Outline

• Insurance, loss triangles & loss reserving
  – broad overview
• Solvency II - implications for loss reserving
• The actuarial spectrum:
  Left (math) wing ------------------------- Right (practice) wing
• Proposed “Solvency II compliant” solution/framework
  – (Statistical) model
  – Dependencies
  – Risk margins
  – Diversification benefits/attribution
• Conclusions, generalizations, qualifications, exhortations

Apologies:
  • Lots of ground is covered
  • Details can be different
## Loss triangles – background/problem

- Expected value of future payments?
- Timing of future payments?
- Risk margin?

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Forecasting method matters!

any variance based risk measure very misleading
Issues & remarks

• What model? (not formula/method)
  – model should justify method not vice versa
  – robust, complete solutions required

• What risk margin/measure? (does it matter – 0.05?)

• Will have many triangles – one corresponding to each line of business (lob, segment)

• Some will be “long” tailed, other “short”
  – timing of payments and timing of variability imp

• Are they related? How, why, consequences?
Solvency II compliant methodology

- sound with respect to selection, fit, and where appropriate, combination of statistical distributions
- produces probability distribution forecasts consistent with methods used to calculate technical provisions.
- sound methodology with respect to dependencies, correlations, aggregations and diversification effects
- admits discounting
- importance of linking reserving activities with the capital measurement function.
- Technical provisions will be estimated as a probability-weighted average of expected future cash flows, taking into account the time-value of money and including a risk margin.
- Need to discount these reserve estimates, requiring projected payment patterns,
- demonstrate an understanding of the uncertainty of those reserves.
Where am I coming from?

• Complete solutions
• Not just exact solutions to tiny parts of the problem
• Robust statistical/practical solutions
• Not going into statistical detail of every step (eg estimation, copulas, refinements, calculations, programs)
• Not going to be held up on small technical details (need to get to end)
The actuarial spectrum

left (mathematics) ------(?)------ right (practice)

- Risk measures
- Copulas
- Loss distributions
- Hierarchical modelling
- Credibility
- Extreme Value Theory
- Ruin Theory
- Estimation Theory

- Risk Margins
- Diversification Benefits
- Dependence/Correlation
- Time profile of payments & discounting
- Robustness
- Communication of results
- Integrate with other “risk management” activities

the middle ground is relatively sparse
we need more middle ground solutions
Implications for the reserving actuary?

• Increased focus on the overall distribution of loss reserves vs. range of reasonable estimates
• Increased need to understand the correlations between reserve segments
• Need to analyze the timing of reserve variability emergence
  – Ultimate variability vs. one-year time horizon
Key differences between Solvency II and current US solvency measures

- Solvency II focus is on capitalizing to a level such that the probability of insolvency over a one year time horizon is remote, 0.5%

- Solvency II is intended to be more aligned with the individual risk profile of a company
  - Formula vs. internal model vs. partial internal model
  - More recognition of risk diversification benefit
    - Line of business and risk module correlations
    - Zero correlation between life and nonlife entities

- Technical provisions for loss reserves are comprised of a discounted best estimate and an explicit risk margin
  - Question around nominal vs. discounted + risk margin
Comments

• Statistical problem with many angles
• Unusual data structure
• Many possible dependencies
• Tradeoff between sophistication and usefulness
• Can’t deal with everything.
Themes

- Dependence between lines of business (lob)
  - How to model?
  - Correlation
  - Copulas

- Implications of dependence for risk margins
  - Diversification benefit
  - Allocation of diversification benefits to lob’s
Philosophy / Apologies

• Cutting corners
  – I want to get to the end – ie a cogent solution
  – Refinements/modifications can come later
    • do they matter?
  – Sound overall framework first – refinements later
  – (“Shoot first – ask questions later”)

Limitations on measuring dependence

• Data limitations
  – sophisticated models cannot be well calibrated with limited data
  – more sophisticated methods need more, less variable, data
  – differences in results between methods tend to decrease with amount of data.

• Future dependence may be different from past
  – often need to inject subjective views/actuarial judgement

• How to measure/model?
  – Best way depends on context
  – correlation? how to apply? varies with where we are in tail?
  – copulas? too sophisticated relative to available data?
  – known sources of dependence? can sources be directly measured?

no single approach is best in all circumstances
Motivation for this project

• Understanding/modelling dependencies between different classes (lines) of business
• Develop an approach to help to quantify these dependencies
• Deploy dependencies in practical applications
  – risk margins analysis,

specialized approach to inter
loss triangle dependence
modelling
Loss triangles used to illustrate and test methods

- \( m = 8 \) lob’s – know little about background
  - catastrophes? inflation? trends, other things?
- Each loss triangles \( n = 10 \) years 1986-1995
- Loss triangles of substantially different “size”
- Loss triangles “related”
- Want to calculate, using observed dependence
  - Risk margins
  - Overall diversification benefit
  - Diversification benefit assigned to each lob.

these are obviously important in any given setting – however today we focus on technique and illustrate what the data is “saying”
US loss triangles used in calculations

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- First of eight triangles
- Cumulatives vs payments
- Exposure/volume indicator
Loss triangles – payments per unit exposure

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Just the first 3 triangles of 8 but you’ll get the idea

payments per unit volume

平均支付

maybe growth rates, different volume
Dependence (association, correlation)

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how are the payments in these different triangles related?
Why/how does dependence arise?

