A correlation sensitivity analysis for non-life underwriting risk module SCR

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Abstract

This paper aims to analyze the impact of using different correlation assumptions between lines of business when estimating the *Solvency Capital Requirement* (SCR). A case study is presented. SCR is calculated according to the *Standard Model* approach. Alternatively we use an *Internal Model* based on a Monte Carlo simulation of the underwriting net result at a one-year horizon, where copulas model the dependence between lines of business. In order to address the impact of model assumptions on SCR we carry out a sensitivity analysis. We examine changes in the correlation matrix between lines of business and we also address the choice of copulas. Using aggregate historical data from the Spanish non-life insurance market between 2000 and 2009, we conclude that modifications of the correlation and dependence assumptions have a significant impact on SCR estimation.

Keywords: Solvency II, Solvency Capital Requirement, Standard Model, Internal Model, Monte Carlo simulation, Copulas

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1 Introduction

The European insurance regulator seeks to obtain a global vision of each insurance company operating in the EU. For that purpose, quantitative tools are used to estimate the economic value of the aggregate risk assumed by a company. The existence of several lines of business within a company is taken into account. Furthermore, lines of business are not necessarily assumed to be statistically independent. Hypothesis can be established on the association between the results of different lines of business within the same company. Significant correlations may arise due to both endogenous or exogenous causes. For instance, when a company has several lines of business that cover risks in a specific geographic region or operates in the same economic environment, some positive correlations between net underwriting results of lines of business at the end of the year are possible.

Our aim is to analyze the influence of the hypothesis made on the correlation matrix between lines of business, and more generally on the dependence structure, when calculating the capital requirement for solvency purposes.

The European Directive 2009/138/EC, generally known as *Solvency II*, provides a common legal frame for those companies based in any of the EU member states to operate in the insurance and reinsurance business. *Solvency II* establishes capital requirements to ensure stability against non expected adverse fluctuations, and so it guarantees policyholders' protection by means of two economic amounts called *Minimum Solvency Capital Requirement* (MSCR) and *Solvency Capital Requirement* (SCR). These economic capitals must be calculated by each company or insurance group according to the so called *Standard Model*, or under regulator's previous authorization, with an *Internal Model*.

Under *Solvency II*, SCR is estimated following a modular structure of those risks related to insurance activity such as underwriting risk, market risk, credit risk and operational risk. In this paper, we focus on non-life underwriting risk, where the directive imposes that capital requirement must consider at least a granularity level by lines of business. The SCR based on *Solvency II Standard Model* is mainly given by some parameters established by the Committee of European Insurance Supervisors. These parameters include premiums and reserves' standard deviations and a correlation matrix between lines of business.

Despite the novelty of this topic, there are some works related with SCR estimation. Pfeifer and Straussburger (2008) deal with the problem on the *Solvency II* SCR global aggregation formula for uncorrelated but dependent risks. They assume *value-at-risk* as a risk measure and also symmetric distributions, concluding that the global aggregation formula underestimates the real SCR under some dependence structures.

Sandström (2007) shows the effect of considering a skewness coefficient in the SCR estimation. Through some examples he shows differences in SCR estimations using calibrated and non-calibrated *Normal Power* distributions. Assuming *value-at-risk* and *tail value-at-risk* as a risk measures he finds that under the *Normal* distribution SRC is also underestimated.

Our contribution focuses on comparing the SCR results between the *Standard Model* and the *Internal Model* approaches assuming several dependence structures and then we also carry out a sensitivity analysis on the correlation matrix assumptions.

An *Internal Model* for solvency purposes may not necessarily be based on a modular structure, although we do keep modules in our model in order to estimate a comparable capital to the one that would be obtained for non-life underwriting risk module under the standard approach. Using linear regression techniques and copulas, and assuming a set of hypothesis on the correlations between lines of business we estimate economic capital requirements under a variety of scenarios. We conclude that the SCR based on *Solvency II Standard Model* overestimates the capital obtained from the *Internal Model* in almost all considered cases.

We believe that our work provides an interesting comparison between alternative modeling approaches, so it can be helpful when discussing the implications of using a *Standard* versus an *Internal Model*. Our sensitivity analysis about the impact of correlation and dependence hypothesis on SCR is illustrated with an empirical application, which confirms many expected results.

