A three dimensional stochastic Model for Claim Reserving

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Claim Reserving – P&C Insurance

Claim reserves must cover all liabilities arising from insurance contracts written in the presence and the past.

Claim reserves are calculated for homogeneous portfolios of insurance contracts

Claim reserves are needed for holistic risk management
→ Realistic modelling of the claim process !
→ Information about the tail of the reserve distribution necessary

Claim Reserving – Classical Models

One example for typical data structure in the framework of classical model:

The run – off – triangle

Occurr. year	Run – off [years]					
	0	1	2	3	4	5
2003	1.101	1.955	2.523	3.088	3.435	3.583
2004	1.113	2.103	2.774	3.422	3.844	
2005	1.265	2.433	3.233	3.977		
2006	1.490	2.873	3.880			
2007	1.725	3.261				
2008	1.889					

- **cumulated payments**
- **basis for reserve estimation methods**

Future payments = reserve

Claim Reserving – Classical Models

Classical Models

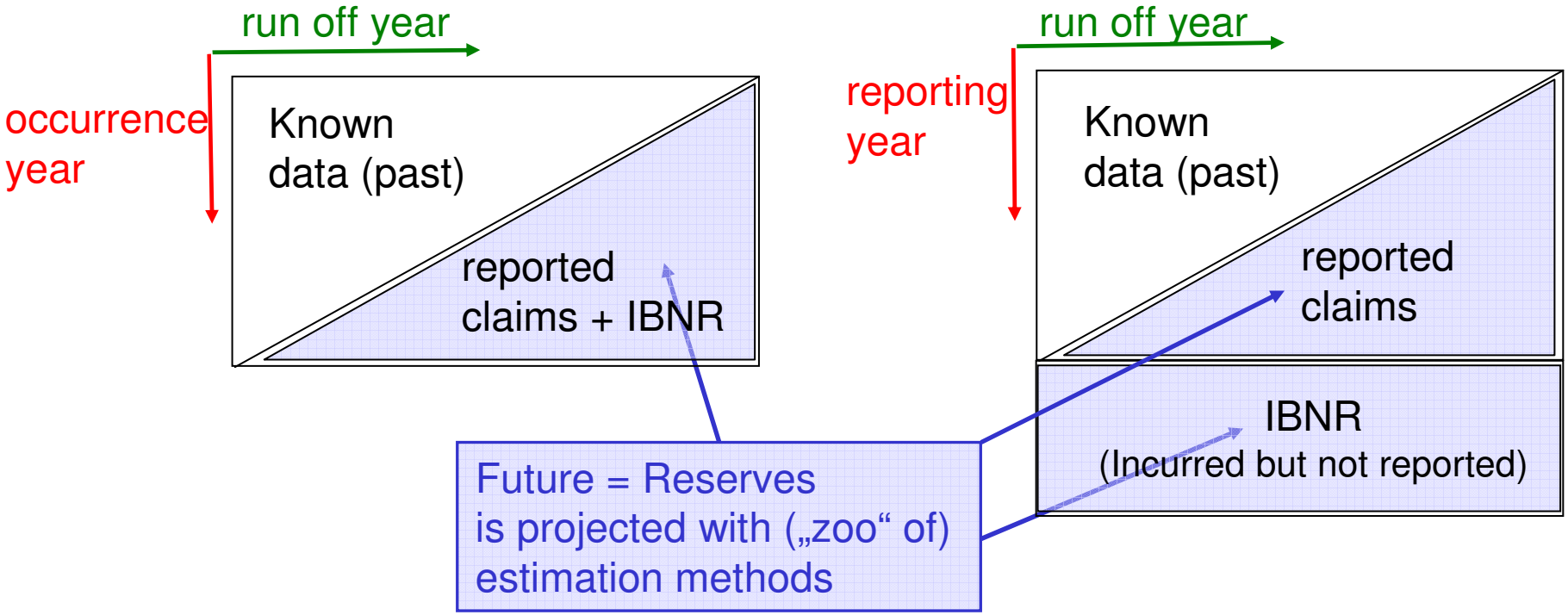
operate on a 2D data structure (cumulated data)

→Projection of complex process to a 2D "world"

→calculate only expectation of reserves (Var under restrictive assumptions)

