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A dynamic analysis of the underwriting cycle in non-life insurance

Agenda

- Introduction
- The Underwriting Cycle: different approaches
- Piecewise Linear Dynamic Systems (PLDS)
- Risk Theory and PLDS: our approach
- Examples
- Final remarks
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Introduction

- Technical specifications in Quantitative Impact Studies 4 (QIS4 – see CEIOPS, 2007), define for Non-Life Underwriting Risk, under one-year time horizon, a capital requirement based on three risks: Pricing, Reserving and Catastrophic Losses, but there is not a specific capital requirement for Underwriting Cycle.
- Meyers [2007]: *The Underwriting Cycle contributes an artificial volatility to underwriting results that lies outside the statistical realm of insurance risk.* For Internal Model development under Solvency II, Underwriting Cycle must be analyzed, because the additional volatility could produce a higher capital requirement.
- Tipically in hard market period, insurance companies tend to increase premium rates, while in soft market premium rates decrease.
- We will present an actuarial model based on the control of safety loading, also considering applications for a long-time horizon.
- It is important to remember the influence of Underwriting Cycle on *Reserving Cycle*. In this paper Reserving Cycle will not be considered.

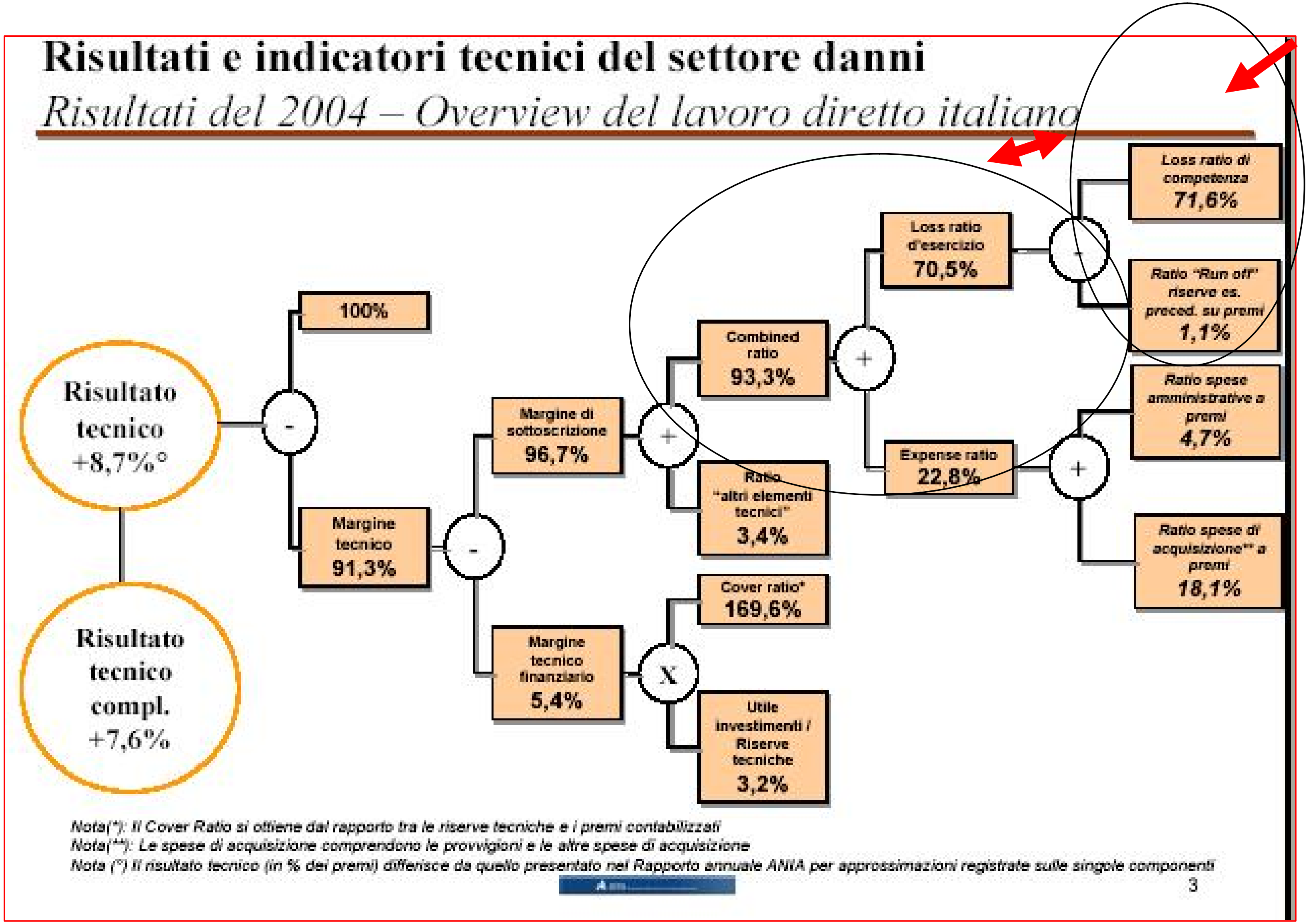
Introduction

- Feldblum [2001] discusses the causes of the underwriting cycle, taking into account insurance industry aspects (product differentiation, cost structures, barriers to entry, etc.). The paper underlines the importance of understand the relationship between competition and profits, which permits to the underwriting cycle to influence insurer solvency.

- The presence and length of cycles could depend on:
  - Position and competitiveness of leader companies in relation to the market
  - Firms' tendency to increase its own market share
  - Internal and External inflation of claim costs
  - Change in premium rates
  - Loyalty changes
  - … .

- The incapacity of obtaining profits at the end of a cycle could produce:
  - Reduction of market share and loss of business
  - Decreasing in the solvency ratio

