REINSURANCE CREDIT RISK MODELLING

DFA APPROACH

ASTIN Colloquium 2009 – *Helsinki, Finland*

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Agenda

- Introduction to Reinsurance Credit Risk (RCR) Modelling
  - RCR - what is it and why do we need to quantify it?
  - RCR vs. Investment Credit Risk;
  - Proposed approach vs existing modelling paradigms;
  - Literature review;
- Proposed modelling approach
  - Model setup;
  - Modelling implications;
  - Numerical illustrations;
- Conclusions

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- Introduction to RCR Modelling -

Reinsurance Credit Risk (RCR)

- What is it?
  - The RCR is the risk of the reinsurance counterparty failing to pay reinsurance recoveries in full to the ceding insurer in a timely manner - *unwillingness* to pay, or even not paying them at all - *inability* to pay;
  - In a wide sense it is the part of company's overall credit risk: *"Credit risk is the risk of loss arising from failure to collect funds from creditors, including reinsurers and intermediaries" – APRA.*
- Why is it important?
  - It is not as significant as insurance risk (e.g. underwriting and catastrophe risks) for insurance companies;
  - However, it must be modelled accurately if one chooses to use an Internal Capital Model according to local regulatory capital requirements or the proposed Solvency II SCR.

RCR vs. Investment Credit Risk

- To some extent they are similar – in both cases the loss can be decomposed into: default *frequency* – *severity* – *recoveries*.
- However, there are differences between them that arise in relation to default frequency, severity (exposure), and recoveries.
  - *Different default events*: under financial distress reinsurers often go into run-off and enter into a commutation agreement with ceding insurers compared to bond issuers that default as a result of shortfall in interest and/or principal payments;
  - *Higher risk concentration*: compared to bond issuers the number of reinsurers is small, and therefore more concentrated exposure;
  - *Higher risk correlation*: RCR exposure is specific to one industry sector (insurance), and therefore correlations are higher than in a diversified bond portfolios;

Existing modelling paradigms

- *Calibration to observed prices*:
  - Shortcoming 1: It relates to model mis-specification, and how that impacts on model derived estimates. If a model is mis-specified, the resulting default rate estimates will be biased;
  - Shortcoming 2: When calibrating to CDS prices, the prices reflect all the dynamics of CDS market most of which are not relevant for the purpose of modelling reinsurance credit default;
- *Calibration to observed parameters* - refers to maximum likelihood estimators based on direct observation of data:
  - In the proposed approach we use direct observation (AM Best asset impairment rates for reinsurers) to estimate unconditional reinsurance default rates.
- *Calibration to 'emergent properties' or 'stylised facts'*:
  - We have, from observation, some views on how model outputs should behave. We calibrate the model such that the desired properties are recovered.

Literature review

- Bulmer et al. (2005) - deterministic factor-based approach to calculating 'bad debt' provisions in reserving area;
  - not ideal for accurate assessment of RCR;
  - however might be practical for well capitalised insurers with little reinsurance on board;
- Flower et al. (2007) and Shaw (2007) - very practical approaches to modelling RCR stochastically. However, they:
  - use corporate bond default rates (Shaw) or Credit Default Swaps (CDS) prices (Flower et al.) to calibrate reinsurance default rates;
  - do not explicitly formulate the modelling of defaults (especially in a multi-period setting) that would incorporate some dependency structure including tail-dependency characteristics.