→No information about the tail of the reserve distribution

<u>Typical data structures for estimation:</u>



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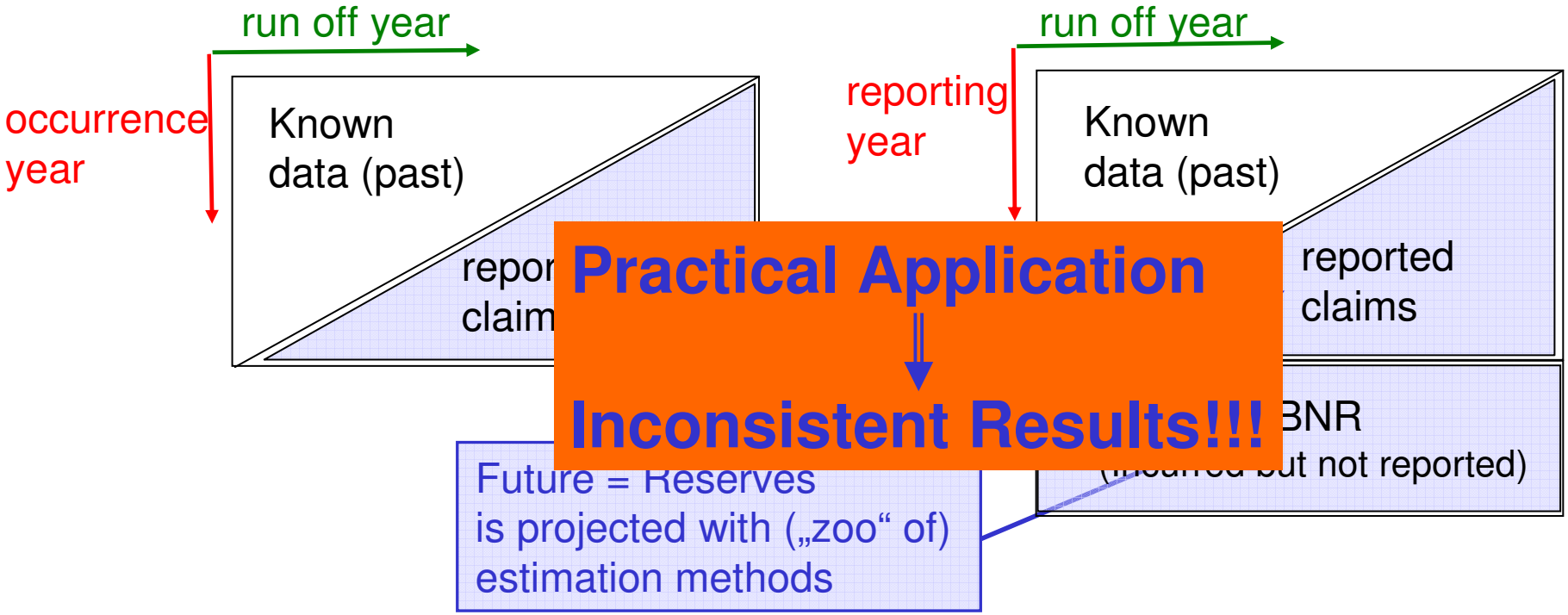


Practical Application

⇓

Inconsistent Results!!!

The 3D Model

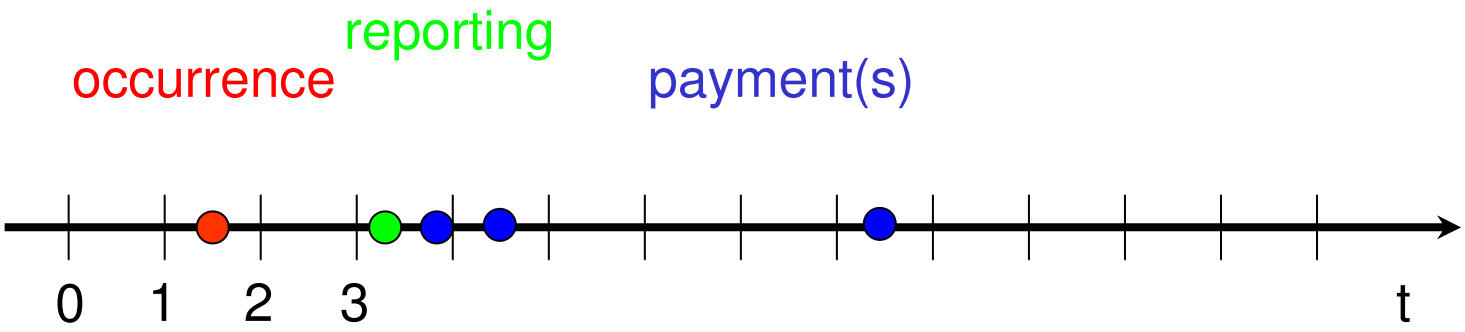
Idea

Given a single claim process

How develop
- Claim Portfolio
- Claim reserves

The 3D Model – the Claim Process

Time Dynamics of (single) claim process



3 dimensional structure:
occurrence (i) // reporting (j) // payment(s) (k)

Examine a portfolio of claims → stochastic model

The 3D Model

Modeled quantities:

1) Number of active claims N_{ijk}

occurred in ith year
reported j year after occurrence
still active k years after reporting

