \right)^2.
$$

If we compare (2) to the MSE of the empirical distribution, which equals $\left( F_X(x) \right)^2 / n$, we conclude that the kernel cdf estimator has less variance than the empirical distribution estimator, but it has some bias which tends to zero as the sample size increases.

Alternatively, let $T(\cdot)$ be a concave transformation, then $Y = T(X)$ and $Y_i = T(X_i), i = 1 \ldots n$ the transformed observed losses, the transformed kernel estimation is:

$$
\hat{F}_Y(y) = \frac{1}{n} \sum_{i=1}^{n} K \left( \frac{y-Y_i}{b} \right) = \frac{1}{n} \sum_{i=1}^{n} K \left( \frac{T(x) - T(X_i)}{b} \right).
$$

In [4] it is shown that the asymptotic mean square error (MSE) of the transformed kernel estimator of cdf is:

$$
\frac{F_X(x)[1-F_X(x)]}{n} - \frac{1}{T'(x)} F_X(x) \left( \frac{1}{b} \right) dt + \frac{1}{T'(x)} \left( \frac{f_X(x)}{T'(x)} \right)^2 \left( \frac{1}{2} f_X(x) \int_{-1}^{1} k(t) dt \right)^2 b^4.
$$

Similarly to the MSE of the classical kernel estimator, which is expressed in (2), the first two terms in (4) correspond to the variance and the last term is the squared bias. The variance
part is smaller than the variance of classical kernel estimator if $T'(x) < 1$ and, on the contrary, the squared bias is larger than the bias of the classical kernel estimator.

In the simulation results shown in [4], the authors concluded that the double transformed kernel estimator asymptotically minimizes (4) by using some interested properties of the Beta distribution.

Terrell (1990) proved that the cdf associated to $Beta(3, 3)$ in the domain $(-1, 1)$ is the one which provides the best fit when using classical kernel estimation, among a set of distribution with known variance. The cdf of $Beta(3, 3)$ in the domain $(-1, 1)$ is:

$$M(x) = \frac{3}{16} x^5 - \frac{5}{8} x^3 + \frac{15}{16} x + \frac{1}{2}, \quad x \in (-1, 1).$$  \hspace{1cm} (5)

The double transformed kernel estimation consists of two steps. A first transformation of the data using a cdf $T(X_i) = Z_i, \ i = 1, \ldots, n,$ is done. Therefore a new transformed variable is obtained that has a distribution around a $Uniform(0, 1)$. Then, we transform $Z_1, Z_2, \ldots, Z_n$ using the inverse of the cdf of a $Beta(3, 3)$ distribution, $M^{-1}(Z_i) = Y_i, \ i = 1, \ldots, n$. The double transformed data are $Y_1, Y_2, \ldots, Y_n$ and they are generated by a distribution that is similar to $Beta(3, 3)$. The double transformed kernel estimation is:

$$\hat{F}_X(x) = \frac{1}{n} \sum_{i=1}^{n} K \left( \frac{M^{-1}(T(x)) - M^{-1}(T(X_i))}{b} \right)$$

$$= \frac{1}{n} \sum_{i=1}^{n} K \left( \frac{y - Y_i}{b} \right),$$  \hspace{1cm} (6)

which corresponds to the classical kernel estimator of the cdf of the transformed variable. We use the Epanechnikov kernel:

$$k(t) = \begin{cases} 
0.75 \left(1 - t^2 \right) & \text{if} \ |t| \leq 1 \\
0 & \text{if} \ |t| > 1.
\end{cases}$$

We can assume that the resulting transformed random sample $Y_1, Y_2, \ldots, Y_n$ is generated by a known distribution and its cdf is defined in (5). This permits to obtain the value of optimal bandwidth $b_T(x)$ at the point where we want to estimate the cdf or, alternatively, for the quantile. In [2] it is shown that the optimality of the kernel estimator of distribution function is also valid for to the quantile estimator obtained using the inverse of the kernel estimation of cdf $x = \hat{F}_X^{-1}(q)$ where $0 \leq q \leq 1$. Also, [2] obtained the optimal bandwidth in the kernel estimation.
of cdf at point $x$. Using the notation in (6) in the expression given in [2] we obtain:

$$b_{T(x)} = \left( \frac{m(y) \left( 1 - \int_1^y K^2(t) \, dt \right)}{4 \left[ \frac{1}{2} m'(y) \int_1^y t^2 k(t) \, dt \right]^2} \right)^{\frac{1}{4}} n^{-\frac{1}{3}},$$

(7)

where $m$ is the pdf associated to $M$ and $m'$ is its first derivative.