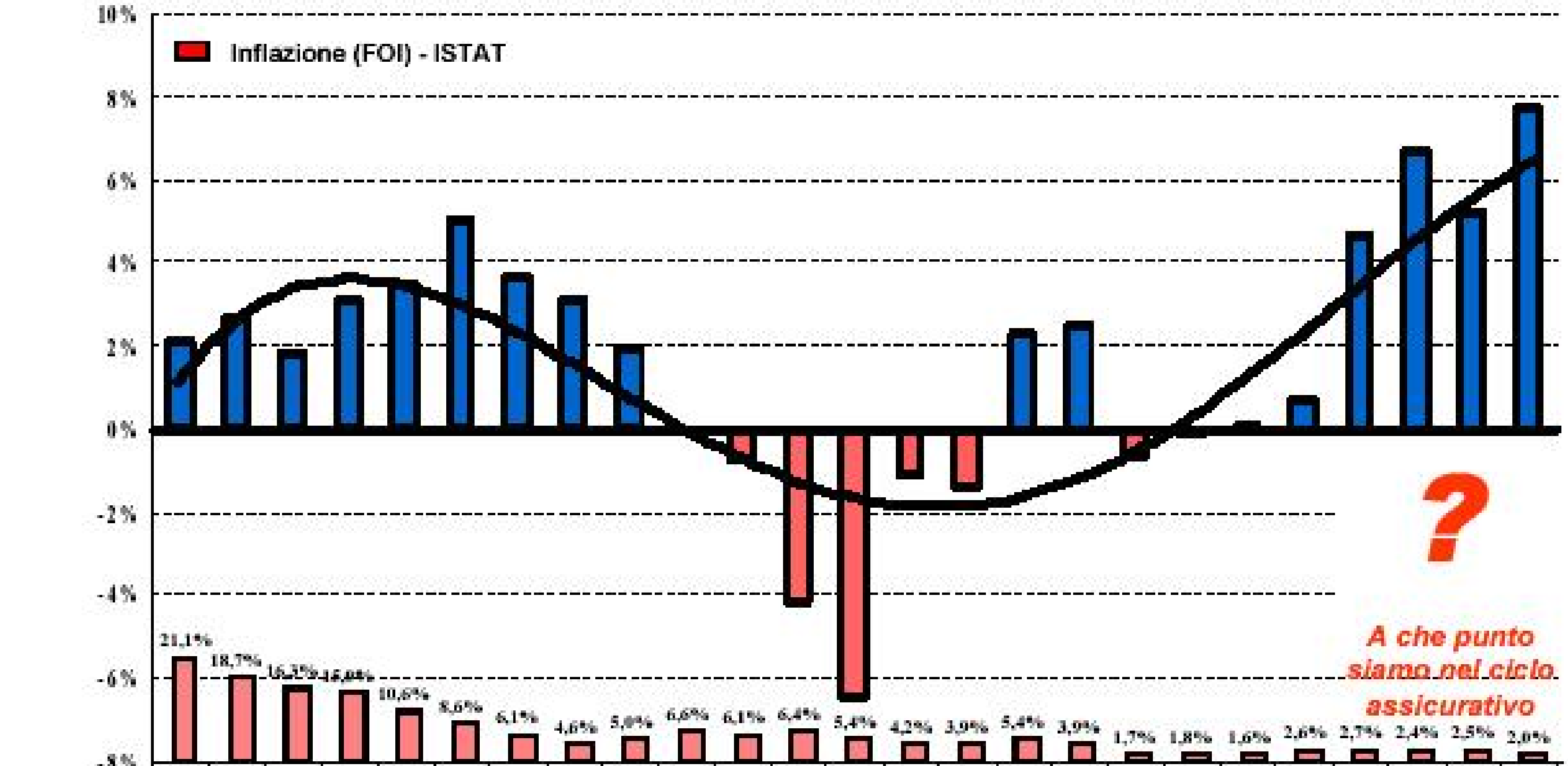
Italian Market Data
(source ANIA)



Cycles in Italian Non-Life Insurance Market (source ANIA)

Risultati e indicatori tecnici del settore danni

Utile su premi contabilizzati*



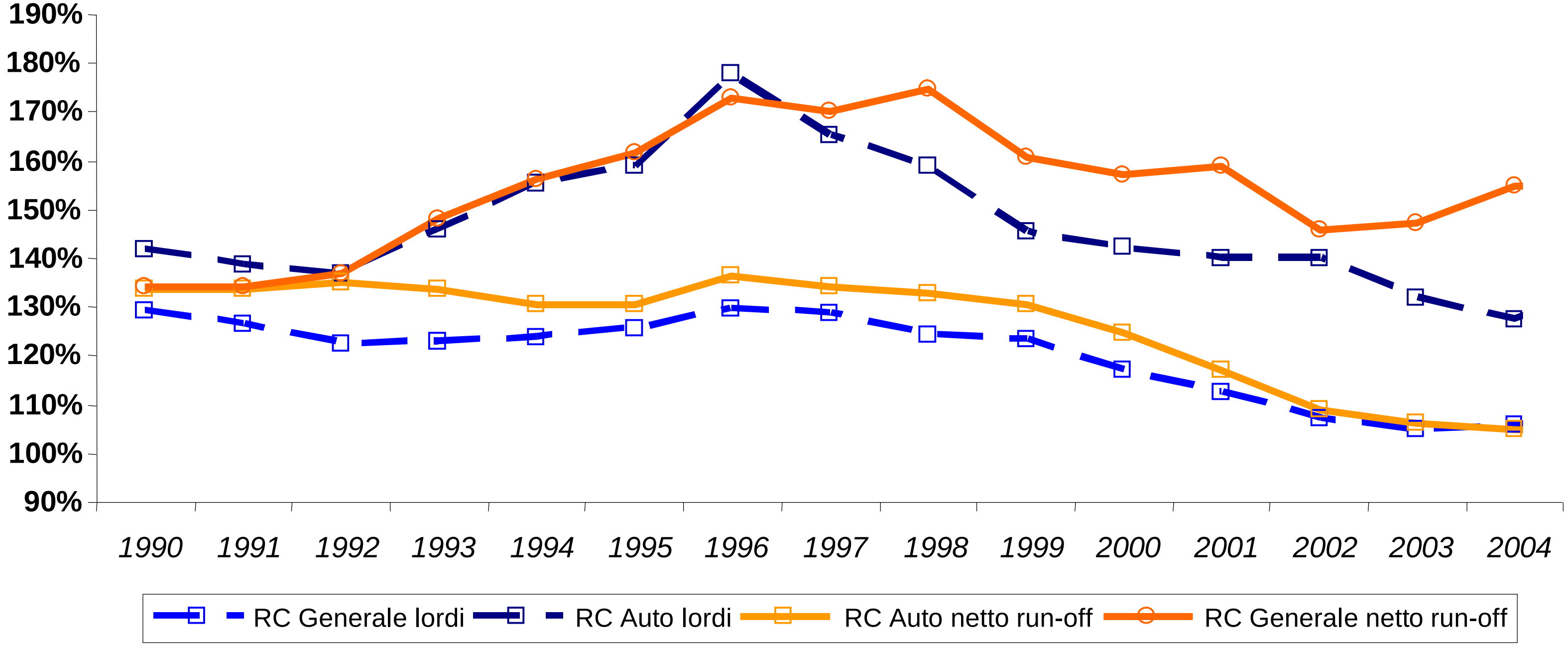
	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004
Utile su premi	2,1%	2,6%	1,8%	3,1%	3,5%	5,0%	3,6%	3,1%	1,9%	0,0%	-0,7%	-4,1%	-6,4%	-1,2%	-1,4%	2,3%	2,4%	-0,7%	-0,1%	0,1%	0,7%	4,7%	6,7%	5,2%	7,7%