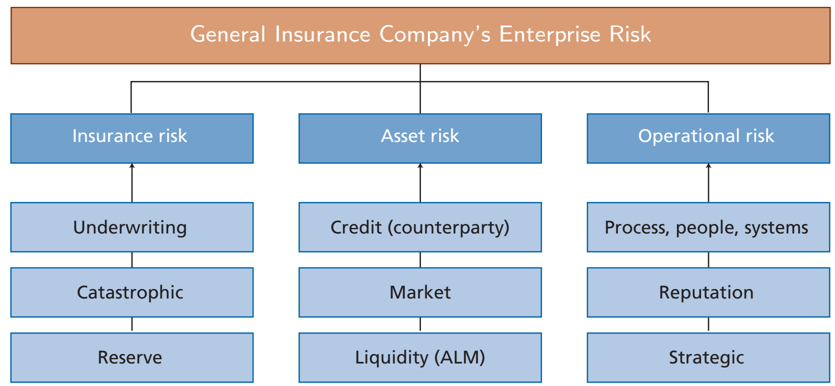
- Proposed modelling approach -

Model setup - *key assumptions*

- *Exposure* - a small number of representative 'proxy reinsurers' is created to capture the company's exposure to RCR. In the event of default, the defaulted proxy reinsurer will be replaced by a new proxy reinsurer of the same credit quality.
- *Default event* - the default of any proxy is assumed to occur at the beginning of any projection time period, assumed here to be a quarter, and is modelled as a binary event using Bernoulli random variable with the default rate dependent on the state - *'normal'* and *'stressed'* - of the global reinsurance market.
- *Cost of default* - the Loss Given Default (LGD) is the product of a recovery rate and exposure to proxy reinsurers, i.e. outstanding reinsurance recoveries, at the end of previous period plus any replacement cost of unexpired reinsurance cover.

Model setup - *DFA model structure*

- *DFA Internal Capital Model* - a customised version of MoSes^{TM} Property-Casualty Financial Model (PCFM).



- The RCR is modelled in the *Reinsurance Default* module, which interacts with other modules:
  - *Cats and Reinsurance* module - by importing potential reinsurance recoverables and unexpired ceded premiums;
  - *Economic Scenario Generator (ESG)* module - by importing rates of economic inflation and discounting rates; and
  - *Accounts* module - by exporting cost of default by business class level to model accounts.

Model setup - *exposure*

- The company's exposure to a proxy reinsurer varies by exposure type, e.g. by catastrophe (cat) and non-cat, small and large cat events. This is defined in the 'Exposure Share Matrix'

BUCKET	CREDIT RATING	< $1.5BN	> $1.5BN	UEP	NON CAT
1	AA	27.5%	29.8%	23.4%	15.8%
2	A+	7.5%	5.0%	4.7%	3.7%
3	A	7.5%	9.0%	7.9%	3.7%
4	AA-	7.5%	4.5%	18.7%	22.1%
5	AA-	10.0%	5.5%	7.1%	2.7%
6	A-	0.0%	0.0%	0.0%	0.0%
7	AA-	5.4%	0.0%	3.4%	0.0%
8	AA-	0.0%	20.5%	0.0%	0.0%
9	A+	1.0%	1.2%	0.4%	0.0%
10	A-	10.0%	1.3%	7.8%	0.0%
11	AA+	2.5%	0.0%	2.4%	15.3%
12	AA-	2.0%	6.0%	1.9%	0.0%
13	A	2.0%	1.7%	1.6%	8.8%
14	A	5.0%	2.5%	3.2%	0.0%
15	A-	4.0%	2.5%	2.7%	0.0%
16	A-	3.0%	0.6%	5.0%	0.0%
17	A+	1.0%	5.4%	5.2%	13.5%
18	A-	1.0%	0.0%	0.8%	12.4%
19	CASH (AAA)	3.1%	4.5%	3.9%	0.6%
20	NOT RATED (BBB)	0.0%	0.0%	0.0%	1.4%
TOTAL		100%	100%	100%	100%

- For each 'proxy reinsurer' the model stores at any time a vector of potential (expected) reinsurance recoveries at that time.

