Economic factors and solvency

Harri Nyrhinen, University of Helsinki

ASTIN Colloquium
Helsinki 2009

Insurance solvency

One of the main concerns in actuarial practice and theory. The companies should have a reasonable capacity to survive

- within a short time horizon (the regulatory point of view)
- within a longer time horizon (the company point of view).

We study the problem from the theoretical point of view,

- in a practical model, basically from **Pentikäinen and Rantala** (1982)
- asymptotically as the initial capital grows.

Our conclusion will be that

- qualitative views can be given by theoretical tools
- quantitative results can be obtained mainly by simulation.

Ruin problem

Instead of survival probabilities, it is equivalent to consider ruin probabilities. Let

$$\begin{aligned} u &= \text{the initial capital of an insurance company,} \\ U_n &= \text{the capital at the end of the year } n, \\ T &= \text{the time of ruin, that is,} \end{aligned}$$

$$T = \begin{cases} \inf\{n \in \mathbb{N}\,;\, U_n < 0\} \\ \infty \text{ if } U_n \geq 0 \text{ for every } n. \end{cases}$$

Consider

$$\mathbb{P}(T \leq x \log u)$$

for a fixed $x > 0$ and large u.

The model

Stochastic submodels for

- real growth
- inflation
- economic cycles
- returns on the investments,

and of course, for

- numbers of claims
- claim sizes
- premiums.

The model for the numbers of claims

For the year n, write

N_n = the accumulated number of claims occured in the years $1, \ldots, n$,
λ = the basic level of the mean of the number of claims in the year,
g_n = the rate of real growth,
q_n = the structure variable describing short term fluctuations in the numbers of claims,
b_n = the variable describing cycles and other long term fluctuations in the numbers of claims.

$N_n - N_{n-1}$ has a mixed Poisson distribution with the (stochastic) parameter

$$\lambda b_n (1 + g_1) \cdots (1 + g_n) q_n .$$

Dependences between consecutive years come via mixing variables, but not otherwise.

The development of the capital

For the year n, let

$$
\begin{aligned}
X_n &= \text{the total claim amount,} \\
P_n &= \text{the premium income,} \\
U_n &= \text{the capital at the end of the year,} \\
r_n &= \text{the rate of return on the investments.}
\end{aligned}
$$

The total claim amounts and the premiums are affected by inflation and by real growth.

Let $U_0 = u > 0$ be the deterministic initial capital of the company. We define

$$U_n = (1 + r_n)(U_{n-1} + P_n - X_n).$$

Results

For appropriate fixed $x > 0$ and a given $\varepsilon > 0$, we have for large u,

$$u^{-R(x)-\varepsilon} \leq \mathbb{P}(T \leq x \log u) \leq u^{-R(x)+\varepsilon},$$

where $R(x)$ is specific, and only depends on the economic factors:

real growth, inflation and returns on the investments.

Also

- suitable variations assumed for short term fluctuations,
- light tails assumed for the claim sizes,
- economic cycles should also affect $R(x)$ in a slightly modified model.

We will measure the **risk** by means of the parameter $R(x)$.

A comparison with the classical model

The key variable in our model is

$$Y = \frac{(1+i)(1+g)}{1+r}$$

where i, g and r describe generic rates of inflation, real growth and investment returns, respectively. Parameters $R(x)$ are determined by Y.

In the classical model, $Y \equiv 1$, and $R(x) = \infty$ for every x. It seems **difficult to come back to classical exponential estimates** since to this end, we should have $Y \leq 1$. By putting $g \equiv 0$, the points for the problems are

- we would need superhedging against inflation by means of investments
- this is difficult, especially since i is not general inflation in the economy

Example 1

Consider two alternative investment strategies: the portfolio consists of

1. stocks only,
2. equal numbers of stocks and associated put options.

In alternative 2, large losses in the investments have been cut so that this strategy should be safer. Our conclusions are

- for **small** x, this can be confirmed by the parameter $R(x)$ ($\mathbb{P}(T \leq x \log u)$ is smaller in alternative 2 for large u)
- for **large** x, a high price of the put option may change the situation, that is, alternative 1 can be preferable.

Example 2

Suppose that the pair

$$(\log(1+i), \log(1+r))$$

has a two-dimensional normal distribution (i is the rate of inflation and r the rate of return on a stock). Two alternative investment strategies:

1. all the money to the stock,
2. all the money to the bank account which yields a fixed interest rate.

Differences can be seen in the parameters $R(x)$. The indication is that for **small** x,

- $\mathbb{P}(T \leq x \log u)$ is smaller in alternative 1 for large u if the correlation between inflation and returns on the stock is high
- $\mathbb{P}(T \leq x \log u)$ is smaller in alternative 2 for large u if the correlation is low.

Thank you for your attention