5

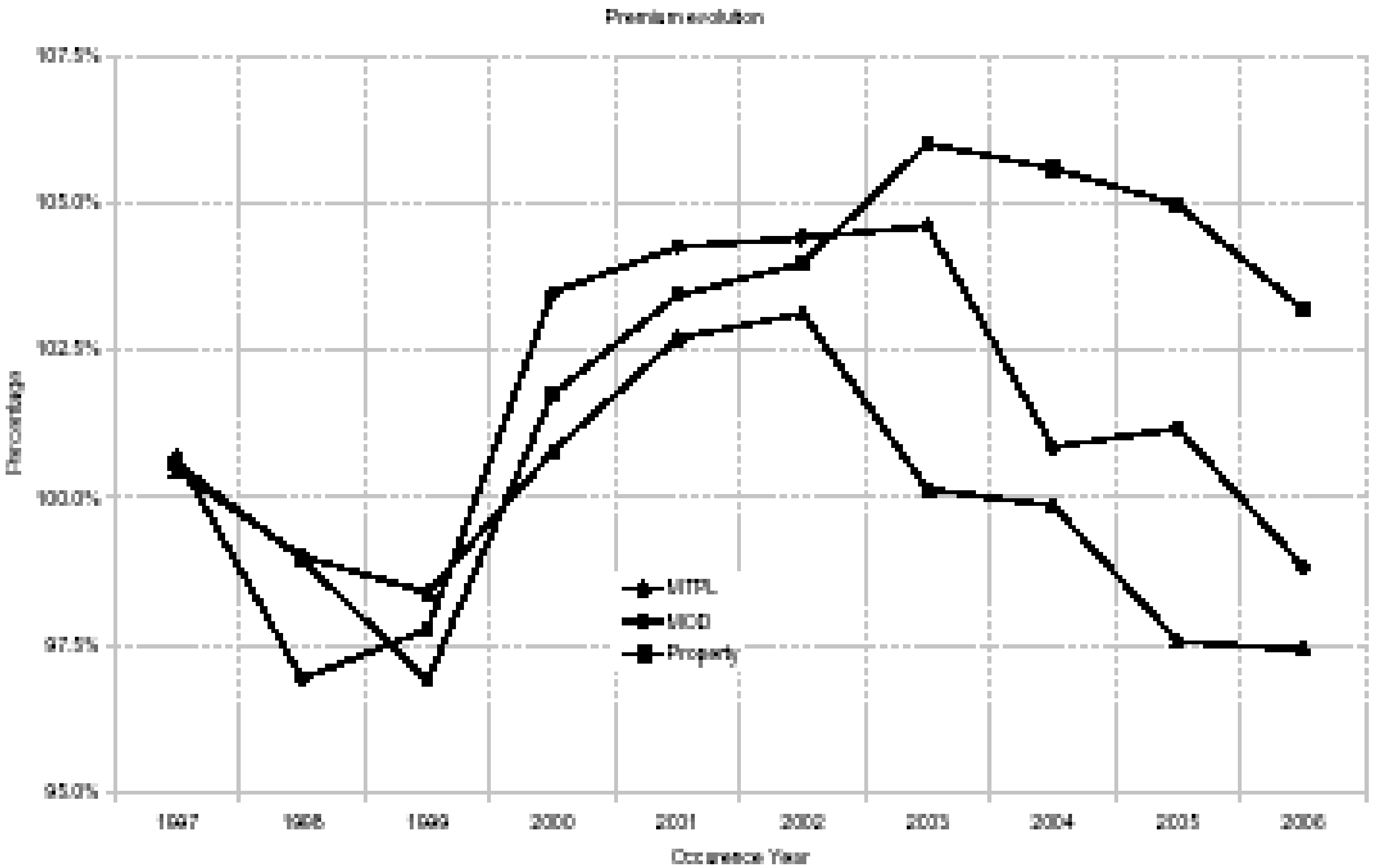
Nota(*): l'utile si riferisce al lavoro complessivo ed esclude le riassicuratrici professionali

Cycles in Italian Non-Life Insurance Market (source ANIA)

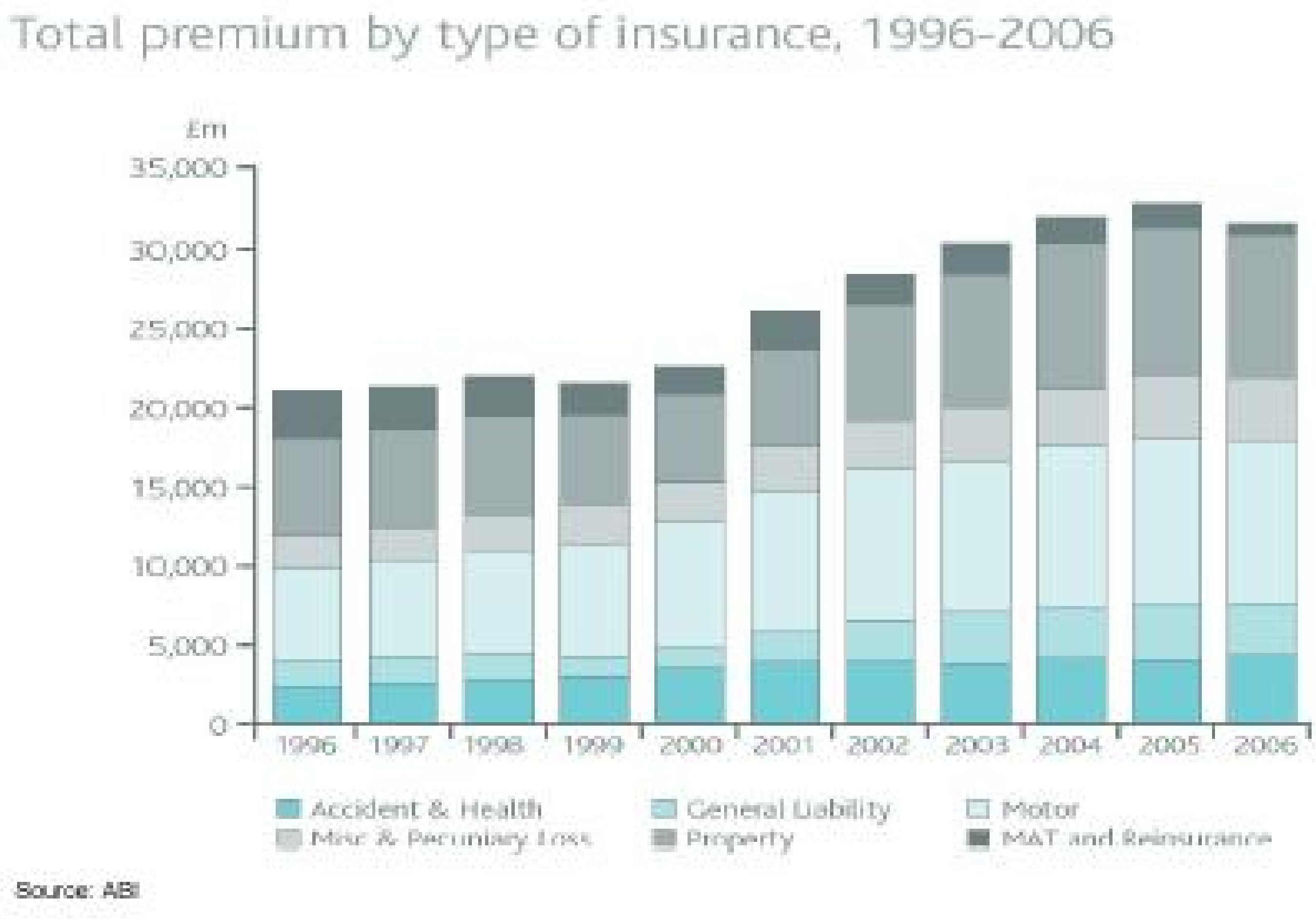
- Combined Ratios (CR) and ratio *Run Off of previous years on Earned Premiums,* for italian LoB (1990-2004).
- This graph shows CR behavior for two of the most important LoB in the Italian Market (MTPL and TPL):
  - CR <u>gross</u> of run-off and gross of reinsurance
  - CR <u>net</u> of run-off and gross of reinsurance