Following [6], we propose to use as first transformation $T(\cdot)$ the cdf of the Champernowne Generalized distribution:

$$T(x) = \frac{(x + c)^\alpha - c^\alpha}{(x + c)^\alpha + (M + c)^\alpha - 2c^\alpha}.$$  

(8)

This transformation was proposed by [9], these authors analyse the properties of this distribution and they concluded that it has a very flexible form, similar to the lognormal in the lower values of the variable and similar to the Generalized Pareto in the higher extreme values. We use the maximum likelihood method as in [9] to obtain the transformation parameters.

The procedure for simulating total operational risk costs in fraud detection has two parts. First, using (6) we fit the cumulative distribution function (cdf) $F_X$ of the random variable $X$, which is equal to cost of a single claim associated to an operational loss. Here a loss is the failure to detect an external fraud. Second, we determine the number of claims with fraud $N_{Fr}$, using the information of an insurer’s database. We suppose that the number of claims $N_{Fr}$ can be deterministic or stochastic. In the stochastic case, $N_{Fr}$ is generated by a Poisson distribution. We simulate the cost of the $N_{Fr}$ claims using the previously fitted cdf. We repeat 10,000 replications. For each replication we calculate the total sum of the $N_{Fr}$ with randomly generated costs, each one cause by an undetected fraudulent claim.

With the double transformed kernel estimation of cdf expressed in (6) we generate the $N_{Fr}$ values $x$, namely the cost of a claim that contains fraud which was not detected:

$$x = \hat{F}_X^{-1}(u),$$

(9)

where $u$ is a random value from a Uniform $(0, 1)$. In order to solve (9) we use a Newton-Raphson algorithm. Then, we obtain the sum of the $N_{Fr}$ generated values from the double transformed kernel estimation of cdf.

Using the 10,000 generated values of the total cost variable, the empirical distribution is calculated and then its value-at-risk at the 99.5% level ($VaR_{0.995}$) is obtained. We compare this
value with the expected cost of the auditing system that the insurer company needs to implement in order to detect fraudulent claims.

3 Fraud detection and fraud control

When aiming to detect external fraud arising from incoming claims, we will take into account two possible courses of action open to companies to the possible existence of suspicious behaviour on the part of policyholders: 1) assume that fraud exists but without initiating any active policy to control, prevent and detect it; and 2) set up specialized departments to fight fraud (known as SIUs, Special Investigation Units), which should be given cases where fraud is suspected so that they can carry out a thorough investigation. As far as the second situation is concerned, many companies in Spain admit that they have no specialist anti-fraud unit, although they have introduced policies to control and detect fraud, normally within appraisal units. In this case, in the same way as in the case of SIUs, we will also take into account the existence of special anti-fraud actions within the company.

The different areas associated with fraud as an operational risk for the company are show in Figure 1. We start from the difference between what we can call internal fraud and external fraud for the company. In general terms, when we talk of internal fraud we refer to that committed by the insurance company’s employees or by others who, although not actually staff members, work for the company. External fraud includes the much more frequent situation where the fraudulent action is carried out by the actual policyholders, trying to obtain wrongful benefit from their insurance policy.

External fraud, which is probably the most studied case in the existing literature and on which companies can find much more information, is committed by policyholders. In fact this type of fraud may actually be committed in collusion with company staff, and therefore in some cases internal fraud and external fraud may coexist. However, most often fraud involves actions carried out independently by policyholders who seek to obtain wrongful compensation under the insurance they have contracted (either by planning the accident, i.e. planned fraud, or by inflating costs, i.e. build-up).

Quantifying the operational risk linked to external fraud means taking into account the anti-fraud policy followed by the company, i.e. whether or not the company has a claims auditing
Figure 1: Sources of fraud for the insurance company

system or a special investigation unit. The dynamics of anti-fraud procedures within an insurance company are shown in Figure 2. As we can see, once a claim is submitted to the company and has undergone the initial audit, if it is not suspicious the process involves following normal procedures (the left-hand part of the chart), but if there are any indications of fraud it will be channelled through a more thorough auditing process (the right-hand part of the chart). However, the procedures shown in Figure 2 do not always form part of companies’ auditing systems. Hence it is not always possible to find special fraud investigation units in companies, many of which often include anti-fraud measures as part of the procedures to be carried out by appraisers, without there being specific acknowledgment of the task.

