Claims reserving & Solvency II

Piet de Jong
Actuarial Studies, Macquarie University
Sydney, Australia

Paper available at ssrn.com

Outline

- Insurance, loss triangles & loss reserving
  – broad overview
- Solvency II - implications for loss reserving
- The actuarial spectrum:

  Left (math) wing ------------------------ Right (practice) wing
- Proposed “Solvency II compliant” solution/framework
  – (Statistical) model
  – Dependencies
  – Risk margins
  – Diversification benefits/attribution
- Conclusions, generalizations, qualifications, exhortations

Apologies:

• Lots of ground is covered

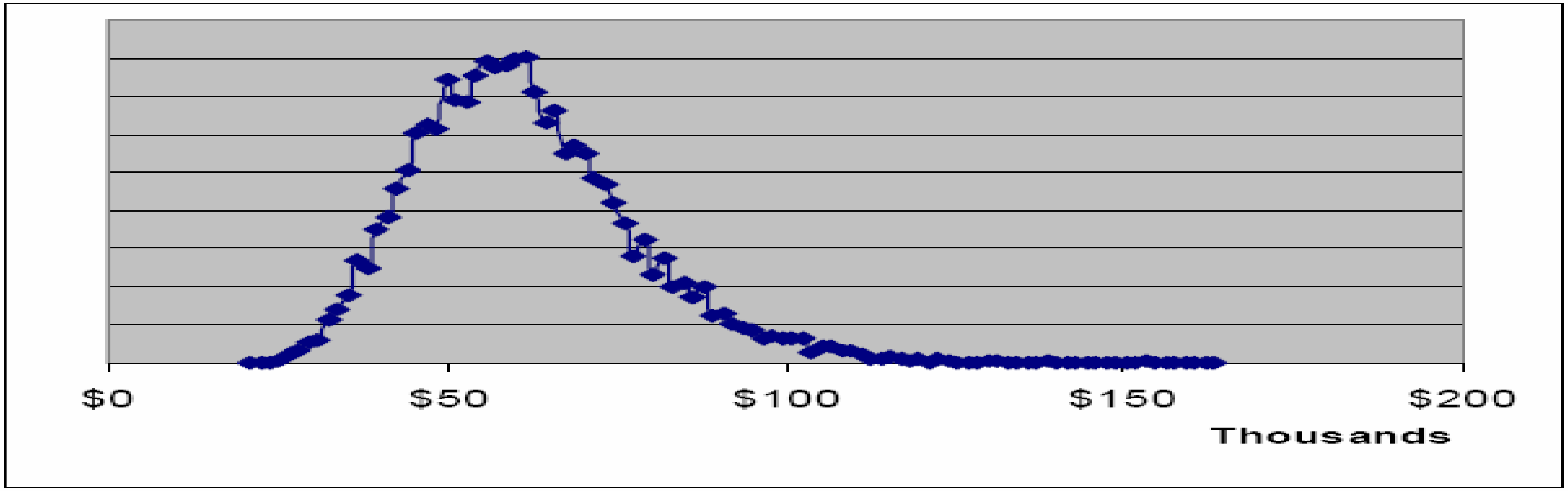
• Details can be different

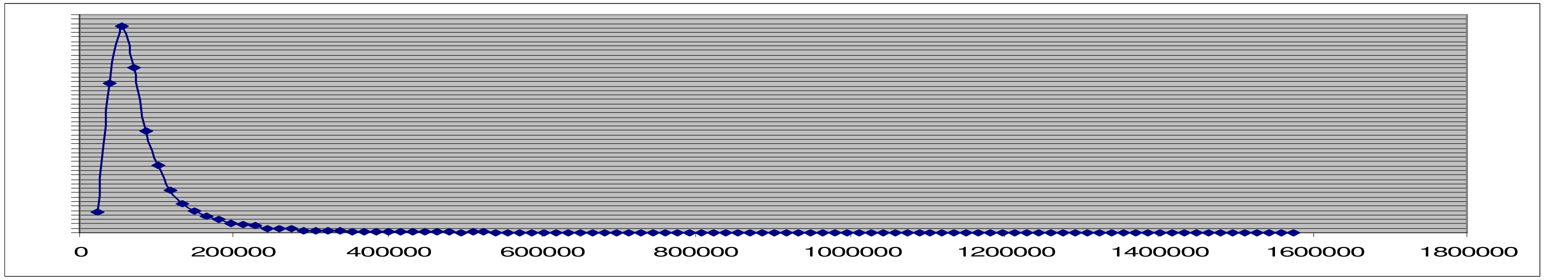
Loss triangles – background/problem

accident year i	development year j									
	0	1	2	3	4	5	6	7	8	9
1	5012	8269	10907	11805	13539	16181	18009	18608	18662	18834
2	106	4285	5396	10666	13782	15599	15496	16169	16704	
3	3410	8992	13873	16141	18735	22214	22863	23466		
4	5655	11555	15766	21266	23425	26083	27067			
5	1092	9565	15836	22169	25955	26180				
6	1513	6445	11702	12935	15852					
7	557	4020	10946	12314			?			
8	1351	6947	13112							
9	3133	5395								
10	2063									

- Expected value of future payments?
- Timing of future payments?
- Risk margin?

Forecasting method matters!





any variance based risk measure very misleading

Issues & remarks

- What model? (not formula/method)
  - model should justify method not vice versa
  - robust, complete solutions required
- What risk margin/measure? (does it matter – 0.05?)
- Will have many triangles – one corresponding to each line of business (lob, segment)
- Some will be "long" tailed, other "short"
  - timing of payments and timing of variability imp
- Are they related? How, why, consequences?

Solvency II compliant methodology

- sound with respect to selection, fit, and where appropriate, combination of statistical distributions
- produces probability distribution forecasts consistent with methods used to calculate technical provisions.
- sound methodology with respect to dependencies, correlations, aggregations and diversification effects
- admits discounting
- importance of linking reserving activities with the capital measurement function.