Cycles in French Non-Life Insurance Market (source Derien, 2008)



Cycles in UK Non-Life Insurance Market (source: Deloitte 2008)



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The Underwriting Cycle: different approaches

- There are several papers on this field. Some studies have been principally based on actuarial models:
  - Deterministic models (trigonometrich functions), as shown in Daykin et al. [1994];
  - Time Series analysis (see Daykin et al. [1994], Cummins and Outreville [1987]);
  - Exogenous impacts: combined use of the previous ones also incorporating external factors (for instance national economy trends or exposition) and simulation models, as shown in Pentilkainen et al. [1989] and Daykin et al. [1994].

- It is worth mentioning also the so-called financial pricing models (based on discounted cash flows). As shown in Derien [2008] :
  - Insurer is risk neutral and has a rational expectation in relation to claims.
  - No cycle should appear with absence of financial market imperfections.
  - Premiums could be correlated negatively with interest rates.
  - Cummins and Outreville [1987], Haley [1995], Leng and Meier [2006] have used the approach based on time series analysis to confirm these assumptions.

The Underwriting Cycle: different approaches

- Another possible approach is the capacity constraint hypothesis, based on the assumption that insurer has always a sufficient capital to meet its liabilities. See Choi and Hardigree [2002], Gron [1994], Winter [1994], Higgins and Thistle [2000] and Derien [2008]. In particular the two last paper considered "regime switching"techniques to allow the diversification of parameters estimate in every phase of the cycle.

- Cummins and Danzon [1997] have extended the previous models, including the assumption that insureds are available to pay a higher premium if insurer has a lower default probability than the other insurance companies.

An example of the mixed approach (deterministic and historical series) for the Underwriting Cycle

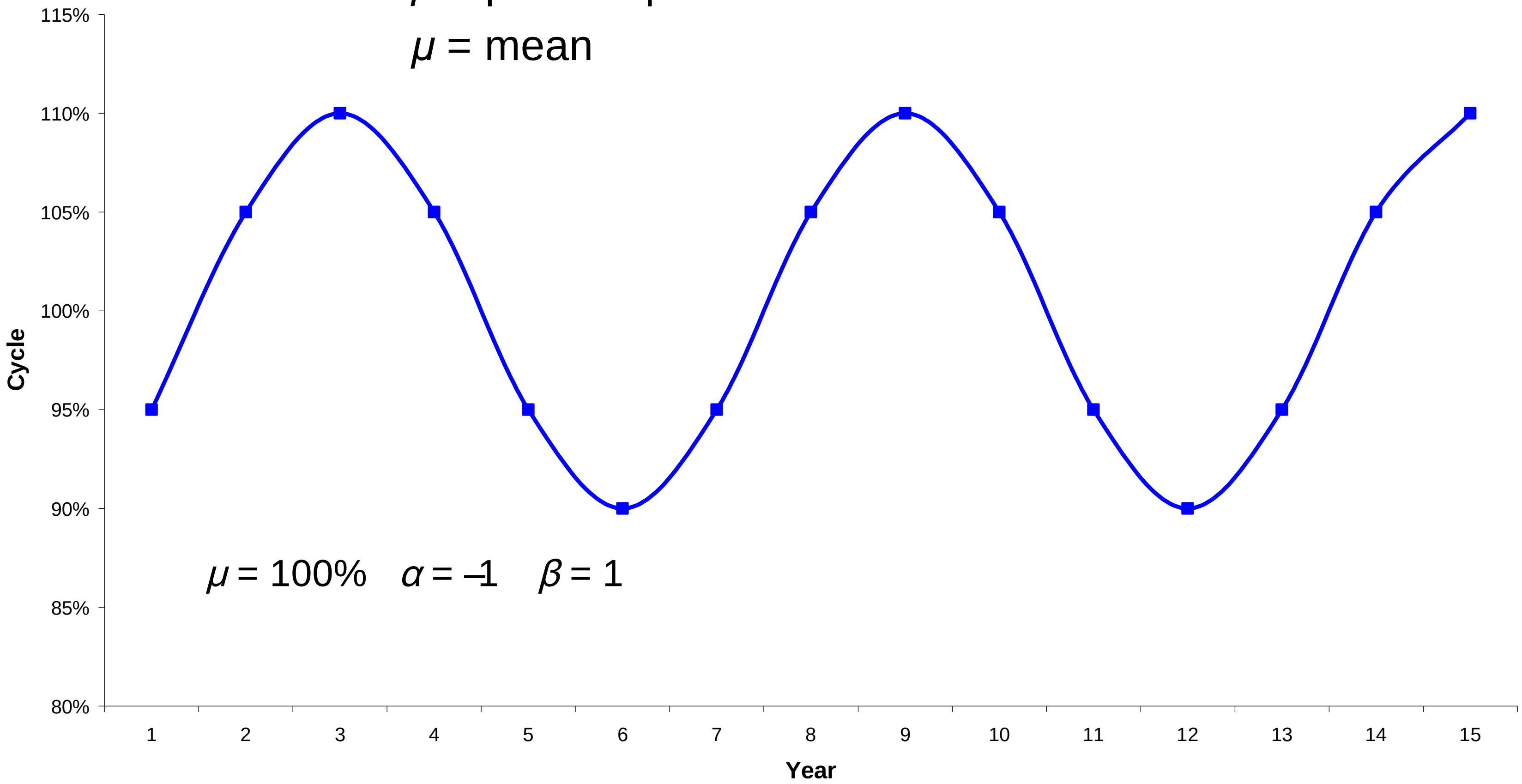
- AR(2):

$$Cy_t = Cy_{t-1} + \alpha\,(Cy_{t-1} - \mu) + \beta\,(Cy_{t-1} - Cy_{t-2})$$

where α = mean reversion (negative)