Model setup - *default event*

- Modelled at the beginning of every quarter by Bernoulli random variable rate of which is dependent on the pre-generated global market state ('normal' or 'stressed'). The duration of 'stress' state is assumed to be two years (was calibrated to industry data and estimated to be in the range from 1 to 4 years). The annualised unconditional default event of each proxy reinsurer is defined by

$$D = D_n(Z-1) + [1 - D_n(Z-1)]\, D_s(5-Z),$$

where for $m \in \{0,1,2,3,4\}$ $D_{n,s}(m) \sim Be\left[1-(1-q_{n,s})^m\right]$ are respectively conditional 'normal' and 'stressed' defaults over the period of m quarters modelled by Bernoulli random variables with quarterly rates q_n and q_s; and $Z \sim TruncGeom[p;4]$ is the Truncated Geometric random variable over the period of four quarters with the quarterly transition rate p. Indicates the index k of the quarter in which the market transits to 'stressed state':

$$\mathbb{P}[Z=k] = \begin{cases} (1-p)^{k-1}p, & k = \overline{1,\ldots,4}; \\ (1-p)^4, & k = 5 \end{cases}$$

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Model setup - *LGD*

- The recovery of potential losses (i.e. total reinsurance recoverables) at default can be modelled either deterministically using a certain recovery rate, or stochastically via introducing a random recovery rate with *Beta* distribution on interval from zero to one.
- In this setup we chose deterministic approach to modelling recoveries, and adopted the following **average loss rates given default** that were derived from the industry study of reinsurance default recovery rates conducted by GIRO Working Group of the UK Institute of Actuaries (Bulmer et al. 2005):

AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	NR
0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.8

Modelling implications - *key points*

- The proposed approach allows for modelling average dependency, and tail dependency between reinsurance default events.
  - The use of a 'stress' scenario that affects all reinsurers is a useful method by which co-dependency between reinsurer defaults can be allowed for.
- Unconditional and conditional ('stressed' and 'normal') default rates are calibrated using the last two modelling paradigms
  - *Use of Paradigm 2:* – recommended annualised unconditional default rates of reinsurers were derived from the AM Best's research study that defines the reinsurer default as an asset impairment event where the value of the reinsurer's net assets falls below a certain threshold.
  - *Use of Paradigm 3:* – under the constraints that the unconditional annual default rates and the market transition rate remain unchanged, the quarterly normal/stressed default rates were calibrated such that the model implied default dependency structure matches the target one – observable dependency structure of reinsurers asset return.

Modelling implications - *calibration*

- The unconditional annual default rate:

$$\mathbb{E}[D] = \mathbb{E}_Z\left[\mathbb{E}\left[D \mid Z\right]\right] = \mathbb{E}_Z\left[1 - (1-q_n)^{Z-1}\,(1-q_s)^{5-Z}\right].$$

- Default dependency structure is defined by the 2×2 contingency table:

$P_{10}^{c_1 c_2}$	$P_{11}^{c_1 c_2}$
$P_{00}^{c_1 c_2}$	$P_{01}^{c_1 c_2}$

where $P_{kl}^{c_1 c_2} = \mathbb{P}\left[D^{c_1} = k;\ D^{c_2} = l\right],\ \ k, l = \overline{0, 1}$ is a joint probability of default/survival for two reinsurers. The joint default probability P_{11} uniquely determines the complement of the contingent table, since $P_{10} + P_{11}$ and $P_{01} + P_{11}$ are marginal default rates that are fixed.

Modelling implications - *calibration*

- Target default dependency – *assuming normality of the distribution of asset return and the base asset return correlation*:

$$P_{11}^{c_1 c_2} = \int_{-\infty}^{R_1} \int_{-\infty}^{R_2} \frac{1}{2\pi\sqrt{1-\rho_A^2}} \exp\left\{-\frac{x_1^2 + x_2^2 - 2\rho_A x_1 x_2}{2(1-\rho_A^2)}\right\} dx_1\, dx_2,$$

where $R_i = \Phi^{-1}\left(\mathbb{E}[D^{c_i}]\right),\ i = \overline{1,2}$; ρ_A is the base asset return correlation. The variable x_i represents asset return for i-th reinsurer. The asset return below a given threshold R_i will lead to default.