- Technical provisions will be estimated as a probability-weighted average of expected future cash flows, taking into account the time-value of money and including a risk margin.
- Need to discount these reserve estimates, requiring projected payment patterns,
- demonstrate an understanding of the uncertainty of those reserves.

Where am I coming from?

- Complete solutions
- Not just exact solutions to tiny parts of the problem
- Robust statistical/practical solutions
- Not going into statistical detail of every step (eg estimation, copulas, refinements, calculations, programs)
- Not going to be held up on small technical details (need to get to end)

The actuarial spectrum

left (mathematics) -------(?)------ right (practice)

- Risk measures
- Copulas
- Loss distributions
- Hierarchical modelling
- Credibility
- Extreme Value Theory
- Ruin Theory
- Estimation Theory

sophistication

- Risk Margins
- Diversification Benefits
- Dependence/Correlation
- Time profile of payments & discounting
- Robustness
- Communication of results
- Integrate with other "risk management" activities

useful

the middle ground is relatively sparse

we need more middle ground solutions

Implications for the reserving actuary?

- Increased focus on the overall distribution of loss reserves vs. range of reasonable estimates
- Increased need to understand the correlations between reserve segments
- Need to analyze the timing of reserve variability emergence
  - Ultimate variability vs. one-year time horizon

Key differences between Solvency II and current US solvency measures

- Solvency II focus is on capitalizing to a level such that the probability of insolvency over a one year time horizon is remote, 0.5%
- Solvency II is intended to be more aligned with the individual risk profile of a company
  - Formula vs. internal model vs. partial internal model
  - More recognition of risk diversification benefit
    - Line of business and risk module correlations
    - Zero correlation between life and nonlife entities
- Technical provisions for loss reserves are comprised of a discounted best estimate and an explicit risk margin
  - Question around nominal vs. discounted + risk margin

Comments

- Statistical problem with many angles
- Unusual data structure
- Many possible dependencies
- Tradeoff between sophistication and usefulness
- Can't deal with everything.