The rest of the paper is structured as follows. Section 2 describes the methodologies used under both the *Standard Model* and the *Internal Model* approach. In Section 3 we present the data and the results for our case study. Finally, a discussion and concluding remarks can be found in Section 4.

2 Methodology

We consider two approaches to estimating solvency capital.

First, we use the implementation suggested by the 5th *Quantitative Impact Study* (QIS-5) standard formula to obtain the premium and reserve risk submodule capital as part of the non-life underwriting risk module. The parameters imposed in the QIS-5 are taken as given. Those parameters include a given correlation matrix between lines of business and premium and reserves standard deviations. Data needed as input in the standard formula are estimates of premium and reserve volumes corresponding at the beginning of the current year. So, we obtain the one-year horizon SCR for the current year. Then we perform a sensitivity analysis of the premium and reserve SCR to changes in the correlation matrix between lines of business.

Second, in order to obtain a comparable capital to the one based in the standard formula, we follow an *Internal Model* approach using historical data. This model is based on the aggregation of the predicted net result by line of business. The predictions of the variables involved in the net result are estimated by a linear regression model approximation. Each line of business predicted result is aggregated to conform a net total. The SCR for the *Internal Model* is estimated as the difference between the estimated 99.5% *value-at-risk* and the estimated expected value after simulating predicted net results. Finally, alternative assumptions on the correlation matrix are used to obtain the SCR and the corresponding results are compared.

Below we present the *Standard Model* approach as it appears in the QIS-5 and our *Internal Model* approach, and we also introduce the notation.

2.1 Standard approach

The *Solvency Capital Requirement* (SCR) under the standard approach is calculated in several submodules. Here we concentrate on the premiums and reserve risk submodule. SCR is obtained by multiplying two terms, namely a volume measure, V, which we will define below and an approximation of the one-year horizon 99.5% *mean-value-at-risk* ($\rho(\sigma)$) of the standardized loss, using a lognormality assumption for the distribution of the underlying random variable. Therefore, the SCR under the standard approach is obtained as follows:

$$SCR = \rho(\sigma) \cdot V. \tag{1}$$

In order to obtain volume measure V, we need to introduce the notation of the required inputs. Let $P_{t,i,j}^{written}$ and $P_{t,i,j}^{earned}$ denote, respectively, the net income written premiums and the net earned premiums corresponding to the *i-th* line of business (LoB), $i = \{LoB_1, \cdots, LoB_n\}$, and the *j-th* geographical area, $j = \{1, \cdots, m\}$ at the beginning of year t. Let $P_{t,i,j}^{PV}$ denote the present value of future net premiums of existing contracts with more than one year maturity corresponding to the *i-th* line of business and *j-th* geographical area, at the beginning of year t. And finally, let $BE_{t,i,j}$ be the *best estimate* of the outstanding claims corresponding to the *i-th* line of business and the *j-th* geographical area, at the beginning of year t, calculated as indicated in the QIS-5. Then, the volume measure is defined as follows,

$$V = \sum_{i=LoB_1}^{LoB_n} V_i = \sum_{i=LoB_1}^{LoB_n} \left(max\{\sum_{j=1}^{m} P_{t,i,j}^{written}; \sum_{j=1}^{m} P_{t-1,i,j}^{written}; \sum_{j=1}^{m} P_{t,i,j}^{earned}\} + \sum_{j=1}^{m} BE_{t,i,j} \right) \left(\frac{3}{4} + \frac{1}{4} W_i \right), \tag{2}$$

where W_i is a geographical diversification coefficient given by

$$W_i = \frac{\sum_{j=1}^{m} \left(max\{\sum_{j=1}^{m} P_{t,i,j}^{written}; \sum_{j=1}^{m} P_{t-1,i,j}^{written}; \sum_{j=1}^{m} P_{t,i,j}^{earned}\} + \sum_{j=1}^{m} BE_{t,i,j} \right)^2}{\left(\sum_{j=1}^{m} max\{\sum_{j=1}^{m} P_{t,i,j}^{written}; \sum_{j=1}^{m} P_{t-1,i,j}^{written}; \sum_{j=1}^{m} P_{t,i,j}^{earned}\} + \sum_{j=1}^{m} BE_{t,i,j} \right)^2}. \tag{3}$$

In order to obtain *mean-value-at-risk* $\rho(\sigma)$, we first need to define the underlying parameter σ, which is called *combined* standard deviation. The term *combined* comes from the way σ is estimated. It is defined as a weighted mean of the specific standard deviations by line of business, where the weights are relative volume measures of each corresponding line of business. So, to obtain an estimate of σ we need first to estimate the standard deviations by line of business, which we call σ_i, $i = \{LoB_1, \cdots, LoB_n\}$.