Quantifying operational risk from the point of view of external fraud has to take into account whether or not the company carries out claim auditing for fraud detection purposes. If the company does not carry out a thorough fraud investigation, operational risk can be quantified by taking into account the expected proportion of fraudulent claims and the total compensation paid by the company for this concept. However, if the company does not have enough practical knowledge of its own regarding fraudulent claims borne, another solution would be to use the proportion of fraud found in the sector as a whole. With insurance against damage to property, therefore, where it is normal to use data for average costs borne by insurance companies in the sector as a whole, one solution would be to quantify operational risk by taking into account the
expected proportion of fraudulent cases and the average compensation paid (calculating both the average accident rate associated with fraud and the product between the expected number of fraudulent cases and the average amount paid). Certainly the difference that exists between the proportion of fraudulent claims borne by the company and the compensation actually paid can be large, and therefore the quantification of operational risk may not be accurate.

In cases where the company follows an active anti-fraud policy, the quantification of operational risk should take into account a series of additional considerations, basically the cost of the investigation and the savings deriving from it. One way of doing this, as we do in this study, is to analyse the company’s loss risk to the detection or non-detection of fraudulent behaviour once an active anti-fraud policy has been introduced. Our aim is to compare the economic costs that the company would have to assume if it did not detect frauds with the costs associated with reducing compensation payments as a result of the frauds detected. The overall economic cost would therefore be calculated based on two concepts: on the one hand, the cost deriving from the payment of the compensation, and on the other, the cost associated with claims auditing. Note that under the approximation presented in this paper we consider that claims auditing includes all those costs associated with the claim that are not part of the actual compensation to the claimant.
4 A case study of fraud in motor insurance

In this section we assess the extent to which the auditing of accident claims reduces the insurer’s loss risk. To this end we use a sample taken from a motor insurance portfolio which was collected by a Spanish insurer and which contains information of 64,642 claims. The claims are about property damages to the vehicle and were reported during 2000; 691 claims are fraudulent, and the rest are not. We assume those that are not found to be fraudulent could still contain some form of undetected fraud.

For each claim we have information about three relevant variables: i) a claimed amount which presumably corresponds to the economic evaluation of property damages to the vehicle, ii) the auditing cost, and iii) the final compensation to the insured. The first variable is related to the total payment that would be paid in order to repair of the damages suffered by the vehicle (or the necessary compensation in the event that the cost of repairing damages exceeds the vehicle value). If fraud does not exist, the claimed amount will be exactly the same as the final compensation to the insured. The second variable, the auditing cost, is related to the inspection by company experts for evaluation of damages. In its minimal value it is the cost for the adjuster evaluating the damages to the vehicle. However, if suspicion of fraud exists a deeper investigation is undertaken and auditing costs can notably be higher. Finally, the third variable is the final compensation to the insured for the damages to the vehicle.

As we have mentioned, if the claim is honest, the compensation to the insured is equal to the evaluation of property damages and to the claimed amount. But, if fraud is detected, compensation is zero in most cases because of the denial of the insurer to pay for the claimed amount. Only in a small number of cases the company agrees to pay a part of damages, which are precisely the ones that are not fraudulent. According to our definitions for the three variables, the total cost of a claim to the insurer is calculated as the sum of two variables, namely, the auditing cost and the final compensation to the insured. Table 1 shows the main descriptive statistics for all these variables.

Table 1 shows that the number of fraudulent claims is very low, around 1.07% of total claims. According to [1] the percentage of undetected fraudulent claims in the insurance portfolio is estimated to be around 5%, even if some special audit exists. However, the average cost of
Table 1: Descriptive statistics of claims costs (in Euros)