Themes

- Dependence between lines of business (lob)
  - How to model?
  - Correlation
  - Copulas
- Implications of dependence for risk margins
  - Diversification benefit
  - Allocation of diversification benefits to lob's

Philosophy / Apologies

- Cutting corners
  - I want to get to the end – ie a cogent solution
  - Refinements/modifications can come later
    - do they matter?
  - Sound overall framework first – refinements later
  - ("Shoot first – ask questions later")

Limitations on measuring dependence

- Data limitations
  - sophisticated models cannot be well calibrated with limited data
  - more sophisticated methods need more, less variable, data
  - differences in results between methods tend to decrease with amount of data.
- Future dependence may be different from past
  - often need to inject subjective views/actuarial judgement
- How to measure/model?
  - Best way depends on context
  - correlation? how to apply? varies with where we are in tail?
  - copulas? too sophisticated relative to available data?
  - known sources of dependence? can sources be directly measured?

no single approach is best
in all circumstances

Motivation for this project

- Understanding/modelling dependencies between different classes (lines) of business
- Develop an approach to help to quantify these dependencies
- Deploy dependencies in practical applications
  - risk margins analysis,

specialized approach to inter loss triangle dependence modelling

Loss triangles used to illustrate and test methods

- m=8 lob's – know little about background
  - catastrophes? inflation? trends, other things?
- Each loss triangles n=10 years 1986-1995
- Loss triangles of substantially different "size"
- Loss triangles "related"
- Want to calculate, using observed dependence
  - Risk margins
  - Overall diversification benefit
  - Diversification benefit assigned to each lob.

these are obviously important in any given setting – however today we focus on technique and illustrate what the data is "saying"

US loss triangles used in calculations

premium	cumulative payments									
208286	28068	65019	93069	112844	123973	135828	139484	141276	141898	142091
255616	32769	74153	105704	131525	152588	169692	177151	179679	179935	
267666	33810	79128	125677	160883	184243	196745	203347	206720		
274526	37663	89434	130432	159928	172597	183801	189586			
268162	40630	96948	153130	185603	201431	209840				
276822	40476	90172	129485	153529	166685					
270215	37128	88110	122264	147719						
280568	41125	94427	134716							
344914	57515	125396								
371139	61553									

?

y_ℓ?

- First of eight triangles
- Cumulatives vs payments
- Exposure/volume indicator

Loss triangles – payments per unit exposure

$i\backslash j$	0	1	2	3	4	5	6	7	8	9
1	13.5	17.7	13.5	9.5	5.3	5.7	1.8	0.9	0.3	0.1
	19.8	21.1	12.6	9.4	5.9	3.3	2.3	-0.2	0.1	0.4
	13.2	16.6	12.4	11.6	7.3	4.9	0.8	0.9	0.4	1.1
2	12.8	16.2	12.3	10.1	8.2	6.7	2.9	1.0	0.1	
	12.4	29.4	10.1	10.6	3.8	1.8	1.6	0.7	0.3	
	11.8	14.8	14.6	10.9	6.2	-10.8	0.3	0.2	-0.2	
3	12.6	16.9	17.4	13.2	8.7	4.7	2.5	1.3		
	14.2	23.7	14.5	6.9	4.4	0.5	1.0	0.8		
	12.3	13.1	14.0	11.1	3.7	4.4	1.6	0.8		
4	13.7	18.9	14.9	10.7	4.6	4.1	2.1			
	21.6	17.3	15.9	7.5	2.8	1.8	1.9			
	11.7	19.6	14.6	6.5	5.0	3.1	0.7			
5	15.2	21.0	21.0	12.1	5.9	3.1				
	13.3	28.8	11.6	5.1	3.2	1.5				
	14.2	19.3	12.7	7.6	5.7	2.6				
6	14.6	18.0	14.2	8.7	4.8					
	10.6	31.8	9.3	3.3	3.9					
	18.2	20.4	16.4	7.8	4.6					
7	13.7	18.9	12.6	9.4						
	7.8	37.1	12.2	6.8						
	13.3	19.7	12.4	13.9						
8	14.7	19.0	14.4							
	14.9	16.0	9.5							
	13.7	17.8	16.3							
9	16.7	19.7								
	14.1	15.5								
	15.5	21.5								
10	16.6									
	16.0									
	16.0									
$\hat{\mu}_{\ell j}$	14.4	18.5	15.0	10.5	6.3	4.9	2.3	1.0	0.2	0.1
	14.5	24.5	12.0	7.1	4.0	1.8	1.7	0.4	0.2	0.4
	14.0	18.1	14.2	9.9	5.4	0.8	0.9	0.6	0.1	1.1
$\hat{\sigma}_{\ell j}$	1.3	1.4	2.7	1.5	1.6	1.2	0.4	0.2	0.1	
	3.8	7.2	2.2	2.3	1.0	0.9	0.5	0.5	0.1	
	1.9	2.6	1.5	2.5	1.1	5.9	0.5	0.3	0.3	

