ASTIN Colloquium Helsinki 2009

Rating without data

How to estimate the loss frequency of loss-free risks

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Agenda

- The loss-free rating situation
- Sample mean amended
- Statistical properties

The loss-free rating situation

Assess the loss frequency of a particular risk

- Data from other risks is not representative
- Own data from past years is (in principle) representative
- Loss record:

no losses in past 7 years

The ideal rating method

- covers all situations
- converges to the empirical loss frequency
- > 0
- Bias nonnegative but small
- monotonic
- smooth renewal: regard premium for next year
  - if year was loss-free then no increase
  - if new loss then increase premium but not too much

ASM: amended sample mean

Sample mean: N / k

N = # losses in observation period

k = # observed years

Define an **amending function** g(n), n = 0, 1, 2, ...

and set

$$\text{ASM} := g(N) / k$$

Good amending functions

- g(n) defined for all n = 0, 1, 2, ...

- $g(n) \to n$, or in particular $g(n) = n$ for $n \geq d$
- $g(n) > 0$
- $g(n) \geq n$
- $g(n+1) > g(n)$

- g(n+1)/g(n) reasonable (smooth)

Desirable smoothness conditions

- $g(n+1)/g(n) \approx (n+1)/n$ for $n>0$
- $g(n+2)/g(n+1) \leq g(n+1)/g(n)$
- $g(n+1)/g(n) \leq (n+1)/n$ for $n>0$
- $g(1)/g(0) \leq 2$

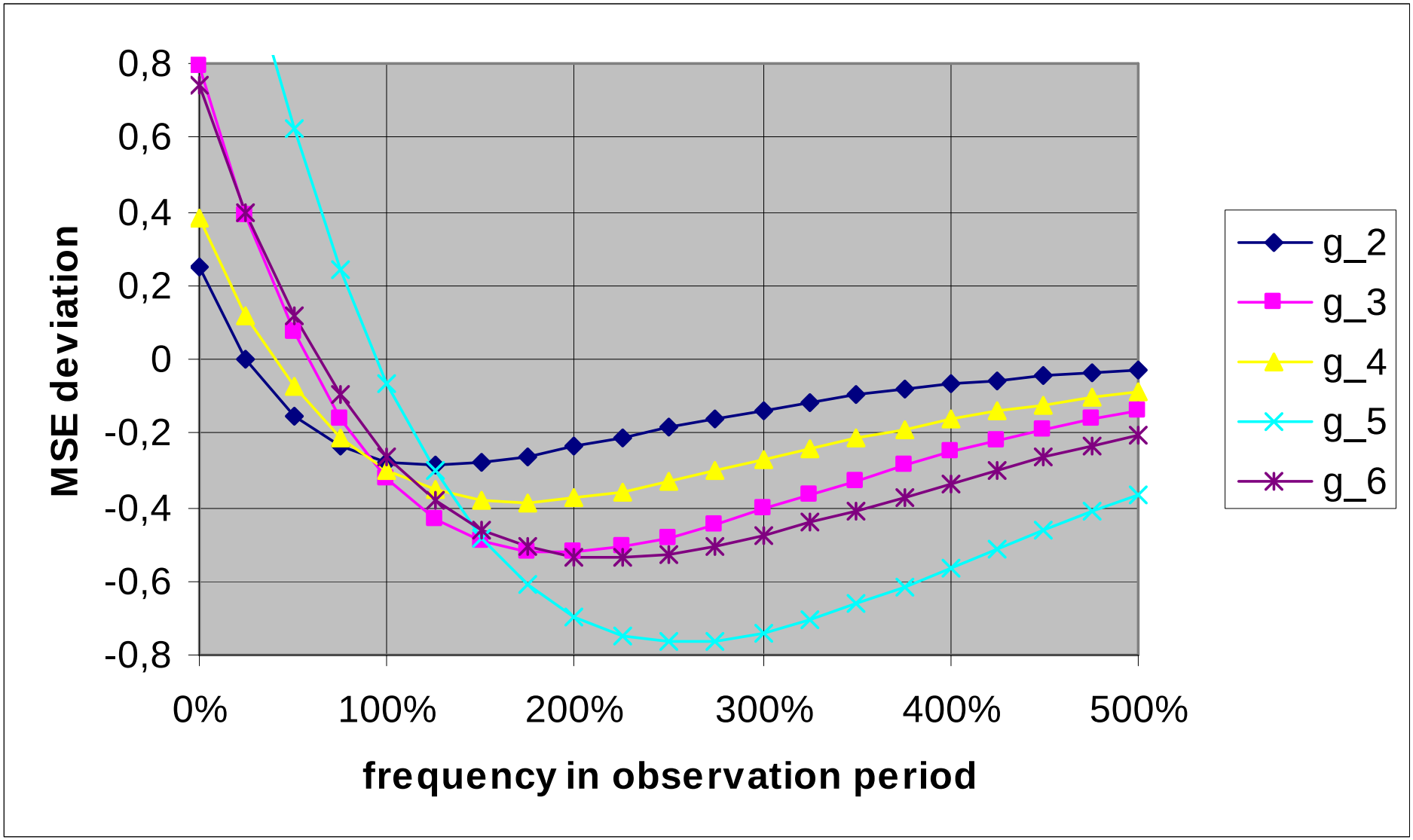
Candidates

n	**0**	**1**	**2**	**3**	**4**	**5**	**6**
$g_1(n)$	1	2	3	4	5	6	7
$g_2(n)$	0.5	1	2	3	4	5	6
$g_3(n)$	0.89	1.33	2	3	4	5	6
$g_4(n)$	0.62	1.19	2	3	4	5	6
$g_5(n)$	1.27	1.69	2.25	3	4	5	6
$g_6(n)$	0.86	1.39	2.11	3	4	5	6

Statistical properties: Bias

- g_1 is too expensive
- g_2 has the lowest Bias (but is not very smooth)
- All other amending functions show a trade-off between bias and smoothness.

Mean Squared Error: beats the sample mean



Conclusion: Use it!

- quick (but not dirty at all)
- systematic, not case by case
- always yields a result
- with many losses same result as other methods
- mathematically consistent, statistically good
- smooth renewal, according to choice of amending function

Thanks

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