The way σ_i is obtained is similar to the way we obtain σ. We weight premium (σ_{pr}^i) and reserve (σ_{res}^i) standard deviations by line of business, where weights are the relative premium and the reserve volume measures by line of business. We assume that both premium and reserve standard deviations by line of business are parameters to be estimated. So,

$$\sigma_i = \frac{\sqrt{\left(\sigma_{pr}^i V_{pr}^i\right)^2 + 2\alpha \sigma_{pr}^i \sigma_{res}^i V_{pr}^i V_{res}^i + \left(\sigma_{res}^i V_{res}^i\right)^2}}{V_{pr}^i + V_{res}^i} \tag{4}$$

and

$$\sigma = \frac{1}{V} \cdot \sqrt{\sum_{k,l} \rho_{kl} \cdot \sigma_k \cdot \sigma_l \cdot V_k \cdot V_l} \tag{5}$$

where $V_{pr}^i = max\{\sum_{j=1}^m P_{t,i,j}^{written}; \sum_{j=1}^m P_{t-1,i,j}^{written}; \sum_{j=1}^m P_{t,i,j}^{earned}\}$ is the volume measure for the premium of the *i-th* line of business and j represents the *j-th* geographical region, $j = \{1, \cdots, m\}$, σ_i indicates the standard deviation of the *i-th* line of business, $i = \{LoB_1, \cdots, LoB_n\}$, σ_{pr}^i is the premium standard deviation of the *i-th* line of business. Analogously, σ_{res}^i denotes the reserve standard deviation of the *i-th* line of business, $V_{res}^i = \sum_{j=1}^m BE_{t,i,j}$ is the reserve volume measure of the *i-th* line of business for all geographical regions, and α is the correlation coefficient between premiums and reserves. Then, σ is the *combined* standard deviation and ρ_{kl} is the correlation coefficient between the *k-th* and the *l-th* line of business.

Once σ is defined, QIS-5 suggests the analytic closed-form expression to approximate the 99.5% *mean-value-at-risk* of a lognormal distribution as follows:

$$\rho(\sigma) = \frac{exp\left(z_{0.995} \cdot \sqrt{log(\sigma^2 + 1)}\right)}{\sqrt{(\sigma^2 + 1)}} - 1, \tag{6}$$

where $z_{0.995}$ is the 99.5-*th* percentile of a standard normal distribution.

In practice and in this paper, premium and reserve standard deviations, parameter α and coefficients ρ_{kl} are taken as fixed and are given by QIS-5, so an insurer just needs to compute volume measures and the *combined* standard deviation to achieve the standard model SCR.

2.2 Internal Model approach

We propose an *Internal Model* based on the simulation of a multivariate random variable, where each marginal function represents the distribution of the random variable $\tilde{R}_{T+1}^i$, which is the net result of the *i-th* line of business, $i = \{LoB_1, \cdots, LoB_n\}$. To approximate the net result for the forthcoming period we use a simple linear regression model for the four components of net results that we consider here, namely, net premiums, net claims, net expenses and other expenses. We do not consider investment incomes neither investment expenses as we believe they are rather related to market risk than to underwriting risk. We assume the four components of the net result to be statistically independent. We then estimate the SCR for this internal model as the difference between 99.55% *value-at-risk* and the expected value of random variable $\tilde{R}_{T+1}^i$. As information is assumed to be available for periods up to T, SCR corresponds to the one-year horizon solvency capital. In order to clarify the proposed model, below we introduce the notation used for the internal approach.

Let $Y_t^{i,s}$ represent the set of historical data at time t, $t = \{0, 1, \cdots, T\}$, for the *i-th* line of business and the *s-th* component $s = \{\text{net premiums, net claims, net expenses, other expenses}\}$. The simple trend model for periods $[0;T]$ is given by

$$\tilde{Y}_t^{i,s} = \beta_0^{i,s} + \beta_1^{i,s} \cdot t + \epsilon_t^{i,s}, \tag{7}$$

where $\epsilon_t^{i,s}$ denotes a random perturbation. We assume that $E(\epsilon_t^{i,s}) = 0$ and $V(\epsilon_t^{i,s})$ is constant over time.