Just the first 3 triangles of 8 but you'll get the idea

payments per unit volume

average payment

maybe growth rates, different volume

Dependence (association, correlation)

0.135	0.177	0.135
0.128	0.162	
0.126		

0.198	0.211	0.126
0.124	0.294	
0.142		

0.132	0.166	0.124
0.118	0.148	
0.123		

how are the payments in these different triangles related?

Why/how does dependence arise?

- Common known factors
  - legislation
  - risk characteristics
  - inflation
  - other causal factors (eg econ conditions)
- Common factors revealed by data
  - generally not directly measured (intelligence, trends)
- “Correlation” - statistical dependence

imperfect dependence $\Rightarrow$ diversification benefits

Simplest form of model

payment

mean

sd

communality

specificity

$$p_{\ell i j} = \mu_{\ell j} + \sigma_{\ell j}(c_\ell \alpha_{\ell t} + s_\ell \epsilon_{\ell i j})$$

calendar year effect
sd=1

noise
sd=1

maybe in other directions

$$c_\ell^2 + s_\ell^2 = 1$$

- ℓ line of business (lob)
- i accident year
- j development year
- $t = i + j$ calendar year

LOB dependence

$$c_\ell^2 + s_\ell^2 = 1 \quad \Rightarrow \quad \text{sd} = 1$$

$$p_{\ell ij} = \mu_{\ell j} + \sigma_{\ell j}\left(c_\ell \alpha_{\ell t} + s_\ell \epsilon_{\ell ij}\right)$$

0.135	0.177	0.135
0.128	0.162	
0.126		

$$\begin{pmatrix} \alpha_{1t} \\ \alpha_{2t} \\ \alpha_{3t} \end{pmatrix} \equiv \alpha_t \sim N(0, R)$$

0.198	0.211	0.126
0.124	0.294	
0.142		

correlation matrix $r_{\ell k}$

correlations moderated by

- communalities c_ℓ
- specificities s_ℓ

0.132	0.166	0.124
0.118	0.148	
0.123		

LOB dependence

$c_\ell^2 + s_\ell^2 = 1 \quad \Rightarrow \quad \text{sd} = 1$

$$p_{\ell ij} = \mu_{\ell j} + \sigma_{\ell j}\left(c_\ell \alpha_{\ell t} + s_\ell \epsilon_{\ell ij}\right)$$

$$\begin{pmatrix} \alpha_{1t} \\ \alpha_{2t} \\ \alpha_{3t} \end{pmatrix} \equiv \alpha_t \sim N(0, R)$$

Parameters to be estimated

- means
- sd's
- correlation matrix
- communalities

not too many parameters

Possible interpretations

- superimposed inflation
- other trends
- company policy

correlation matrix $r_{\ell k}$

correlations moderated by

- communalities c_ℓ
- specificities s_ℓ

Correlation between lob's

correlations $R = (r_{\ell k})$

	1	2	3	4	5	6	7	8
1	0.66	-0.09	0.74	0.58	0.27	0.74	-0.26	0.27
2	-0.06	0.54	-0.02	-0.08	0.02	-0.24	0.02	0.23
3	0.47	-0.01	0.63	0.42	0.69	0.66	-0.11	0.13
4	0.36	-0.05	0.26	0.59	0.43	0.40	0.17	-0.25
5	0.18	0.01	0.45	0.27	0.67	0.16	0.55	-0.34
6	0.42	-0.12	0.37	0.21	0.09	0.49	-0.47	0.44
7	-0.16	0.01	-0.07	0.10	0.33	-0.25	0.55	-0.52
8	0.14	0.11	0.07	-0.12	-0.18	0.20	-0.25	0.41

correlations $c_\ell c_k r_{\ell k}$

squared communality c_ℓ^2

These are correlations measured from triangles:

- **adjust using actuarial judgment** to reflect expected future scenarios
- **further analyse** to disentangle sources of dependence
- **interpret** to see if they allign with known systematic features

Next steps

- Have measured “correlation” between triangles but what do we do with it?
  - analyse
    - what is the structure? – can we simplify?
    - does it align with anything we know about?
    - is the data consistent with our model?
  - time pattern of payments
  - effect on risk margins/diversification benefit?
    - what size of diversification benefit can we claim? (Depends on risk margin? VaR, CTE?)
    - time profile of payments?
    - attribution of diversification of payments?

(PC) Analysis of lob dependence

variability in lob ℓ payments

calendar year variability due to common factors

1	54	1	0
2	1	0	47
3	50	1	2
4	24	11	1
5	16	37	4
6	36	6	1
7	3	40	1
8	2	26	4
	39	28	14

calendar year variability unique to lob ℓ

1	11
2	6
3	9
4	24
5	9
6	6
7	11
8	9
	19

1	34
2	46
3	37
4	41
5	33
6	51
7	45
8	59

is this superimposed inflation?

correlation explained by 3 common factors

correlation not explained by 3 common factors

100%

the rest - specificity s_ℓ^2

Future payments

- Simulate future payments from the model
  - payments for each cell under estimated correlation structure $p_{\ell ij}$ (allow for volume)
  - total payments at each point of time for each loss triangle $i + j = t$
  - aggregate future payment y_ℓ for each triangle ℓ
  - aggregate payments at each point of time aggregating across all triangles (time profile)
  - total payments aggregating across all future time periods and all triangles

$$\sum_{\ell=1}^{m} y_\ell$$

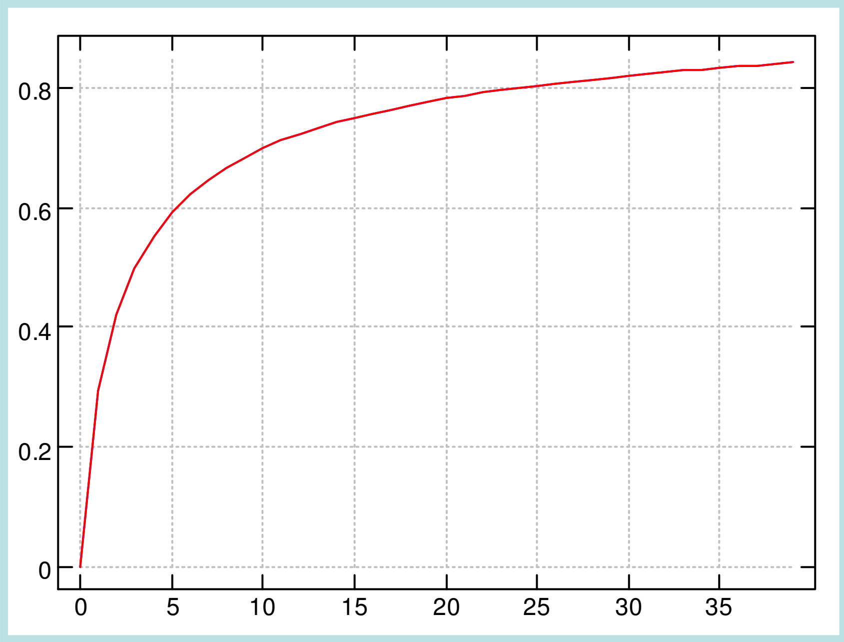
Get whole distribution for for each of these

Diversification calculations

- If m lob's are "independent" then diversification benefit is proportional to

$$1 - \frac{1}{\sqrt{m}}$$

- m = # lob
- equal sds
- sd based



- Copula ?