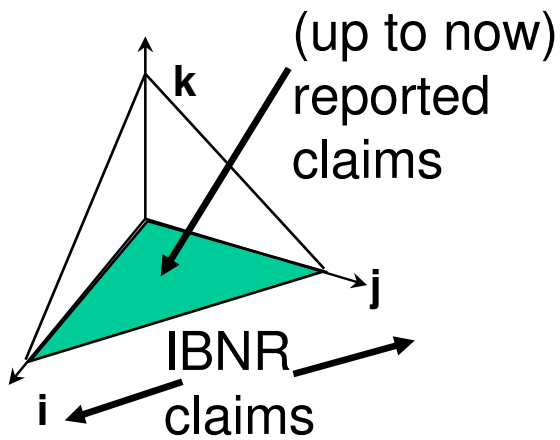
2) Total claim payments Z_{ijk}

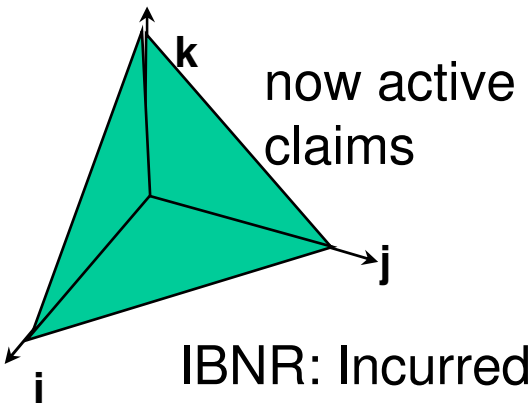
in kth year after reporting arising from all active claims with occurrence year i reported with j years delay.

The 3D Model

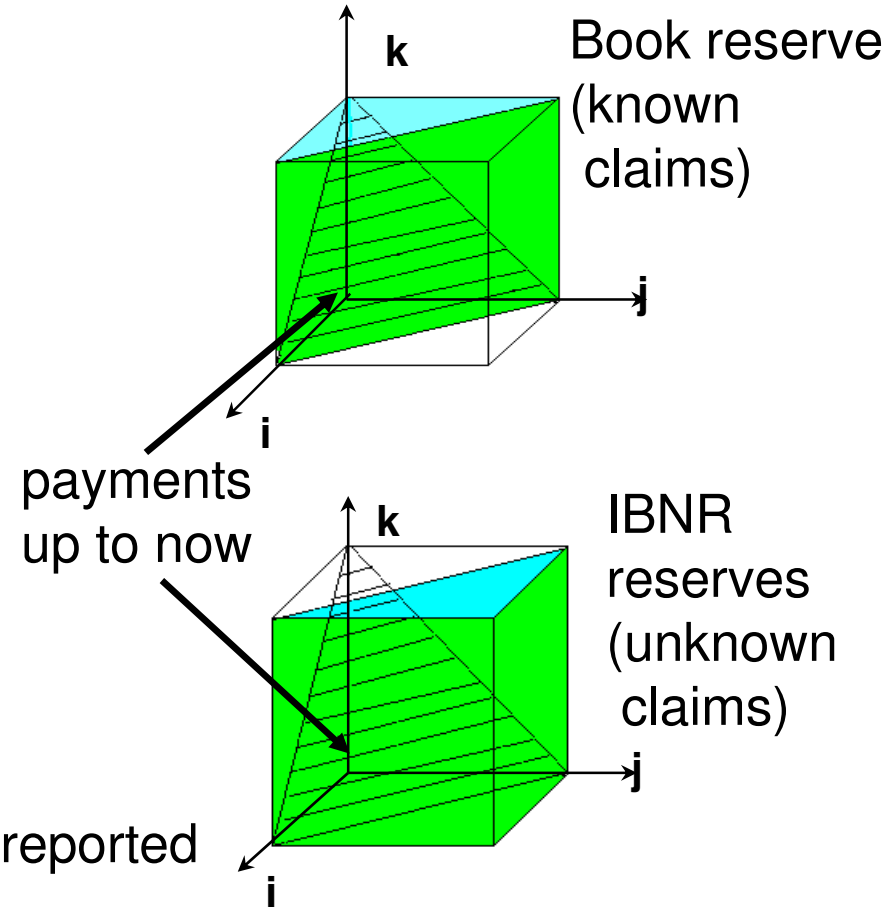
Visualising the model

Number of active claims N_{ijk}





Payments Z_{ijk}



IBNR: Incurred but not reported

Connection 3D and 2D Model

Up to now reported claims: N_{ij0} with $i + j \leq I_{\max}$

Number of IBNR Claims: $$\hat{N}_{IBNR} = \sum_{\{i,j \mid i+j > I_{\max}\}} \hat{N}_{ij0}$$

IBNR Reserve: $$\hat{R}_{IBNR} = \sum_{\{i,j,k \mid i+j+k > I_{\max} \vee i+j < I_{\max}\}} \hat{Z}_{ijk}$$

Total Reserve: $$\hat{R}_{total} = \hat{R}_{IBNR} + \sum_{\{i,j,k \mid i+j > I_{\max}\}} \hat{Z}_{ijk}$$

The run off triangle “payments: occurrence versus run off year” :

$$S_{mn}^{(1)} = \sum_{\{j,k \mid j+k=n \vee m+j+k \leq I_{\max}\}} Z_{mjk}$$

The run off triangle “payments: reporting versus run off year ” :

$$S_{mn}^{(2)} = \sum_{\{i,j \mid i+j=m \vee i+j+n \leq I_{\max}\}} Z_{ijn}$$

The 3D Model

Modeling the number of active claims

$$N_i = \sum_j N_{ij0}$$ (number of claims with occurrence year =i)

$$N_i \propto Poisson\left(\overline{N}_i\right)$$

Claim numbers mult. nom. dist. Along reporting years

with parameters $\lambda_j \quad j \in \{1,2,..,I_{\max}\}$ and $\sum_j \lambda_j = 1$