By extrapolating model (7) we can easily see that the expectation of random variable $\tilde{Y}_{T+1}^{i,s}$ can be predicted from model estimation. Ordinary least squares (OLS) can be used to obtain parameter estimates and so $\hat{Y}_{T+1}^{i,s} = \hat{\beta}_0^{i,s} + \hat{\beta}_1^{i,s} \cdot (T+1)$, where $\hat{\beta}_0^{i,s}$ and $\hat{\beta}_1^{i,s}$ correspond to OLS estimators. The expectation of $\tilde{Y}_{T+1}^{i,s}$ can be estimated by $\hat{Y}_{T+1}^{i,s}$ and its variance $Var[\tilde{Y}_{T+1}^{i,s}] = Var[\epsilon_t^{i,s}]$ can also be estimated using the OLS variance estimation of the error term in (7).

Then, we are able to define first and second moments of the random variable considered in the multivariate model $\tilde{R}_{T+1}^i$, provided that it is defined as the sum of four independent components. Then,

$$E[\tilde{R}_{T+1}^i] = \sum_{\forall s} E[\tilde{Y}_{T+1}^{i,s}] \tag{8}$$

and

$$Var[\tilde{R}_{T+1}^i] = \sum_{\forall s} Var[\tilde{Y}_{T+1}^{i,s}]. \tag{9}$$

The expectation and variance of $\tilde{R}_{T+1}^i$ can then be trivially estimated for each line of business, given the initial observed data and the estimation of model (7). More assumptions are needed in order to find the distribution of $\tilde{R}_{T+1}^i$.

The multivariate problem arises when aggregating the net result of several lines of business. For that purpose, we consider two families of multivariate distributions. We first introduce some basic ideas about the generalization of multivariate distributions and copulas. Then we briefly comment on some notes about the copulas we used in this study, namely, the *Gaussian copula* and the *t-Student copula*.

A copula is the distribution function of a random vector in $\mathbb{R}^d$ with uniformly distributed margins, or alternatively a copula is whatever function $C : [0;1]^d \to [0,1]$ which meets the following properties:

1. $C(x_1, \ldots, x_d)$, is increasing in each component x_i.

2. $C(1,\ldots,1,x_i,1,\ldots,1)=x_i; \forall i \in \{1,\ldots,d\}, x_i \in [0;1]$.

3. $\forall (a_1,\ldots,a_d)', (b_1,\ldots,b_d)' \in [0;1]^d$ with $a_i \le b_i$ we have:

$$\sum_{i_1=1}^{2} \cdots \sum_{i_d=1}^{2} (-1)^{i_1+\cdots+i_d} C(x_{1i_1},\ldots,x_{di_d}) \ge 0 \qquad (10)$$

where $x_{j1}=a_j$ y $x_{j2}=b_j$ for all $j \in \{1,\cdots,d\}$.

For our study we choose two families of copulas belonging to the so-called elliptical family, the *Gaussian copula* and the *t-Student copula*, and we set two families of margins, the *Gaussian* and the *t-Student* margins. So we examine four possibilities corresponding to a *Gaussian copula* with *Gaussian* margins, a *Gaussian copula* with *t-Student* margins, a *t-Student copula* with *Gaussian* margins and a *t-Student copula* with *t-Student* margins. The parameter set of both the *Gaussian copula* and the *t-Student copula* is the linear correlation matrix between random variables represented by the margins. In our case, margins correspond to random variable $\tilde{R}^i_{T+1}$, $i=\{LoB_1,\cdots,LoB_n\}$, the net estimated result of the *i-th* line of business, so the parameters of the copulas must be the linear correlation matrix between the net results of two lines of business.

Let $Z \in \mathbb{R}^n$ represents the n-dimensional random vector whose components correspond to the random variables $\tilde{R}^i_{T+1}$. We can fit *Gaussian* margins to each component of Z given $E[\tilde{R}^i_{T+1}]$ and $Var[\tilde{R}^i_{T+1}]$, such that its *Gaussian copula* shall be:

$$C_P^{Ga}(Z)=C\left(F^{\leftarrow}_{R^1_{T+1}}(u_1),\ldots,F^{\leftarrow}_{R^n_{T+1}}(u_n)\right), \qquad (11)$$

with a $n \times n$ correlation matrix, P, and *Gaussian* distribution functions, $F_{R^i_{T+1}}$, with mean $E[\tilde{R}^i_{T+1}]$ and variance $Var[\tilde{R}^i_{T+1}]$, and $F^{\leftarrow}_{R^i_{T+1}}$ is the generalized inverse function of $F_{R^i_{T+1}}$.