<table>
<thead>
<tr>
<th></th>
<th>Average</th>
<th>Std. Dev.</th>
<th>Minimum</th>
<th>Median</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Non fraudulent claims</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(N=63,951)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Claimed property damages (A)</td>
<td>612.45</td>
<td>1,178.79</td>
<td>0.00</td>
<td>333.30</td>
<td>63,959.71</td>
</tr>
<tr>
<td>Auditing cost (B)</td>
<td>38.94</td>
<td>26.65</td>
<td>0.00</td>
<td>32.42</td>
<td>394.77</td>
</tr>
<tr>
<td>Final compensation (C)</td>
<td>612.45</td>
<td>1,178.79</td>
<td>0.00</td>
<td>333.30</td>
<td>63,959.71</td>
</tr>
<tr>
<td>Total cost of claim (B+C)</td>
<td>651.39</td>
<td>1,199.02</td>
<td>0.00</td>
<td>369.59</td>
<td>64,011.70</td>
</tr>
<tr>
<td><strong>Fraudulent claims</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(N=691)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Claimed property damages (A)</td>
<td>1,116.96</td>
<td>1,486.42</td>
<td>49.52</td>
<td>696.00</td>
<td>17,251.83</td>
</tr>
<tr>
<td>Auditing cost (B)</td>
<td>223.09</td>
<td>127.12</td>
<td>0.00</td>
<td>189.91</td>
<td>882.45</td>
</tr>
<tr>
<td>Final compensation (C)*</td>
<td>7.55</td>
<td>88.35</td>
<td>0.00</td>
<td>0.00</td>
<td>1888.11</td>
</tr>
<tr>
<td>Total cost of claim (B+C)</td>
<td>230.64</td>
<td>155.87</td>
<td>0.00</td>
<td>189.98</td>
<td>2,271.56</td>
</tr>
<tr>
<td><strong>All claims</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(N=64,642)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Claimed property damages (A)</td>
<td>617.85</td>
<td>1,183.63</td>
<td>0.00</td>
<td>335.90</td>
<td>63,959.71</td>
</tr>
<tr>
<td>Auditing cost (B)</td>
<td>40.91</td>
<td>35.13</td>
<td>0.00</td>
<td>32.56</td>
<td>882.45</td>
</tr>
<tr>
<td>Final compensation (C)</td>
<td>605.99</td>
<td>1,174.16</td>
<td>0.00</td>
<td>330.58</td>
<td>63,959.71</td>
</tr>
<tr>
<td>Total cost of claim (B+C)</td>
<td>646.90</td>
<td>1,193.49</td>
<td>0.00</td>
<td>366.74</td>
<td>64,011.70</td>
</tr>
</tbody>
</table>

*Only in 8 cases the final compensation to the insured was different from zero
fraudulent claims is reduced by 79% if these claims were audited. If all the claims are taken together, after auditing the average cost increases by 4.7%. However, we have to take into account that the distribution of claims costs has a marked asymmetry towards the right, a fact that can be deduced by comparing the average and the median of the variables. In all the analysed cases (fraud, non fraud and total) the median is lower than the mean. This implies the existence of extreme values, i.e. very high costs that could severely affect the company’s operational risk profile and consequently its solvency. It is therefore essential to analyse the extent to which auditing reduces the company’s operational loss risk.

5 Results

We have calculated an estimate of the 99.5% quantile of the total cost of undetected fraud. This is the basis for estimating the solvency capital requirement due to operational risk arising from the incapacity to detect fraudulent claims.

In order to see the impact of the amount of fraud on operational risk evaluation, we have set a series of scenario levels ranging from 0.1% to 10% corresponding to the proportion of fraudulent claims in the total number of claims. There are methods to estimate the percent of undetected fraudulent claims, but here we have not implemented those here and we have just assumed that a certain proportion of all claims reported to the company is fraudulent. Obviously, the larger the proportion, the larger the expected cost of fraudulent claims to the insurer, and consequently the larger quantile is. This result is clearly shown on Table 2 as all columns present increasing amounts.

The second column in Table 2 refers to an approximation of the value-at risk assuming that the aggregate cost of fraudulent claims follows a normal distribution. The third column in Table 2 indicates the estimate of the Value-at-Risk where the simulation approach has been used and no assumption on the parametric distribution of the cost of fraudulent claims is made. Here, the number of fraudulent claims is deterministic, as it is the proportion given in the first column time the total number of claims, i.e. 64,642. In the last column of Table 2 we present the estimates of the Value-at-risk of the aggregate cost of fraudulent claims using a simulation approach where both the number of fraudulent claims is random and the cost of each individual claim is simulated using the nonparametric estimate of the distribution function of the cost of a
Table 2: Estimated Quantile for Operational Risk (in thousand Euros)