Closing the claims along years after reporting (Binomial process):

$\eta_k \quad k \in \{1,2,..,K_{\max}\}$ with $\eta_0 = 1$ and $\eta_{k+1} < \eta_k$

$$N_{ijk} \propto BiNom\left[N_{ijk-1}, \eta_k\right]$$

Expectation: $E\left[N_{ijk}\right] = \overline{N}_i \cdot \lambda_j \cdot \eta_k$

The 3D Model

Modeling the total claim payments: Collective Model

Number of single payments

$$Z_{ijk} = \sum_{l=1}^{\nu_{ijk}} X_l^{(ijk)}$$

Size of single payment

Probability of payment for an active claim

$$\nu_{ijk} \propto Binom\left[N_{ijk}, p_{ik}\right]$$

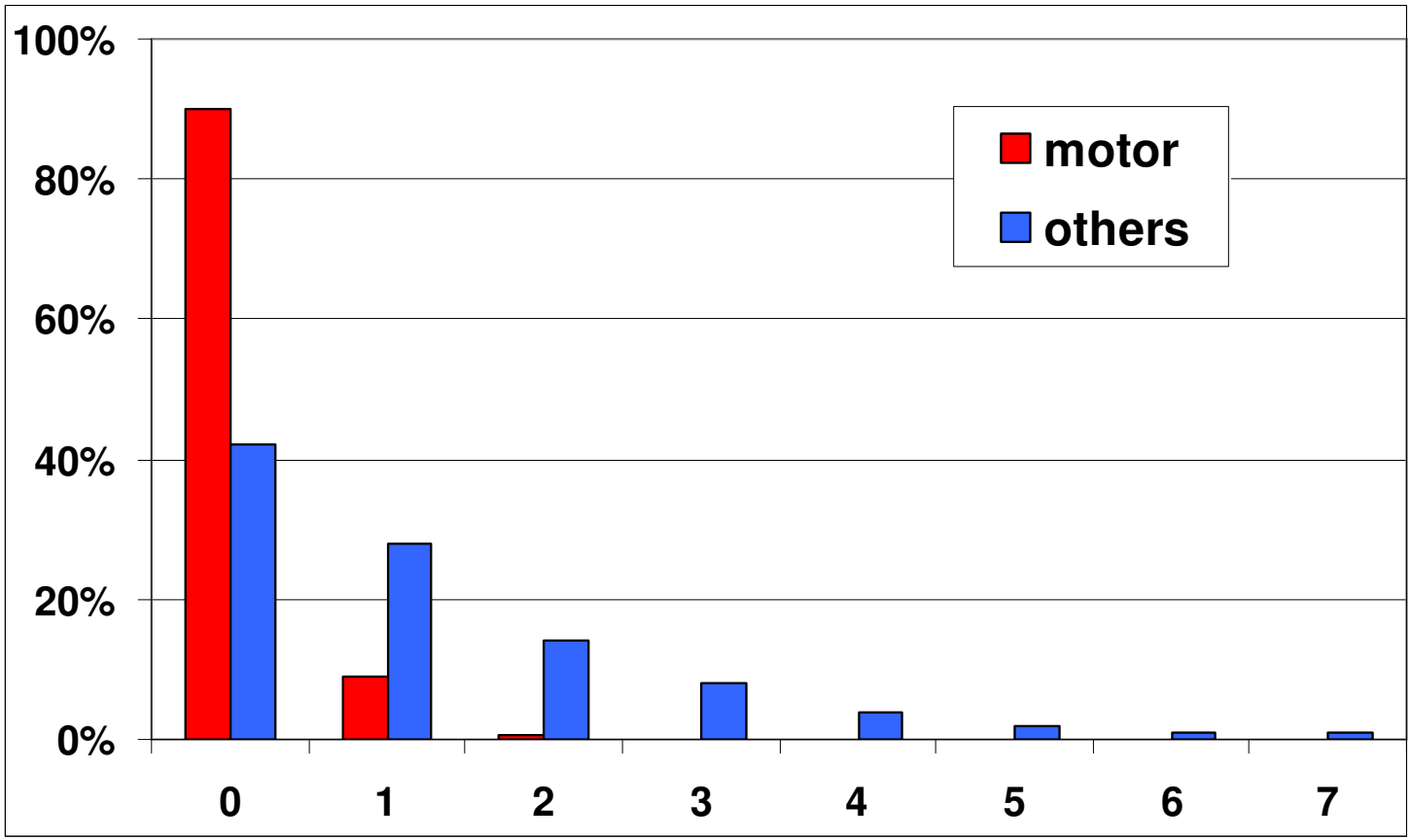
$$X_{ijk} \propto \Gamma\left[EW_{ijk}, Var_{ijk}\right]$$