The *t-Student copula* has one more parameter to be considered, i.e. the degrees of freedom. Our aim is to set a joint distribution such that the behavior on tails shall be heavier than the multivariate *Gaussian* case, so we need to assume a *t-Student* distribution with a low number of degree of freedom. The higher the degree of freedom the closer the behavior of a multivariate *t-Student* distribution to a multivariate *Gaussian* distribution. Given that the considered random variable $\tilde{R}^i_{T+1}$ is not centered in zero, we encountered some computational difficulties in practice. We were not able to work directly with a *t-Student copula* such that its *t-Student* margins have expected value $E[\tilde{R}^i_{T+1}]$ and variance $Var[\tilde{R}^i_{T+1}]$ and a prefixed degree of freedom υ, so we had to base our model on a *t-Student copula* with *t-Student* margins with mean zero and variance given by $\frac{\upsilon}{\upsilon-2}$ and then we rescaled the random sample to get margins with the desired expectation and variance.

Let $Q \in \mathbb{R}^n$ represent n-dimensional random whose components are univariate *t-Student* distributed random variables with υ degree of freedom, mean zero and variance equal to $\frac{\upsilon}{\upsilon-2}$. Given the linear correlation matrix P, the *t-Student copula* is given by

$$C^t_{\upsilon,P}\left(Q\right) = t^n_\upsilon\left(t^{\leftarrow}_\upsilon(u_1), \ldots, t^{\leftarrow}_\upsilon(u_n)\right). \tag{12}$$

where $t_\upsilon(u)$ is the univariate zero-centered *t-Student* distribution with υ degrees of freedom and $t^{\leftarrow}_\upsilon(u)$ its generalized inverse function.

Once we have simulated a multivariate random sample from the *t-Student copula* as mentioned above, we rescaled the values to obtain the original marginal location and dispersion while preserving pair-wise correlations.

The two additional hypothesis considered in our analysis, i.e. a *Gaussian copula* with *t-Student* margins and a *t-Student copula* with *Gaussian* were simulated in a way that is similar to the procedure explained above.

3 A case study

Using historical aggregate data from the Spanish non-life underwriting market corresponding to the period 2000-2009 he have computed 2010-SCR for the whole Spanish non-life market under two approaches, the *Standard model* and the *Internal model* approach. Our aim is to compare both approaches and then to perform a sensitivity analysis of SCR to changes given alternative assumptions on the choice of dependence structures and alternative correlation assumptions between lines of business.

3.1 The Data

The data come from the files available on the *Dirección General de Seguros y Fondos de Pensiones* (DGSFP) website[1] (in section *Documentación Estadístico Contable*). We have used the aggregated information from the profit and losses accounts. Since this information it is available, according the Spanish legislation, in twenty-one insurance branches, we reclassified the available

[1] http://www.dgsfp.meh.es

information in the lines of business that are established for the QIS-5 purposes. As recommended, we have used the guidelines that UNESPA[2] were given to the QIS-5 Spanish insurance companies that participated in the study on how to reclassify insurance branches into lines of business. Finally, we considered the twelve lines of business specified in QIS-5. A complete and detailed description about lines of business is available in the QIS-5 technical specifications.

Table 1 shows the necessary inputs for applying the *Standard Model*. First, Table 1 shows the volume measures in thousand of million Euros. Lines of business I to IX volumes are net of reinsurance while lines of business X to XII are the accepted reinsurance volumes. We assume a non proportional reinsurance for them. Furthermore, we assume that the best estimates are calculated as required in QIS-5. Finally, we also assume that earned premium and written premium volumes are equal, all geographical diversification coefficients in all line of business are set to one and that all existing contracts are single-premium, so that the present value of net premiums of existing contracts are null. Second, Table 1 shows the values provided in QIS-5 for premium and reserves standard deviation by line of business and then σ_i is calculated according to (4) with $\alpha = 0.5$.