- Default dependency implied by the DFA model:

$$\begin{aligned} P_{kl}^{c_1 c_2} &= \mathbb{P}\left[D^{c_1} = k;\ D^{c_2} = l\right] \\ &= \sum_{z=1}^{5} \mathbb{P}\left[D^{c_1} = k \,|\, Z = z\right] \mathbb{P}\left[D^{c_2} = l \,|\, Z = z\right] \mathbb{P}\left[Z = z\right],\quad k, l = \overline{0,1}, \end{aligned}$$

where $\{D^{c_i} \,|\, Z = z\} \sim Be\left(1 - (1 - q_n^{c_i})^{z-1}\,(1 - q_s^{c_i})^{5-z}\right),\quad i = \overline{1,2}.$

Modelling implications - *calibration*

- The base asset correlation is assumed to be at 25%, which incorporates 17.5% mean sample estimate of asset return correlation taken from the KMV empirical study (QIS 3 Study) plus a margin to deal with potentially increased correlation from the reinsurance industry.
- The DFA model uses the *diagonal calibration*, in which for a given credit rating the normal/stressed default rates were set such that the dependency structure of two distinct proxy reinsurers with the same credit rating matches the target one. Such diagonal approach does not consume as much computational resources as the full calibration would.

Numerical illustrations

- Smoothed unconditional default rates (in %):

YEAR	AAA	AA+	AA	AA-	A+	A	A-	BBB+	NR
1	0.063	0.192	0.318	0.446	0.587	0.702	0.836	1.506	4.124
2	0.101	0.232	0.350	0.476	0.609	0.719	0.879	1.586	4.335
3	0.116	0.248	0.361	0.487	0.617	0.725	0.895	1.617	4.413
4	0.131	0.264	0.373	0.497	0.626	0.731	0.912	1.648	4.494
5	0.146	0.279	0.384	0.508	0.634	0.737	0.928	1.678	4.571
6	0.162	0.296	0.396	0.519	0.643	0.743	0.945	1.710	4.655
7	0.182	0.317	0.412	0.533	0.654	0.751	0.967	1.752	4.762
8	0.200	0.336	0.426	0.546	0.664	0.758	0.987	1.790	4.859

Numerical illustrations

- Calibrated conditional 'normal' default rates (in %):

YEAR	AAA	AA+	AA	AA-	A+	A	A-	BBB+	NR
1	6.9×10^{-5}	0.038	0.089	0.148	0.218	0.277	0.350	0.744	2.513
2	8.3×10^{-3}	0.053	0.103	0.162	0.229	0.287	0.374	0.794	2.665
3	0.012	0.059	0.108	0.167	0.233	0.290	0.383	0.813	2.722
4	0.017	0.066	0.113	0.172	0.237	0.293	0.393	0.832	2.780
5	0.022	0.072	0.118	0.177	0.242	0.296	0.402	0.851	2.836
6	0.027	0.079	0.124	0.183	0.246	0.299	0.411	0.872	2.897
7	0.034	0.088	0.131	0.190	0.252	0.304	0.424	0.898	2.975
8	0.041	0.097	0.138	0.197	0.258	0.308	0.435	0.922	3.046

- Calibrated conditional 'stressed' default rates (in %):

YEAR	AAA	AA+	AA	AA-	A+	A	A-	BBB+	NR
1	0.987	2.466	3.688	4.839	6.016	6.926	7.951	12.589	27.081
2	1.472	2.870	3.984	5.092	6.189	7.058	8.270	13.107	28.098
3	1.644	3.023	4.087	5.180	6.256	7.103	8.390	13.302	28.473
4	1.816	3.179	4.192	5.271	6.324	7.150	8.512	13.501	28.856
5	1.976	3.325	4.292	5.357	6.389	7.194	8.628	13.689	29.219
6	2.146	3.482	4.400	5.450	6.460	7.242	8.754	13.894	29.611
7	2.360	3.681	4.537	5.569	6.550	7.303	8.914	14.154	30.110
8	2.551	3.859	4.661	5.676	6.632	7.359	9.059	14.390	30.561

Numerical illustrations

- Model implied base asset correlation (in %):