MC Simulation – model parameters

Lag distribution

Third party liability / person injuries

Percentage of reported claims

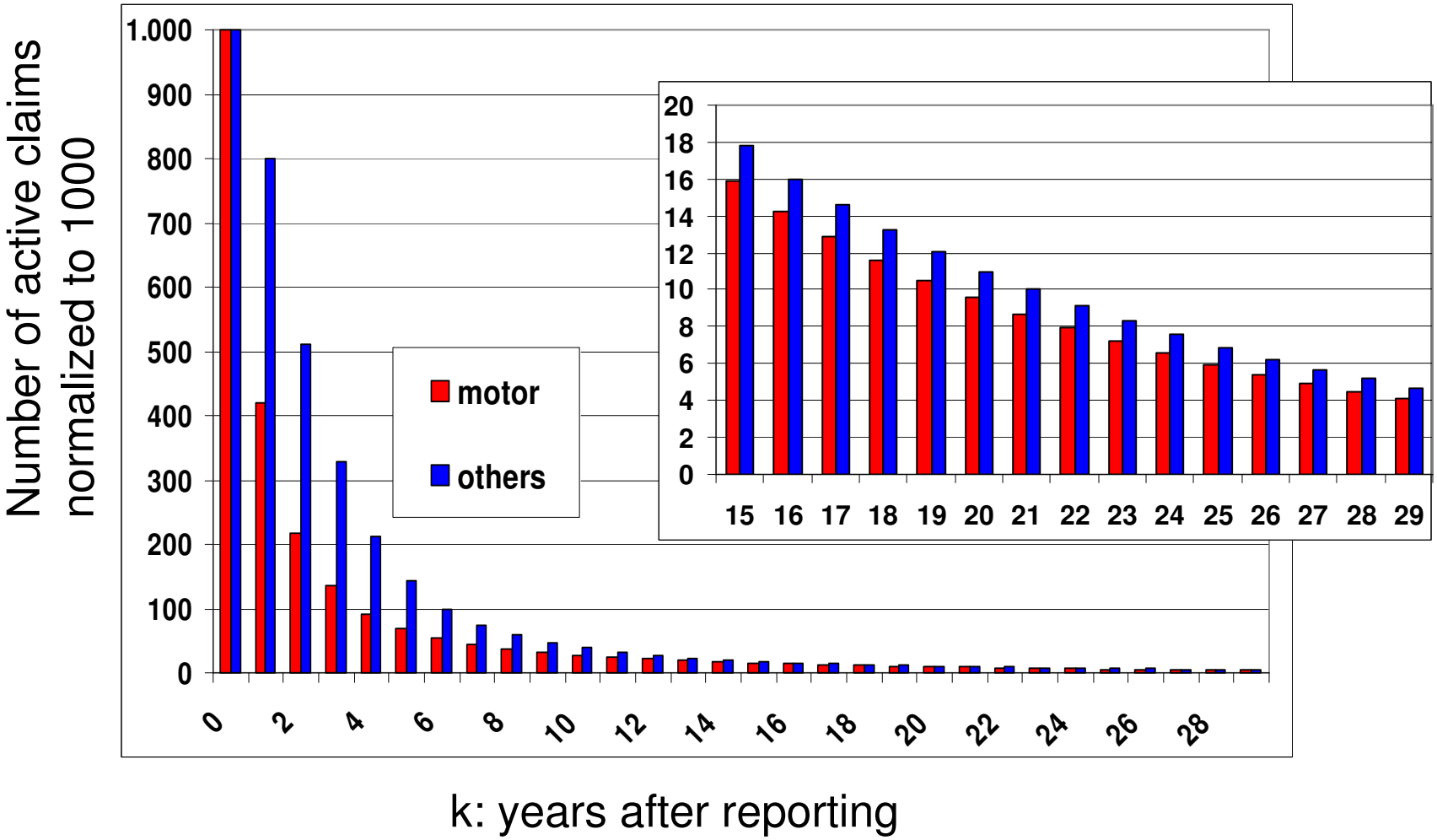


j: reporting delay (lag) [years]