β = positive parameter

μ = mean



Actuarial Models

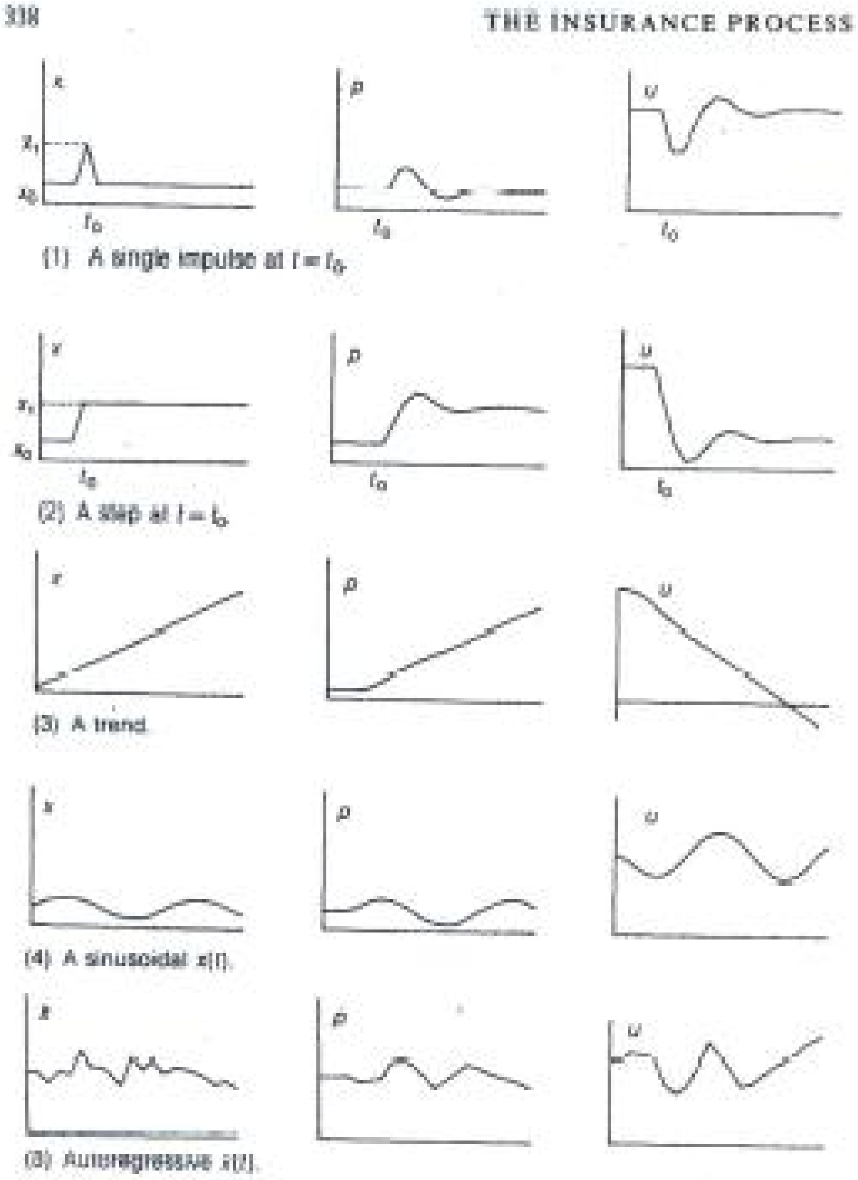


Figure 12.3.1 *System responses for alternative impulses.*

See Daykin et al. [1994]

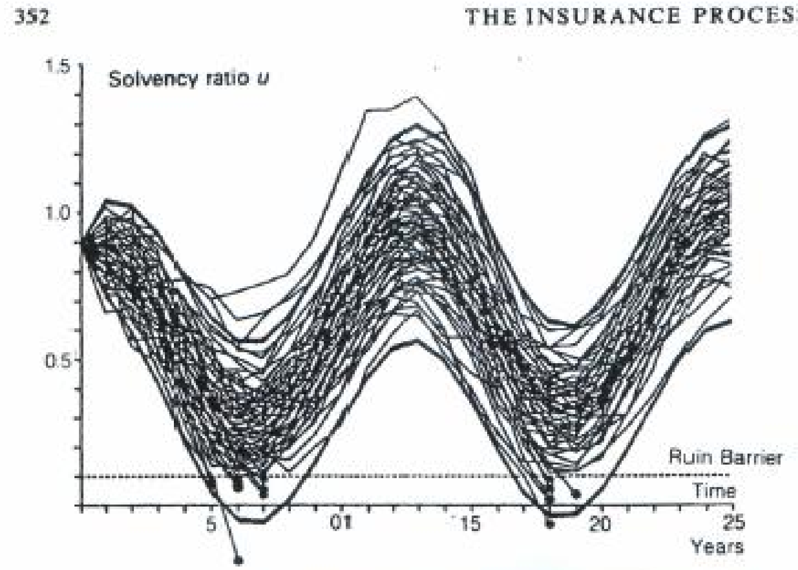


Figure 12.4.3 *The same as in Figure 12.4.2 but with a cycle.*

- Safety loading represents a "buffer" to allow an acceptable ruin probability.
- This model has characterized from a relation:
  - Direct with the variance of claims costs
  - Inverse with the capital.

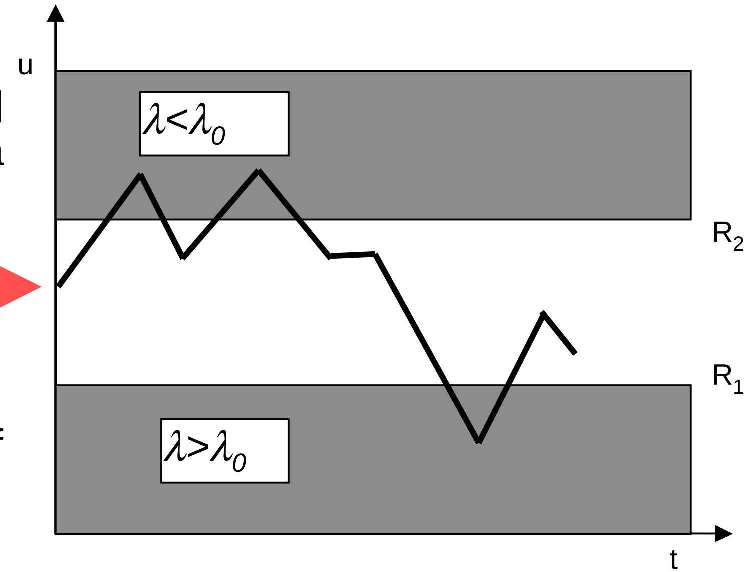
Underwriting Cycle and Actuarial Models: Dynamic Control on Safety Loading

- One of the goals of Risk Theory is the definition of dynamic control and decisional rules for solvency margin calculations, investment strategy and other key issues of Risk Management (or recently ERM).