• Common known factors
  – legislation
  – risk characteristics
  – inflation
  – other causal factors (e.g., economic conditions)
• Common factors revealed by data
  – generally not directly measured (intelligence, trends)
• “Correlation” - statistical dependence

imperfect dependence $\Rightarrow$ diversification benefits
Simplest form of model

\[ p_{\ell ij} = \mu_{\ell j} + \sigma_{\ell j} \left( c_{\ell} \alpha_{\ell t} + s_{\ell} \epsilon_{\ell ij} \right) \]

- \( p_{\ell ij} \): payment
- \( \mu_{\ell j} \): mean
- \( \sigma_{\ell j} \): standard deviation
- \( c_{\ell} \): communality
- \( s_{\ell} \): specificity
- \( \alpha_{\ell t} \): calendar year effect (sd=1)
- \( \epsilon_{\ell ij} \): noise (sd=1)

\[ c_{\ell}^2 + s_{\ell}^2 = 1 \]

- \( \ell \): line of business (lob)
- \( i \): accident year
- \( j \): development year
- \( t = i + j \): calendar year

maybe in other directions
LOB dependence

\[
\begin{pmatrix}
\alpha_{1t} \\
\alpha_{2t} \\
\alpha_{3t}
\end{pmatrix}
\equiv \alpha_t \sim \mathcal{N}(0, R)
\]

\[
p_{\ell ij} = \mu_{\ell j} + \sigma_{\ell j} \left( c_{\ell} \alpha_{\ell t} + s_{\ell} \epsilon_{\ell ij} \right)
\]

\[
c_{\ell}^2 + s_{\ell}^2 = 1 \quad \Rightarrow \quad sd = 1
\]

correlation matrix \( R_{\ell k} \)

correlations moderated by
- communalities \( c_{\ell} \)
- specificities \( s_{\ell} \)

\[
\begin{array}{ccc}
0.135 & 0.177 & 0.135 \\
0.128 & 0.162 & \\
0.126 & \\
0.198 & 0.211 & 0.126 \\
0.124 & 0.294 & \\
0.142 & \\
0.132 & 0.166 & 0.124 \\
0.118 & 0.148 & \\
0.123 & \\
\end{array}
\]
LOB dependence

\[ p_{\ell ij} = \mu_{\ell j} + \sigma_{\ell j} \left( c_{\ell} \alpha_{\ell t} + s_{\ell} \epsilon_{\ell i j} \right) \]

\[ \begin{pmatrix} \alpha_{1t} \\ \alpha_{2t} \\ \alpha_{3t} \end{pmatrix} \equiv \alpha_t \sim N(0, R) \]

Parameters to be estimated
- means
- sd’s
- correlation matrix
- communalities
not too many parameters

Possible interpretations
- superimposed inflation
- other trends
- company policy

correlation matrix \( r_{\ell k} \)
correlations moderated by
- communalities \( c_{\ell} \)
- specificities \( s_{\ell} \)

\[ c_{\ell}^2 + s_{\ell}^2 = 1 \Rightarrow \text{sd} = 1 \]
## Correlation between lob’s

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These are correlations measured from triangles:

- **adjust using actuarial judgment** to reflect expected future scenarios
- **further analyse** to disentangle sources of dependence
- **interpret** to see if they align with known systematic features

**Correlation between lob’s**

\[ R = (r_{\ell k}) \]

**Correlations**

\[ C_{\ell} C_{k} R_{\ell k} \]

**Squared communality**

\[ C_{\ell}^{2} \]
Next steps

• Have measured “correlation” between triangles but what do we do with it?
  – analyse
    • what is the structure? – can we simplify?
    • does it align with anything we know about?
    • is the data consistent with our model?
  – time pattern of payments
  – effect on risk margins/diversification benefit?
    • what size of diversification benefit can we claim? (Depends on risk margin? VaR, CTE?)
    • time profile of payments?
    • attribution of diversification of payments?
(PC) Analysis of lob dependence

variability in lob $\ell$ payments

<table>
<thead>
<tr>
<th>Year</th>
<th>Lob 1</th>
<th>Lob 2</th>
<th>Lob 3</th>
<th>Lob 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>54</td>
<td>1</td>
<td>0</td>
<td>47</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0</td>
<td>47</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>50</td>
<td>1</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>24</td>
<td>11</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>16</td>
<td>37</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>36</td>
<td>6</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>40</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>26</td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

calendar year variability due to common factors

calendar year variability unique to lob $\ell$

<table>
<thead>
<tr>
<th>Year</th>
<th>Lob 1</th>
<th>Lob 2</th>
<th>Lob 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>24</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>9</td>
<td></td>
<td></td>
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<tr>
<td>6</td>
<td>6</td>
<td></td>
<td></td>
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<tr>
<td>7</td>
<td>11</td>
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<td></td>
</tr>
<tr>
<td>8</td>
<td>9</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