MC Simulation – model parameters

Closing of open claims

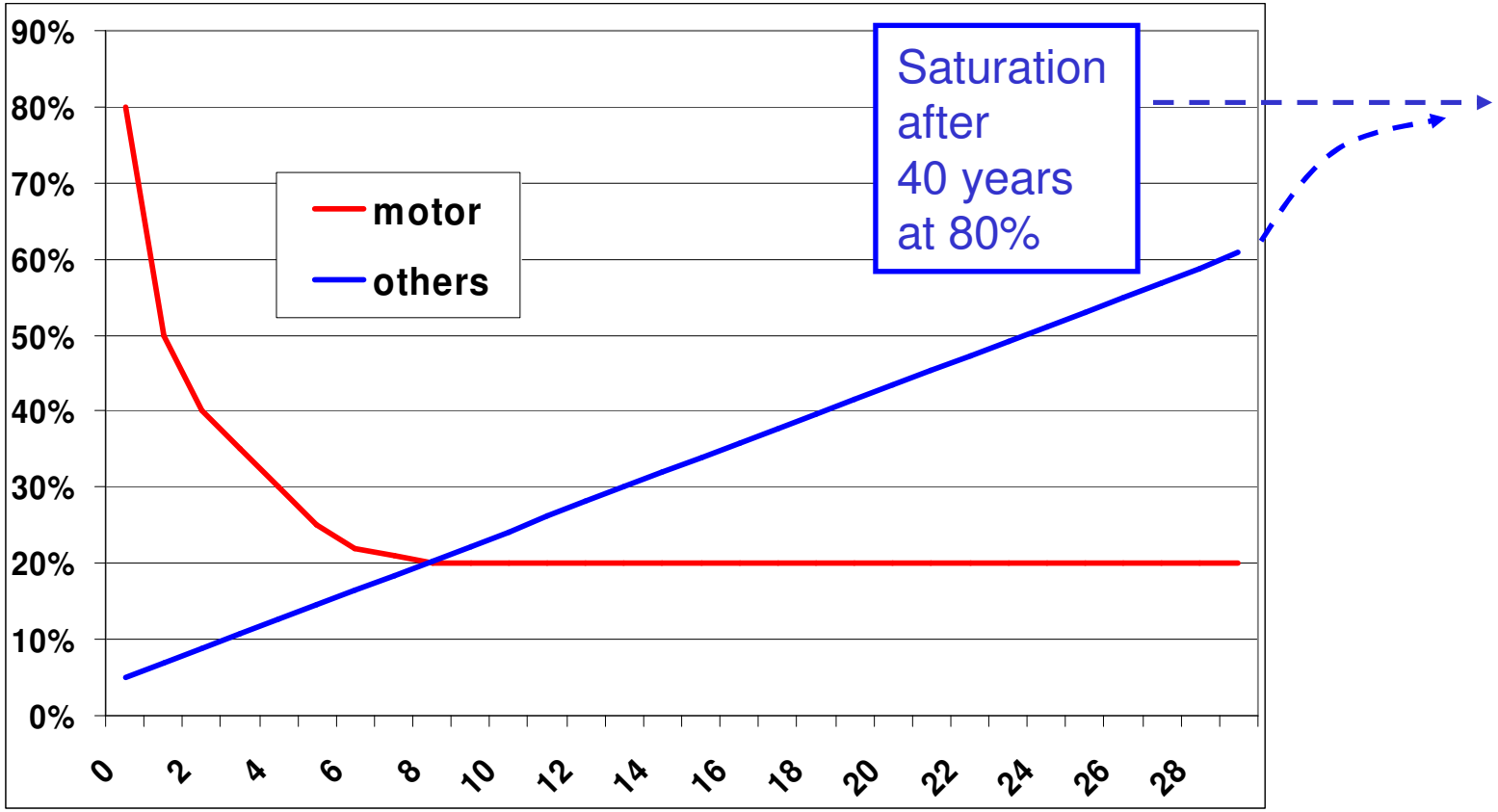
Third party liability / person injuries



MC Simulation – model parameters

Third party liability / person injuries

Probability of payment for an active claim

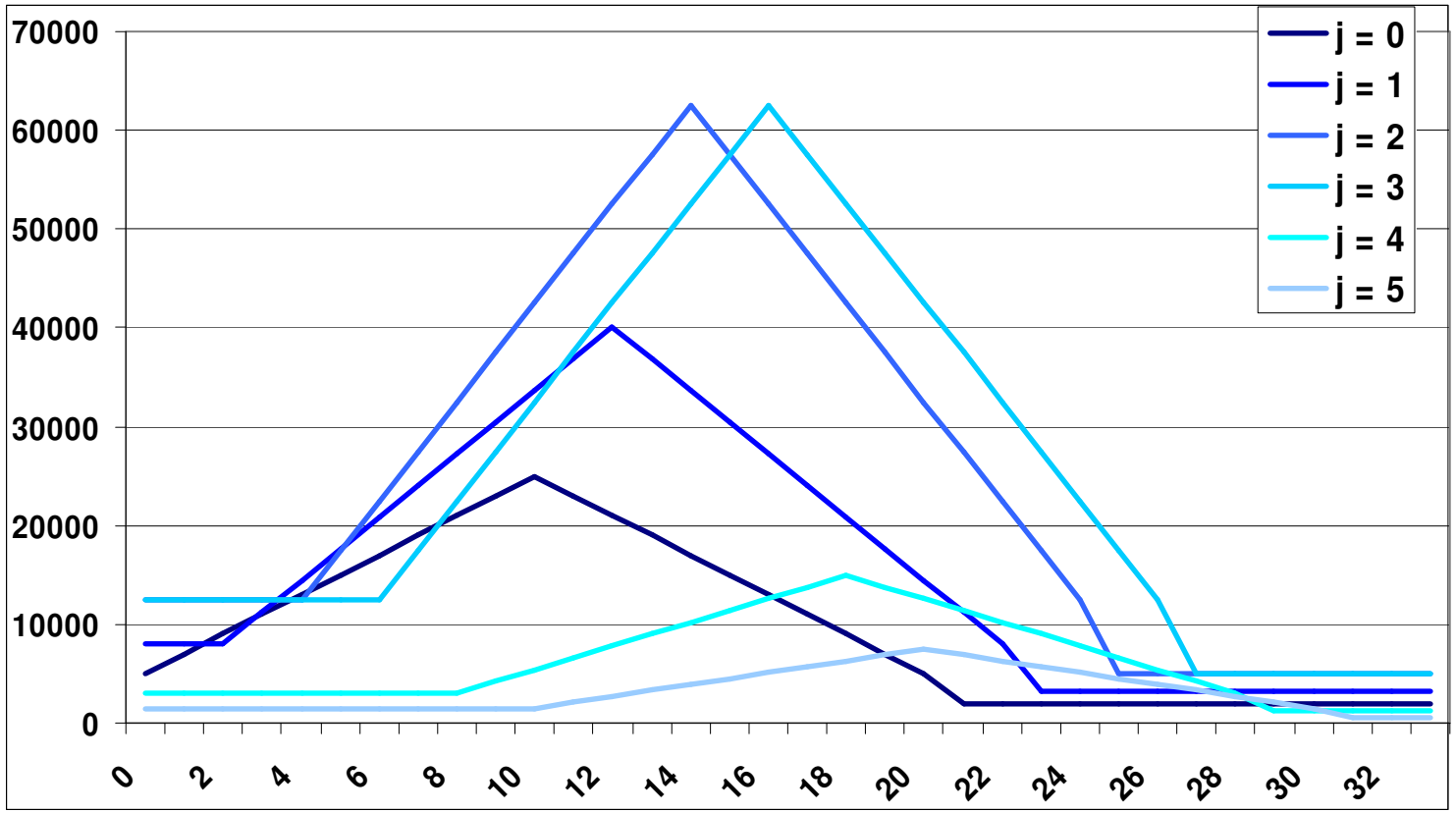


k: years after reporting

MC Simulation – model parameters

Third party liability - others / person injuries

Expectation value of single payment
for claims reported j years after occurrence



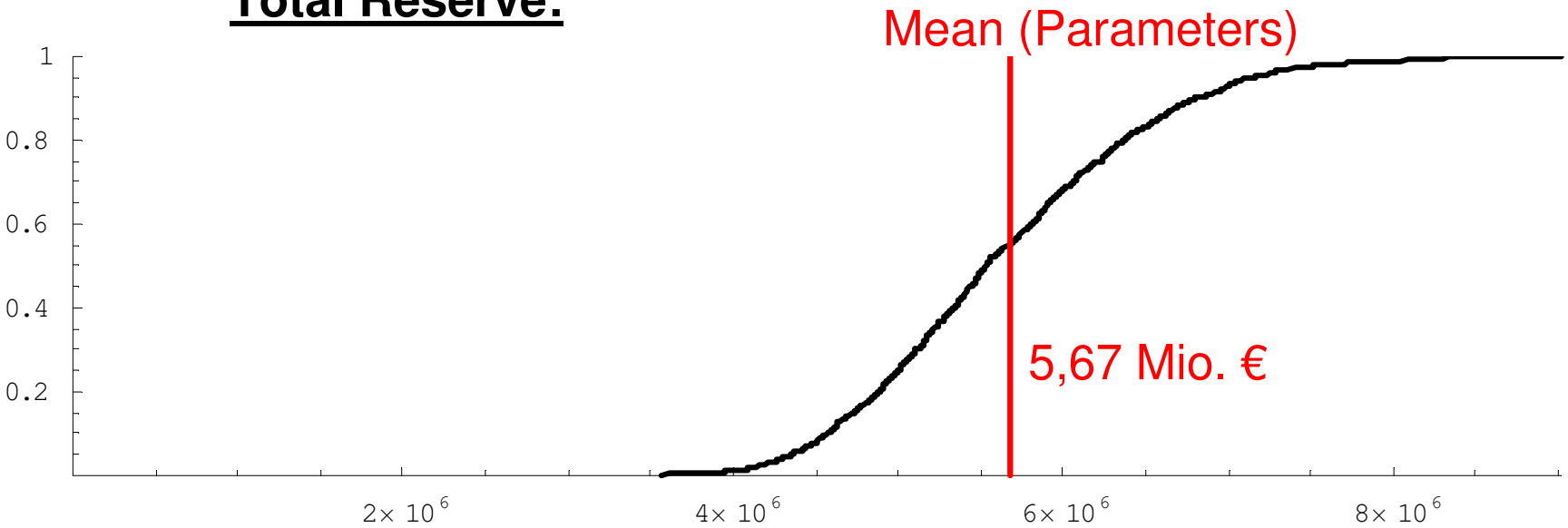