- For solvency analysis, it is important to understand how to maintain the solvency ratio *u(t)* (Risk reserve on Premiums) within a given range.

- A simple dynamic control rule of premium can be achieved by incorporating the safety loading $\lambda(t)$, which gives a better understanding of the decisions to make:

$$\lambda(t+1) = \lambda_0 + c_1 \max[0, R_1 - u(t)] - c_2 \max[0, u(t) - R_2]$$



Source Daykin et al. [1994]

- Hence the loading will be increased if the solvency ratio u(t) is under a certain level R1, whereas it will be reduced if u(t) goes over the threshold R2.

- Obviously with this control, range of variation of u(t) will be constrained. This rule could be adopted for dividends, expenses, etc.

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Piecewise Linear Dynamic Systems

- Piecewise Linear Dynamic Systems (linear map in intervals)
- Bifurcation is a change in the dynamic behaviour
- In our model, we detect the so called "Fold" Bifurcations in points where the map is non-differentiable: "Border Collision" Bifurcations, BCB (see Di Bernardo et al. [2008])
- It is worth mentioning that an equilibrium (or steady state or fixed point or attractor) u* of a unidimensional map is a solution of the algebraic equation:

$$u^* = f(u^*)$$

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The actuarial model using the Piecewise Linear Dynamic Systems on a long time horizon

$$u(t+1) = r\,u(t) + [1+\lambda(t+1)]p(t+1) - x(t+1)$$

$$r = \frac{1+j}{(1+g)(1+i)}$$

- j = interest rate
- g = portfolio growth rate
- i = inflation rate

- Solvency Ratio = Risk Reserve on Risk Premium
- Aggregate loss distribution doesn't change along the time (*p(t+1) = 1* assumption varying of t)
- Considering now the model of Daykin et al. [1994]:

$$u(t+1) = r\,u(t) + \{1 + \lambda_0 + c_1 \max[0, R_1 - u(t)] - c_2 \max[0, u(t) - R_2]\} - x(t+1)$$

- The last equation become a *piecewise linear* unidimensional map in the state variable *u(t).* It is firstly possible to develop analitically a deterministic analysis of solvency ratio, by letting *x(t+1)* = *x,* so that the aggregate loss can be regarded as a parameter.
- In particular we dedect the so called *Border-collision bifurcations* (*Fold type - see Di Bernardo et al., 2008*), when, by varying a model parameter, the crossing of the *u(t+1)* map trajectory pass into regions where the map definition changes.
- So a Fold Border-collision bifurcation occurs as a fixed point collides with a border where the definition of the map changes and the map is not differentiable there.

Deterministic dinamyc of solvency ratio and equilibrium analysis

$$u(t+1) = f(u(t)) = r\,u(t) + 1 + \lambda_0 - x + \begin{cases} c_1\left(R_1\text{-}u(t)\right) & \text{if } u(t) < R_1 \\ 0 & \text{if } R_1 \le u(t) \le R_2 \\ c_2\left(R_2 - u(t)\right) & \text{if } u(t) > R_2 \end{cases}$$

- We can observe two points where map definition change (R_1 and R_2).
- The number of equlibria and their dynamic properties depend on the given parameters configuration.
- **A)** If c1=c2 = 0 (no control enforced), the dynamical system reduces to a linear map, whose the unique equilibrium is globally asimptotically stable for *r < 1* and unstable otherwise:

$$u_M{}^* = \frac{1+\lambda_0 - x}{1-r}$$

- **B)** If *r > max[c1,c2]* (also for *r>1*), u(t+1) is strictly increasing. In general, if we consider c1,c2 Є (0,1], the map is piecewise linear (in different intervals). So the number of different equilibria and their dynamic property depend on the parameter configuration and are obtained according to the branch of the map that intersects the identity map. In this model we have exactly three equlibria (see the proof in our paper):

$$u_L{}^* = \frac{1+\lambda_0 + c_1 R_1 - x}{1-r+c_1} \qquad u_M{}^* = \frac{1+\lambda_0 - x}{1-r} \qquad u_H{}^* = \frac{1+\lambda_0 + c_2 R_2 - x}{1-r+c_2}$$

- In next slide we developed different considerations and derived some control rules depending on values of parameter *r.*

Deterministic dinamyc of solvency ratio and equilibrium analysis

- *Proposition*. If ***0 < r < 1***, then the dynamic system has a unique equilibrium point u*, that results asymptotically and globally stable (as shown also in Daykin et al. [1994]).

- Introducing dynamic control rules in some cases it is possible to develop a long time analysis of solvency ratio also in presence of *r > 1*, when the process doesn't diverge (Daykin et al. [1994]).

- In particular, the previous three equilibria exist (simultaneously and distinct) if ***r > 1*** and moreover if:

$$R_2 > \frac{1+\lambda_0 - x}{1-r} > R_1 > -\frac{1+\lambda_0 - x}{c_1}$$

$$r - c_2 < 1$$

- If ***r = 1*** the analysis could be developed in a similar way.