<table>
<thead>
<tr>
<th>Fraud Frequency</th>
<th>( N_{Fr} ), deterministic</th>
<th>( N_{Fr} ), deterministic</th>
<th>( N_{Fr} ), stochastic</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001</td>
<td>102.99</td>
<td>560.47</td>
<td>663.62</td>
</tr>
<tr>
<td>0.003</td>
<td>269.93</td>
<td>1,581.71</td>
<td>1,737.66</td>
</tr>
<tr>
<td>0.005</td>
<td>429.85</td>
<td>2,590.33</td>
<td>2,802.31</td>
</tr>
<tr>
<td>0.010</td>
<td>819.37</td>
<td>5,059.18</td>
<td>5,371.40</td>
</tr>
<tr>
<td>0.030</td>
<td>2,334.68</td>
<td>14,824.58</td>
<td>15,368.61</td>
</tr>
<tr>
<td>0.050</td>
<td>3,827.80</td>
<td>24,555.51</td>
<td>25,168.01</td>
</tr>
<tr>
<td>0.060</td>
<td>4,570.60</td>
<td>29,387.41</td>
<td>30,098.21</td>
</tr>
<tr>
<td>0.070</td>
<td>5,311.73</td>
<td>34,208.80</td>
<td>34,947.18</td>
</tr>
<tr>
<td>0.080</td>
<td>6,051.54</td>
<td>39,072.85</td>
<td>39,868.76</td>
</tr>
<tr>
<td>0.100</td>
<td>7,528.09</td>
<td>48,701.35</td>
<td>49,780.99</td>
</tr>
</tbody>
</table>

\( N_{Fr} \) is the number of fraudulent claims

fraudulent claim. Note the difference between the normal approximation and the ones based on a nonparametric approach. The normal approximation substantially underestimates the quantile. For instance, if 5% of the claims were fraudulent, the normal approximation would estimate Value-at-Risk at 99.5% level is 3,827.80 thousand Euros, which is much lower than the figures produce by the simulation method. For the stochastic approach, the Value-at-Risk if 5% of the claims were fraudulent equals 25,168.01 thousand Euros. So, this means that the insurer could pay more than 25 million Euros with probability 0.5% dues to fraudulent claims.

Table 3 shows the estimates of auditing costs in the different scenarios of proportion of fraudulent claims. The total auditing cost if 5% of the claims were fraudulent, and 5% were audited with a special investigation, is expected to be equal to 3,231.39 thousand Euros. The aggregate cost of special audits would then be equal to 721.02 thousand Euros.

There are obvious implications from the previous results. For instance, assume our company has a 6% proportion of fraudulent claims. Implementing a special audit scheme like the
Table 3: Auditing cost (in thousand Euros)

<table>
<thead>
<tr>
<th>Fraud Frequency</th>
<th>Total auditing cost</th>
<th>Fraud auditing cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001</td>
<td>2,636.23</td>
<td>14.28</td>
</tr>
<tr>
<td>0.003</td>
<td>2,755.19</td>
<td>43.06</td>
</tr>
<tr>
<td>0.005</td>
<td>2,874.33</td>
<td>72.06</td>
</tr>
<tr>
<td>0.010</td>
<td>2,993.29</td>
<td>144.11</td>
</tr>
<tr>
<td>0.030</td>
<td>3,112.43</td>
<td>432.57</td>
</tr>
<tr>
<td>0.050</td>
<td>3,231.39</td>
<td>721.02</td>
</tr>
<tr>
<td>0.060</td>
<td>3,350.35</td>
<td>865.13</td>
</tr>
<tr>
<td>0.070</td>
<td>3,469.49</td>
<td>1,009.25</td>
</tr>
<tr>
<td>0.080</td>
<td>3,588.45</td>
<td>1,153.58</td>
</tr>
<tr>
<td>0.100</td>
<td>3,707.59</td>
<td>1,442.03</td>
</tr>
</tbody>
</table>

one observed in the data would reduce the proportion of fraudulent claims that go undetected approximately by 1%. The expected cost of auditing that 1% equals 144.11 thousand Euros. However, value-at-risk at the 99.5% level is reduced from 30,098.21 to 25,168.01 thousand Euros, see the third column of Table 2 for a frequency of 6% and 5%, respectively.

6 Conclusions

The non-detection of fraudulent behaviour can be considered a source of operational risk within an insurance company. Although different areas can be associated with fraud in the company, external fraud includes the much more frequent situation where the fraudulent action is carried out by the actual policyholders, trying to obtain wrongful benefit from their insurance policy, in our case in the context of car damages.

Quantifying operational risk from the point of view of external fraud has to take into account whether or not the company carries out claim auditing for fraud detection purposes. We chose value-at-risk as the risk measure, then carried out a non-parametric estimation of the loss risk vis-à-vis the detection or non-detection of fraudulent claims. We also address auditing costs.

According to our results, the auditing of claims considerably reduces operational risk when there has been fraud. Fraudulent claims auditing reduces operational losses but the relative
gain depends on the proportion of fraud that is not detected. The results obtained justify the introduction of active fraud detection policies in insurance companies.

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