k: years after reporting

MC Simulation - Results

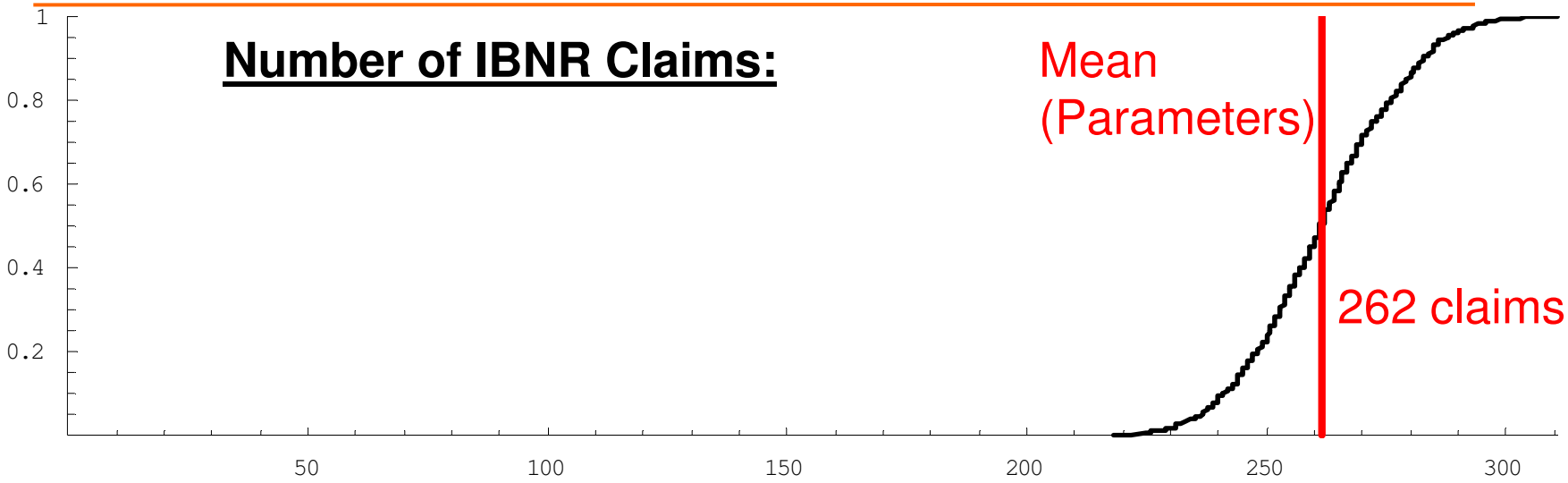
Parameter Set:
Third party liability - others / person injuries
(as seen)
For 15 occurrence years with mean claim number per occ. year:

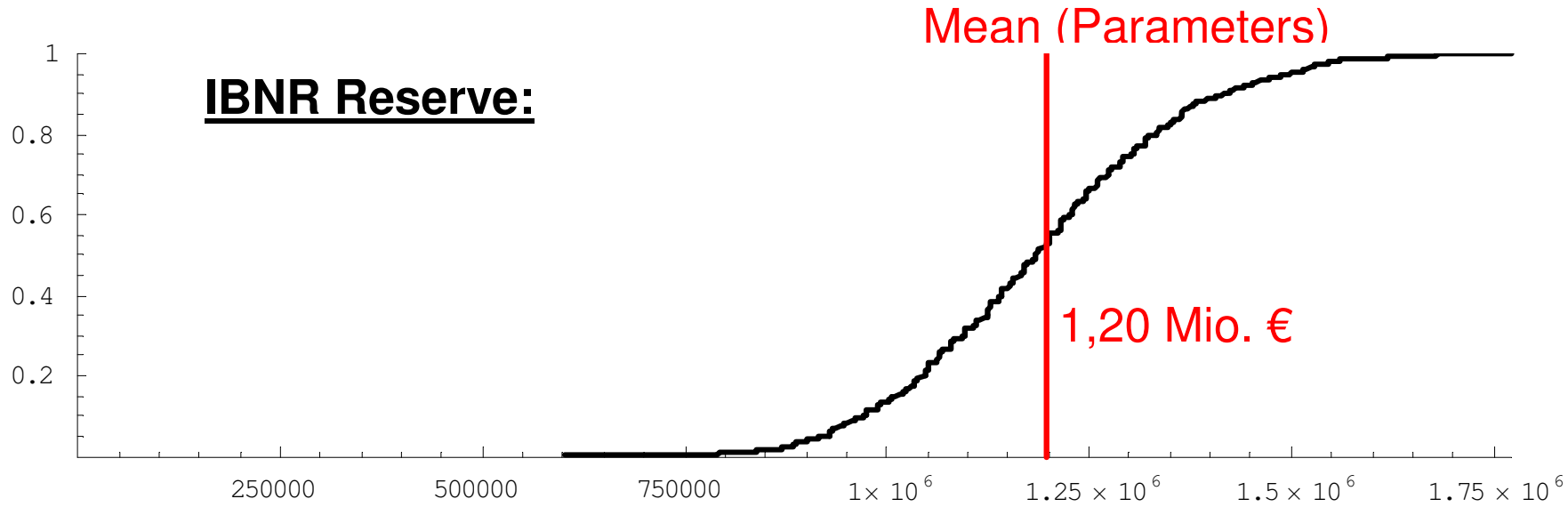
$$\overline{N}_i = 150 \cdot (1.03)^{(i-1)} \qquad \text{for} \quad i = 1,2,\ldots 15$$

Total Reserve:



MC Simulation - Results





Overview and Outlook

- 3D model is extension of classical 2D models
- Calibration of 3D model via standard methods of data analysis
- Expectations and variance of reserves are analytic functions of model parameters
- MC Simulation → reserves distribution → all measures of risk accessible
- Microscopic parameters (claim process) → "global" macroscopic reserve of portfolio

Outlook:

Develop Method to decide on the basis of claim data if 2D model is sufficient or not.

Parameter Estimation: robust estimators, Bayes inference to include expert opinion.

Include a reserve process in the 3D model.

Thank you for your attention !

I look forward to your questions and our discussion!