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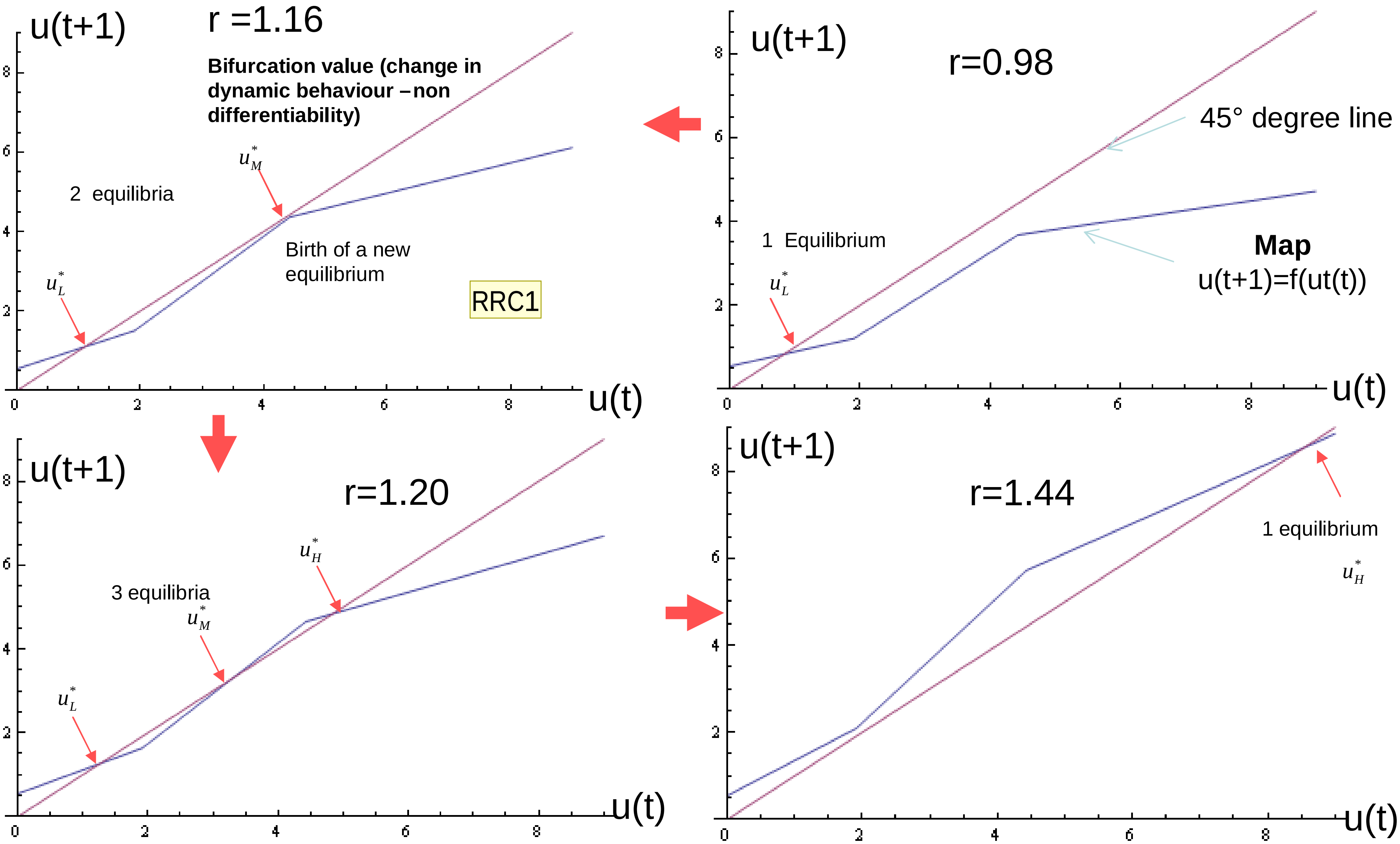


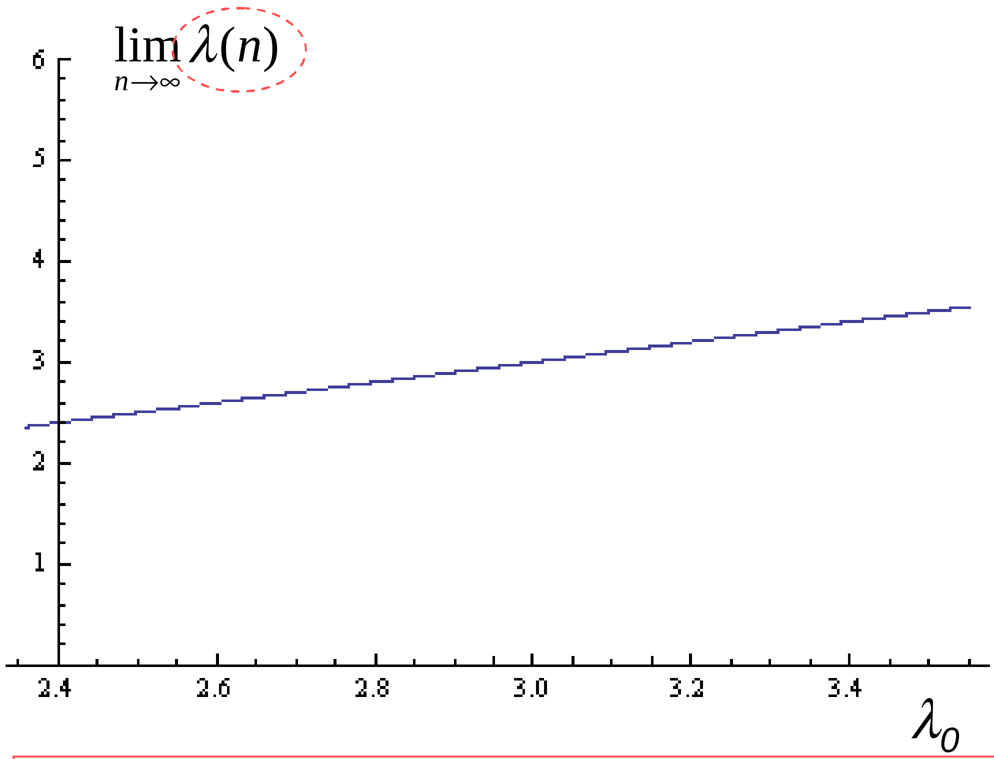
- Our assumption: c1=0.632; c2=0.754; R1=1.91; R2=4.43; x=4.5; λ_0**=2.79**
- Double ‘border collision bifurcation’ obtained by increasing *r*: from one attractor with ‘low’ u to one attractor with a ‘high’ u with a phase characterized by coexistence of attractors.

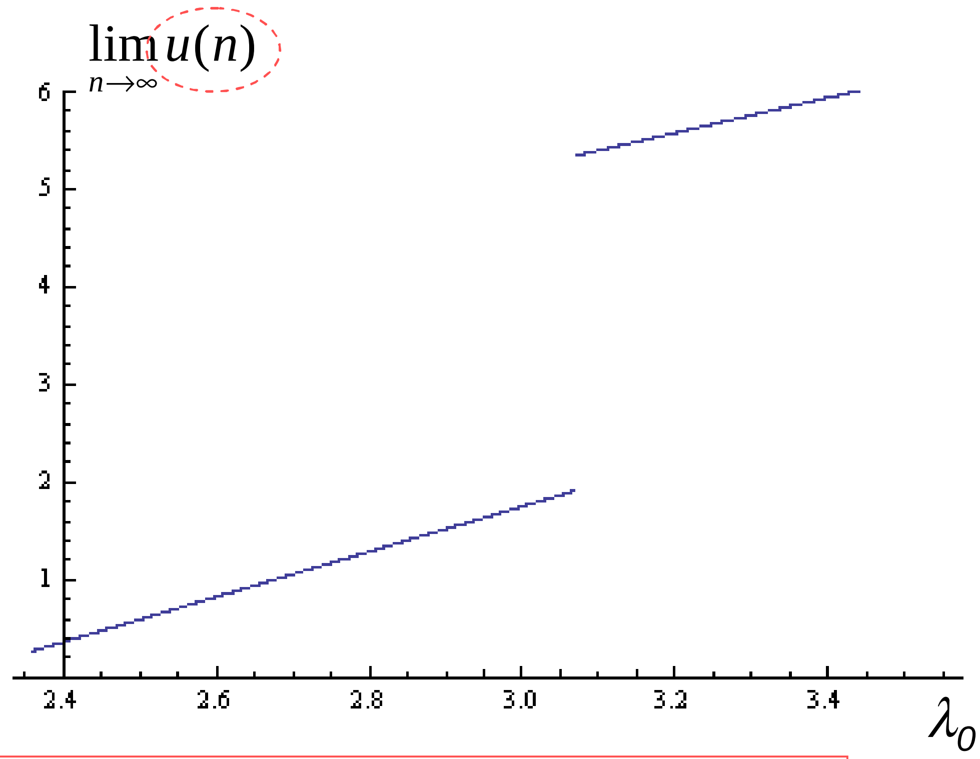
RRC1 Un equilibrio è un punto di intersezione tra la retta a 45° e la mappa u(t+1)=f(u(t)) che è la funzione lineare a tratti in blu.