Table 1: Standard model inputs

	LoB	$P_i^{2009,written}$	$P_i^{2010,written}$	$BE_i^{2010}(*)$	(%) σ_{pr}^i	(%) σ_{res}^i	(%) σ_i
I	Motor vehicle liability	5.78	5.15	5.22	10	9.5	8.5
II	Other motor	4.81	4.54	1.00	7	10	6.8
III	Marine, Aviation, Transport	0.42	0.30	0.59	17	14	13.2
IV	Fire	6.87	5.86	2.65	10	11	9.1
V	3rd. party liability	1.21	1.05	4.33	15	11	10.6
VI	Credit, Suretyship	0.49	0.41	0.90	21.5	19	17.3
VII	Legal expenses	0.16	0.16	0.12	6.5	9	6.3
VIII	Assistance	0.67	0.61	0.06	5	11	5
IX	Miscellaneous	1.89	1.90	0.21	13	15	12.51
X	N.P. Property	1.85	0.41	0.00	17.5	20	16
XI	N.P. Casualty	0.07	0.03	0.00	17	20	15.9
XII	N.P. MAT	0.23	0.10	0.00	16	20	16.2

Source: DGSFP / (*)*Best Estimate*

For the *Internal Model* approach we took the 2000-2009 time series of profit and losses account from the Spanish non-life underwriting market. Data used in the model were deflated

[2] UNESPA, Unión Española de Entidades Aseguradoras y Reaseguradoras is an association representing more than 96% of the Spanish insurance market. http://www.unespa.es

in order to obtain 2009 constant currency unit values[3].

The net underwriting result by line of business is the result of considering net premiums, net claims, net expenses and other expenses. As already mentioned, we did not consider investment incomes neither investment expenses.

We must distinguish between the lines of business I to IX and the lines of business X to XII. While net premiums and net claims are direct insurance magnitudes in lines of business I to IX, lines of business X to XII include premiums coming from accepted reinsurance plus the variation of reserves for non earned premiums and current risks, and also claims coming from accepted reinsurance plus the variation of reserves for claims. As expenses refers to operating expenses in lines of business I to IX, those expenses are conformed by commissions coming from accepted reinsurance. Additionally, we consider other type of expenses coming from agreements between companies, assets depreciations and so on, while we do not consider this kind of expenses in lines of business X to XII.

Table 2: Internal model inputs

	LoB	$\hat{Y}^{i,pr}_{2010}$	$\hat{Y}^{i,cl}_{2010}$	$\hat{Y}^{i,exp}_{2010}$	$\hat{Y}^{i,o.exp}_{2010}$	(%) CV^i_{pr}(*)	(%) CV^i_{cl}	(%) CV^i_{exp}	(%) $CV^i_{o.exp}$
I	Motor vehicle liability	6.82	5.13	1.26	-0.01	10	6	9	53
II	Other motor	5.41	3.86	0.92	0.03	6	2	4	21
III	Marine, Aviation, Transport	0.48	0.32	0.12	0.008	6	11	8	18
IV	Fire	7.63	4.71	2.10	0.12	3	3	4	10
V	3rd. party liability	1.63	0.90	0.33	0.02	14	21	9	18
VI	Credit, Suretyship	0.55	1.03	0.37	0.02	3	71	39	55
VII	Legal expenses	0.19	0.09	0.04	0.0009	9	9	7	42
VIII	Assistance	0.74	0.53	0.13	0.01	6	8	6	8
IX	Miscellaneous	1.96	0.77	0.69	0.04	1	2	4	11
X	N.P. Property	1.84	0.69	0.34	-	5	41	32	-
XI	N.P. Casualty	0.07	0.03	0.02	-	8	12	6	-
XII	N.P. MAT	0.23	0.46	0.39	-	8	1.38	1.75	-

Source: DGSFP / (*)$CV^i_s = \left(\sqrt{\frac{\sum_{t=1}^{T}(Y_t^{i,s}-\hat{Y}_t^{i,s})^2}{T-1}}\right) / \left(\frac{\sum_{t=1}^{T}\hat{Y}_t^{i,s}}{T}\right)$ which estimated $\sqrt{Var(\epsilon_t^{i,s})}/E[\hat{Y}_t^{i,s}]$

In Table 2 we summarize the inputs that are required to apply the *Internal Model* approach. First, the predicted values in thousand of million Euros for 2010 and by line of business of all the components considered here for the net result. Second, although we use standard deviation of the prediction error in our internal approach, we display coefficients of variation by line of

[3] We used a serial time deflater available in INE, Instituto Nacional de Estadística. http://www.ine.es

business in order to make comparisons with standard approach more understandable. They are estimated as the squared root of the variance divided by the expectation, as usual.