is this superimposed inflation?
correlation explained by 3 common factors
correlation not explained by 3 common factors

the rest - specificity $S_\ell^2$

100%
Future payments

- Simulate future payments from the model
  - payments for each cell under estimated correlation structure $p_{ij}$ (allow for volume)
  - total payments at each point of time for each loss triangle $i + j = t$
  - aggregate future payment $y_{\ell}$ for each triangle $\ell$
  - aggregate payments at each point of time aggregating across all triangles (time profile)
  - total payments aggregating across all future time periods and all triangles

$$\sum_{\ell=1}^{m} y_{\ell}$$

Get whole distribution for each of these
Diversification calculations

• If $m$ lob’s are “independent” then diversification benefit is proportional to

\[
1 - \frac{1}{\sqrt{m}}
\]

- $m =$ # lob
- equal sds
- sd based

• Copula ?
### Expected payments and risk margins

<table>
<thead>
<tr>
<th>line of business</th>
<th>expected</th>
<th>stand alone</th>
<th>diversified</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>372.41</td>
<td>21.40</td>
<td>17.03</td>
</tr>
<tr>
<td>2</td>
<td>96.49</td>
<td>8.97</td>
<td>0.46</td>
</tr>
<tr>
<td>3</td>
<td>116.05</td>
<td>15.74</td>
<td>13.05</td>
</tr>
<tr>
<td>4</td>
<td>254.41</td>
<td>15.98</td>
<td>10.97</td>
</tr>
<tr>
<td>5</td>
<td>289.75</td>
<td>15.30</td>
<td>10.45</td>
</tr>
<tr>
<td>6</td>
<td>6.54</td>
<td>0.96</td>
<td>0.48</td>
</tr>
<tr>
<td>7</td>
<td>5.06</td>
<td>1.49</td>
<td>0.13</td>
</tr>
<tr>
<td>8</td>
<td>2.23</td>
<td>0.46</td>
<td>0.01</td>
</tr>
<tr>
<td>total</td>
<td>1142.93</td>
<td>80.30</td>
<td>52.57</td>
</tr>
</tbody>
</table>

- no discounting
- risk measure?
- how attributed?
- ignore time profile
- how robust? (to model, risk measure, estimation)

\[
\begin{align*}
\mathbb{E}(y_\ell) & \quad \sigma_\ell \dot{M}_\ell(y_\ell) \quad \sigma_\ell \ddot{M}_\ell(y_\ell) \\
\sigma_\ell & = \text{Var}(y_\ell)
\end{align*}
\]
Risk margins
- based on usual risk measures:

\[ M_\ell = \sigma \phi \text{cor} \{ y_\ell, \phi(u_\ell) \} \]

<table>
<thead>
<tr>
<th></th>
<th>Value-at-Risk at 0 ≤ q ≤ 1</th>
<th>E(max) r ≥ 1 ind. copies</th>
<th>CTE (TVaR,..) at 0 ≤ q ≤ 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi(u) )</td>
<td>( (u = q) )</td>
<td>( ru^{r-1} )</td>
<td>( \frac{(u&gt;q)}{1-q} )</td>
</tr>
</tbody>
</table>

Choo, Weihao and De Jong, Piet, Loss Reserving Using Loss Aversion Functions. Available at SSRN
Diversified risk margins

\[ \tilde{M}_\ell = \sigma \phi \text{cor} \{ y_\ell, \phi(u_+) \} \]

% Saving = \[ 1 - \frac{\text{P} \left( \sum_\ell y_\ell \right)}{\tilde{M}_\ell} \]

= \[ 1 - \frac{\text{cor} \{ y_\ell, \phi(u_+) \}}{\text{cor} \{ y_\ell, \phi(u_\ell) \}} \]

You are now basing aversion on the extremeness of the total.
Total saving due to diversification

\[ \sum_{\ell} \sigma_{\ell}(\dot{M}_{\ell} - \ddot{M}_{\ell}) = \dot{T} \sum_{\ell} \frac{\sigma_{\ell} \dot{M}_{\ell}}{\dot{T}} \left(1 - \frac{\ddot{M}_{\ell}}{\dddot{M}_{\ell}}\right) \]

- Total stand alone margin $80.30
- % total stand alone margin due to lob
- % saving lob
- % share 35%
- \( \text{cor}\{y_{\ell}, \phi(u_+)\} \)
- \( \text{cor}\{y_{\ell}, \phi(u_{\ell})\} \)
Diversification benefits

- Risk measure: expected worst in 10 years
- Dependence: estimated from loss triangles
Remarks

• Outlined a (simple) model for dependence between loss triangles
  – basis for more complicated versions allowing for dependence based on accident or development years

• Model can be used to quantify/study dependence between triangles

• Model suitable for simulating future outcomes
  – suitable for discounting & distribution calculations
Not discussed today – see manuscript

- Actuarial judgement and it’s importance
- Value of sophisticated approach
  - Transformation $\phi$
  - Nongaussian copulas
- Estimation
- Residual analysis & its importance

De Jong, Piet, Modeling Dependence between Loss Triangles Using Copulas. Available at SSRN