Per valori di $r<1$ c'è solo un equilibrio sempre, mentre per valori di $r>1$ possiamo averne due come ad es. per $r=1.2$.
Cerchiara; 1.9.2008

Bifurcation Diagram

Long-terms values of the system as a function of bifurcation parameter λ_0

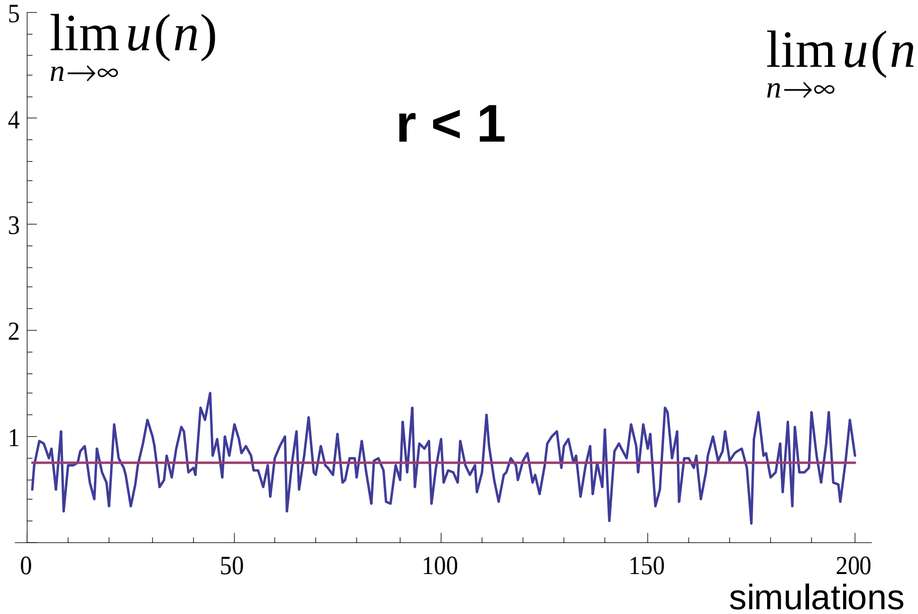


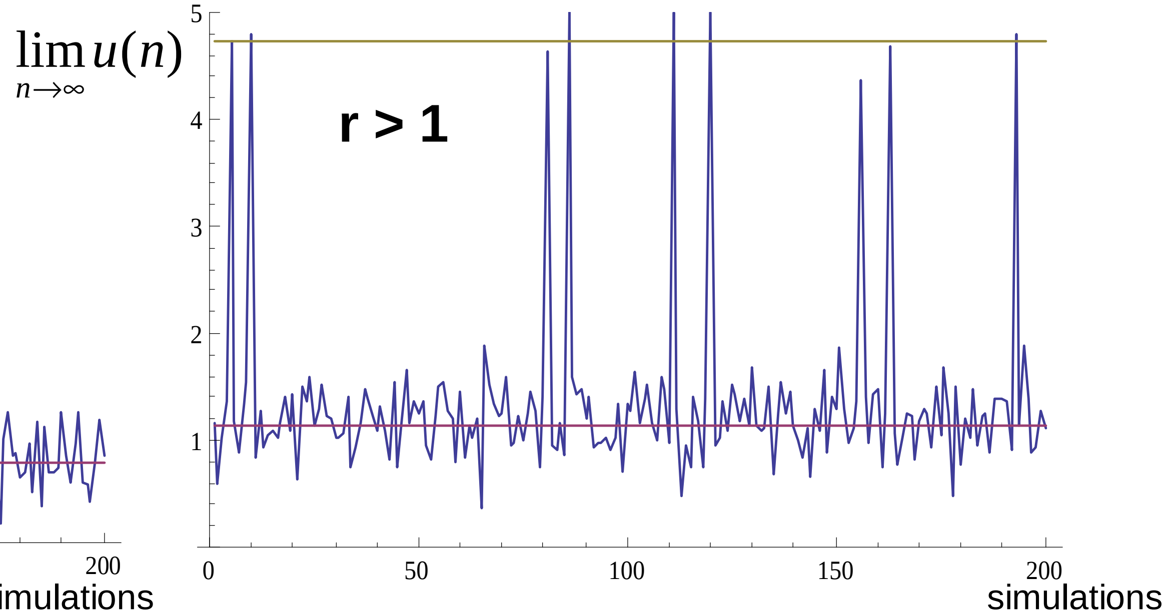


- Same parameters of previous slide, but with r =1.2; **λ_0 from 2.36 to 3.55**

- **λ(t)** is continue in λ_0, so equilibrium is increasing and continuos in λ_0
- **u(t)** presents a jump discontinuity, whose formation is due to the change of the stable attractor after the second bifurcation takes place. So we have double "border collision bifurcation" for increasing λ_0: from one attractor with a "low" u to one attractor with a high u.

An example of stochastic application





- c1=0.632; c2=0.754; R1=1.91; R2=4.43; λ_0=2.79;
- **x=4.5z; z distributed according a lognormal with μ=0 and σ =0.05**

- Trajectory with 200 independent simulations.
- The limit $\lim_{n\to\infty} u(n)$ has been obtained from a same initial condition for all simulations.
- r <1: the trend varies around the mean (horizontal line).
- r >1: presence of two attractors, one for a lower level of u and one for a higher level, to which converge more seldom. It depends on the parameters values considered and on initial condition.

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Final Remarks

- Remarks:
  - Within the proposed model it is possible to define analytical control rules by setting the parameters c_1, c_2, R_1, R_2 and, consequently, to fix the safety loading level.
  - With this approach it is possible to "guarantee" prefixed levels of equilibrium of the solvency ratio and so of the capital requirements of the insurer.
  - This model could represent an alternative (or a complementary) tool to the traditional techniques used in actuarial application, such as simulation, approssimation formulas, etc.
  - These slides represent a work in progress, to be extended with the following developments:
    - Stochastic implementation for aggregate losses.
    - Parameters estimation using real data.

- Acknowledgements
  - The authors are grateful to T. Pentikainen for the several contributes that inspired our research.

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