With the information displayed on Table 2, we can obtain an estimate the expected value and the standard deviation of the component random variable considered, and then the predicted net result for 2010 by line of business $\tilde{R}^i_{2010}$ follows from the sum of expectations.

3.2 Results

Table 3 shows the different capital requirements obtained after computing the *Standard Model* and the *Internal Model* approaches. For the *Standard Model* approach we have used the inputs and the parameters presented in Table 1 and the QIS-5 line of business correlation matrix. In order to obtain all plausible values for the SCR, we have focused on the capital requirements calculated assuming independence and comonotonicity between lines of business. We do not consider negative correlations between lines of business since it is difficult to justify them.

Table 3: SCR. Standard Model *versus* Internal approach

Independence correlation matrix					
	Standard model	Gaussian magins		t-Student margins	
d.f.		Gaussian copula	t-Student copula	Gaussian copula	t-Student copula
4	4.15	4.15	4.62	4.72	5.28
10	-	-	4.39	4.31	4.56
35	-	-	4.22	4.17	4.25
QIS-5 correlation matrix					
	Standard model	Gaussian margins		t-Student margins	
d.f.		Gaussian copula	t-Student copula	Gaussian copula	t-Student copula
4	7.18	6.74	7.30	7.65	8.53
10	-	-	7.03	7.06	7.38
35	-	-	6.83	6.84	6.94
Comonotonicity correlation matrix					
	Standard model	Gaussian margins		t-Student margins	
d.f.		Gaussian copula	t-Student copula	Gaussian copula	t-Student copula
4	11.03	10.22	10.25	12.93	12.92
10	-	-	10.25	11.22	11.28
35	-	-	10.22	10.47	10.49

Source: Own source

As shown in Table 3, *Solvency Capital Requirement* obtained with the *Standard Model* approach underestimates almost those obtained with the proposed *Internal Model* approach except in the cases involving *t-Student* distributions margins with more than ten degrees of freedom in the QIS-5 and comonotonicity correlation matrix cases. When we assume independence between the lines of business, the standard approach provides an estimate of SCR of 4.15 thousand millions Euros, similar to the *Gaussian copula* with *Gaussian* margins case obtained with the *Internal Model* approach. The independence assumption, as expected, always provides the smallest value given one particular model, i.e. either the *Standard Model* or the *Internal Model* with a copula structure.

Correspondingly, the comonotonicity always leads to the highest SCR estimate in each particular model. In the *Standard Model* approach, the comonotonicity assumption almost triples the SCR with respect to the one obtained under the independence assumption between lines of business, while the QIS-5 correlation assumption is somehow in between the two.

For the *Internal Model* approach we have computed SCR under the four alternative dependence structures based on copulas. First, we present the results from the *Gaussian copula* with *Gaussian* margins and then with *t-Student* margins. Then, we show the results of the *t-Student copula* with *Gaussian* margins and with *t-Student* margins. All copulas including a *t-Student* distribution have been considered with 4, 10 and 35 degrees of freedom.

To establish comparisons within the *Internal Model* approach, we have also examined the independence case, the comonotonicity case and the one with the QIS-5 correlation matrix.

For all the cases considered here, cases where the SCR is calculated with copulas involving *t-Student* margins produce larger estimates than those obtained with copulas using *Gaussian* margins. In particular, the smallest SCR values are obtained when using a *Gaussian Copula* with *Gaussian* margins and then the value for the *t-Student copula* with *Gaussian* margins. Then, the *Gaussian Copula* with *t-Student* margins and the *t-Student copula* with *t-Student* margins produce increasingly higher results. We see the evidence that the selection of margins influences capital calculations and the effect of considering heavy-tailed marginal distributions becomes noticeable even in the case of the *Gaussian copula.*

Note also that when the number of degrees of freedom used in the *t-Student* margins increases, the SCR obtained for those copulas with *t-Student* margins decreases compared to margins with a smaller number of degrees of freedom and the results converges to the capital obtained with the *Gaussian copula* with *Gaussian* margins. This influence of the number of degrees of freedom was expected and it is observed in all correlation assumptions, namely, independence, QIS-5 correlation matrix and comonotonicity.