Expected payments and risk margins

line of business	expected	stand alone	diversified
1	372.41	21.40	17.03
2	96.49	8.97	0.46
3	116.05	15.74	13.05
4	254.41	15.98	10.97
5	289.75	15.30	10.45
6	6.54	0.96	0.48
7	5.06	1.49	0.13
8	2.23	0.46	0.01
total	1142.93	80.30	52.57

- no discounting
- risk measure?
- how attributed?
- ignore time profile
- how robust? (to model, risk measure, estimation)

$\mathrm{E}(y_\ell)$

$\sigma_\ell \dot{\mathrm{M}}_\ell(y_\ell)$

$\sigma_\ell \ddot{\mathrm{M}}_\ell(y_\ell)$

$\sigma_\ell = \mathrm{Var}(y_\ell)$

Risk margins

- based on usual risk measures:

$$\dot{\mathrm{M}}_\ell = \sigma_\phi \mathrm{cor}\left\{y_\ell, \phi(u_\ell)\right\}$$

stand alone

risk margin in # of sd's

defined by risk measure

$\mathrm{P}(y_\ell)$

"aversion" function

sd & cor not so bad after all

	Value–at–Risk at $0 \le q \le 1$	E(max) $r \ge 1$ ind. copies	CTE (TVaR,..) at $0 \le q \le 1$
$\phi(u)$	$(u = q)$	ru^{r-1}	$\frac{(u>q)}{1-q}$

Choo, Weihao and De Jong, Piet, Loss Reserving Using Loss Aversion Functions. Available at SSRN

Diversified risk margins

stand together

$$\ddot{\mathrm{M}}_\ell = \sigma_\phi \mathrm{cor}\left\{y_\ell, \phi(u_+)\right\}$$

risk margin in # of sd's

defined by risk measure

$\mathrm{P}\left(\sum_\ell y_\ell\right)$

You are now basing aversion on the extremeness of the total

% Saving = $1 - \ddot{\mathrm{M}}_\ell / \dot{\mathrm{M}}_\ell$

$$= 1 - \frac{\mathrm{cor}\{y_\ell, \phi(u_+)\}}{\mathrm{cor}\{y_\ell, \phi(u_\ell)\}}$$

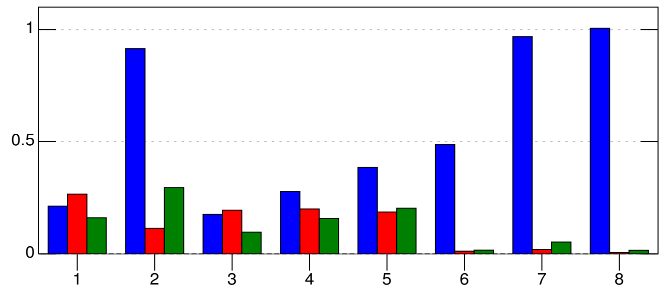
Total saving due to diversification

$$\sum_{\ell} \sigma_\ell(\dot{\mathrm{M}}_\ell - \ddot{\mathrm{M}}_\ell) = \dot{\mathrm{T}} \sum_{\ell} \frac{\sigma_\ell \dot{\mathrm{M}}_\ell}{\dot{\mathrm{T}}} \left(1 - \frac{\ddot{\mathrm{M}}_\ell}{\dot{\mathrm{M}}_\ell} \right)$$