	AAA	AA+	AA	AA-	A+	A	A-	BBB+	NR
AAA	25.0								
AA+	25.0	25.0							
AA	25.1	25.0	25.0						
AA-	25.3	25.1	25.0	25.0					
A+	25.4	25.2	25.1	25.0	25.0				
A	25.5	25.2	25.1	25.0	25.0	25.0			
A-	25.7	25.3	25.1	25.1	25.0	25.0	25.0		
BBB+	26.2	25.7	25.4	25.3	25.2	25.1	25.1	25.0	
NR	28.0	26.9	26.4	26.1	25.9	25.7	25.6	25.3	25.0

- Using the *diagonal calibration* of conditional default rates results in slightly biased value of base asset return correlation for pairs of proxy reinsurers with different credit rating. Although, the differences in the base asset return correlation are within the simulation error bounds when using a model run with 10,000 simulation trials.

Modelling implications – *yet another thing*

- **The risk of reinsurance default in run-off** (i.e. after quarter four) is captured through increasing the quarter four default rate.
- The required overlay of the default rate is calculated as an expected rate of default occurred in the runoff, weighted by relative exposure to reinsurance recoveries (both cat and non-cat) at the beginning of quarter five, and is added to the quarter four normal/stressed default rates.
- In essence, the actual timing of the default in run-off (i.e. beyond one year period) is not modelled in the DFA model. The run-off default risk is rather condensed over the run-off time and brought back to quarter four through overlaying its quarterly conditional normal/stressed default rates.
- On the other hand, calibrating the quarter four default rate overlays will require explicit modelling of the timing of the default in runoff.

Modelling implications – *RCR in run-off*

- The default of a proxy reinsurer in run-off is modelled using the 'life contingency' approach, in which the proxy reinsurer's survival path depends on random times of market transition into stressed state.

- Assuming run-off pattern of RCR exposure - percentages $w(k)$, $k \geq 5$, of the initial exposure to reinsurance recoverables at the beginning of quarter 5, the expected default rate over the RCR run-off period can be modelled in the following way:

$$\theta = \mathbb{E}_\delta \left[\sum_{k \geq 5} \left(\prod_{j=1}^{k-5} [1 - q_s(j+4)]^{\delta(j+4)} [1 - q_n(j+4)]^{1-\delta(j+4)} \right) \times [q_s(k)]^{\delta(k)} [q_n(k)]^{1-\delta(k)} w(k) \right],$$

where $\prod_{j=1}^{0} \triangleq 1$; $q_{n,s}(k)$ are calibrated conditional quarterly forward default rates in quarter k; $\delta(k)$ is the '$0/1$' process generated via a Geometric r.v. of the timing of market transition into stressed state.

Numerical illustrations

- Modelled quarter four default rate overlay θ (in %):

AAA	AA+	AA	AA-	A+	A	A-	BBB+	NR
0.203	0.920	0.834	1.168	1.272	1.275	1.667	2.858	13.179

- Need to be added to the quarter four normal/stressed default rates.

Numerical illustrations - *key output*

- Distribution of the total reinsurance default cost (in % of Minimum Capital Requirements (MCR)):

50th percentile	75th percentile	90th percentile	95th percentile	99th percentile	99.5th percentile	99.9th percentile
0.000	0.059	0.996	1.615	3.525	3.525	3.525

- Other key stats of the distribution:

Average cost of default (% of MCR)	Chance of incurring default cost over 1yr (%)	Average cost of default given default (% of MCR)
0.29	36.70	0.80

- Conclusions -

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Conclusions

- **This approach has some advantages over other existing approaches to modelling reinsurance credit risk that are based on ideas borrowed from the investment banking industry:**
  - it uses reinsurers' asset impairment rates rather than corporate bond default rates or rates calibrated to CDS prices which are inappropriate in reinsurance credit risk modelling;
  - it offers explicit modelling of reinsurance defaults with embedded dependency between them.
- **Further improvements:**
  - use of t or Extreme Value copulae will result in much reacher dependency structure for reinsurance default compared to the assumed Gaussian dependency structure of reinsurers asset return.

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