4 Discussion

Since the early discussions of the *Solvency II* project until its approval by the European Parliament in November 2009 a lot of debate has been taking place related to solvency requirements to be adopted by insurance companies in the EU. With the *Quantitative Impact Studies* the regulator has tested which would be the impact of implementing *Solvency II* principles. Since the early stages, participating insurance companies have also reported their total or partial internal model results in order to provide the regulators with enough information to improve solvency principles implementation when *Solvency II* shall be mandatory.

Our work contributes to the understanding of the methodology involved in solvency capital requirement calculation purposes. First, using data from the Spanish non-life underwriting market we have obtained the *Solvency Capital Requirement* under the *Standard Model* approach for the whole market. As Ferri *et al.* (2011) noted, this capital must be understood as if the whole market was operating as a single insurance company and it can only be used by market agents, the regulator and Spanish companies, as a market *benchmark* tool. Companies may compare their own capital estimation with the market position. Regulators may use the aggregated market calculation to allocate capital to companies and to compare the allocated capital with the capital they are effectively estimating individually. These authors noted that the *Standard Model* for non-life premium and reserve risk SCR submodule calculation is a too rigid system when using QIS-5 parameters depending on volume measures. They also criticized that this approach does not take into account the premium security margin but underwriting premiums, which could lead to wrong conclusions when comparing two companies with the same premium volumes and different premium security margin.

Second, we have built an *Internal Model* approach in order to calculate the solvency requirement through this approach and then compared it with the one obtained with the standard approach. A first difference between the internal and the standard approach is the use of the underlying random variable involved in our analysis. We do not consider a mixture of premium and reserves standard deviations by line of business but the predicted net result for the forthcoming period by line of business. We believe that the predicted net result reflects the meaning of premium and reserves risk. It indicates the insufficient resources to cover claims and expenses and so it translates into positive or negative results the profit and loss account. This fact justifies our choice. We used a *Gaussian copula* and *t-Student copula* with the same family marginal distributions because those random variables are defined in all real numbers. A second difference between the internal and the standard approach is the way our capital estimation is obtained. While the standard approach is based on past data, i.e. last written premium volume by line of business, the internal approach requires the prediction of the distribution of the net results. We believe that given the SCR *Solvency II* definition, the SCR should be based on the future evolution of the random variable rather than on past behavior. So we base our prediction on linear regression trend which provides information on the expected path of each random variables and its prediction error can be assessed.

Our last conclusion compares the results for both the *Standard Model* and the *Internal Model* approaches. We have observed that the SCR based on the standard approach underestimates almost all corresponding SCR obtained with our internal approach for QIS-5 and comonotonicity correlation hypothesis. In the case of comonotonicity between lines of business, except in those copulas involving *t-Student* distributions with four degrees of freedom, the standard approach produces SCR higher than the capital obtained with internal approaches. In the case of independence correlation matrix SCR obtained with *Gaussiam copula* with *Gaussian* margins equals that obtained with the standard approach. The rest of cases involving independence correlation case produces higher and convergent results to the *Gaussian copula* with *Gaussian* margins case. We believe that this may be a consequence of the influence of volume measures on the underlying random variable involved on the *Standard Model*, since the parameters fixed in QIS-5 lead to a potential overestimation of dispersion, compared to the *Internal Model* approach.

Comparing the results obtained from the four dependence and distributional assumptions that were used here, we have established a sort of lowest to highest capital result. We observe that, besides the copula selection, a key point is the hypothesis on margins. The lowest capital would be obtained with those copulas with *Gaussian* margins.

Solvency Capital Requirement estimation procedures need to be improved. From our point of view, more disaggregated data is needed and so more frequency and longer time series would increase the overall accuracy. When using copulas, one key point is margins selection and estimation in order to get a plausible fit, given the data. Furthermore, estimation of correlations is another key step on both internal and standard approaches, and the better the data base is the stronger the statistical properties of the correlation estimations. In our opinion a deeper dynamic analysis is needed in order to assess the *Standard Model*. We think that some comparisons and sensitivity analyses would be useful. For instance, addressing the impact of some other parameters such as dispersion and correlations, and using technical rather than underwriting premiums.

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