total stand alone margin $80.30

% total stand alone margin due to lob

% saving lob



% share

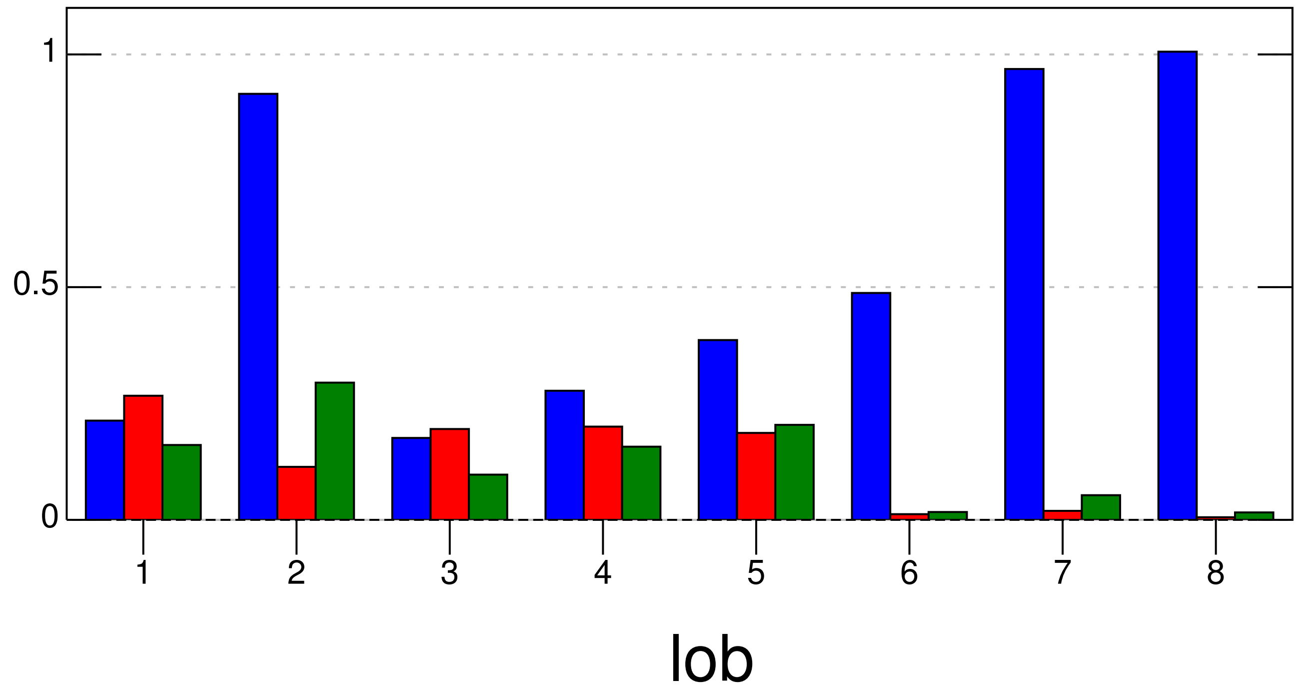
35%

$$\frac{\mathrm{cor}\{\mathrm{y}_\ell, \phi(\mathrm{u}_+)\}}{\mathrm{cor}\{\mathrm{y}_\ell, \phi(\mathrm{u}_\ell)\}}$$

Diversification benefits

- Risk measure: expected worst in 10 years
- Dependence: estimated from loss triangles

- % Div saving
- % Standalone
- % Div share

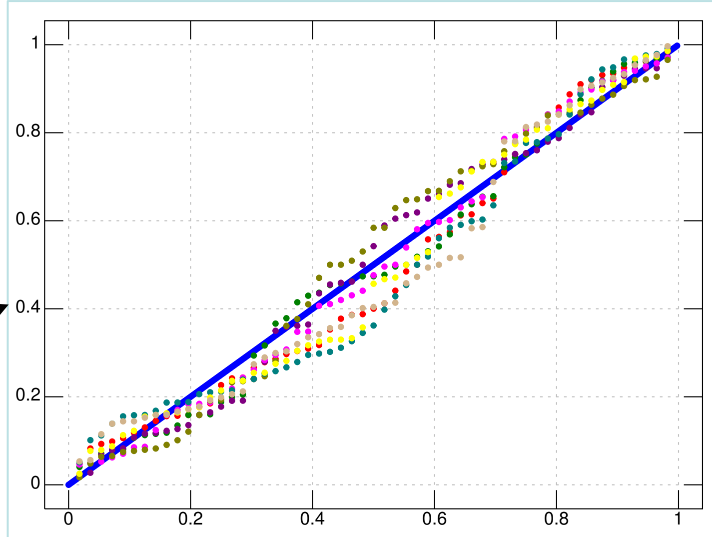


Remarks

- Outlined a (simple) model for dependence between loss triangles
  - basis for more complicated versions allowing for dependence based on accident or development years
- Model can be used to quantify/study dependence between triangles
- Model suitable for simulating future outcomes
  - suitable for discounting & distribution calculations

Not discussed today – see manuscript

- Actuarial judgement and it's importance
- Value of sophisticated approach
  - Transformation φ
  - Nongaussian copulas
- Estimation
- Residual analysis & its importance



De Jong, Piet, Modeling Dependence between Loss Triangles Using Copulas. Available at SSRN