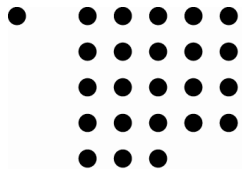




# A three dimensional stochastic Model for Claim Reserving

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Stochastic claims reserving, M. Schiegl



Institut für Versicherungswesen

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# Claim Reserving – P&C Insurance

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Claim reserves must cover all liabilities arising from insurance contracts written in the presence and the past.

Claim reserves are calculated for homogeneous portfolios of insurance contracts

Claim reserves are needed for holistic risk management

→ Realistic modelling of the claim process !

→ Information about the tail of the reserve distribution necessary

# Claim Reserving – Classical Models

One example for typical data structure in the framework of classical model:

The run – off – triangle

Occurr. year	Run – off [years]					
	0	1	2	3	4	5
2003	1.101	1.955	2.523	3.088	3.435	3.583
2004	1.113	2.103	2.774	3.422	3.844	
2005	1.265	2.433	3.233	3.977		
2006	1.490	2.873	3.880			
2007	1.725	3.261				
2008	1.889					

•cumulated payments

•basis for reserve estimation methods

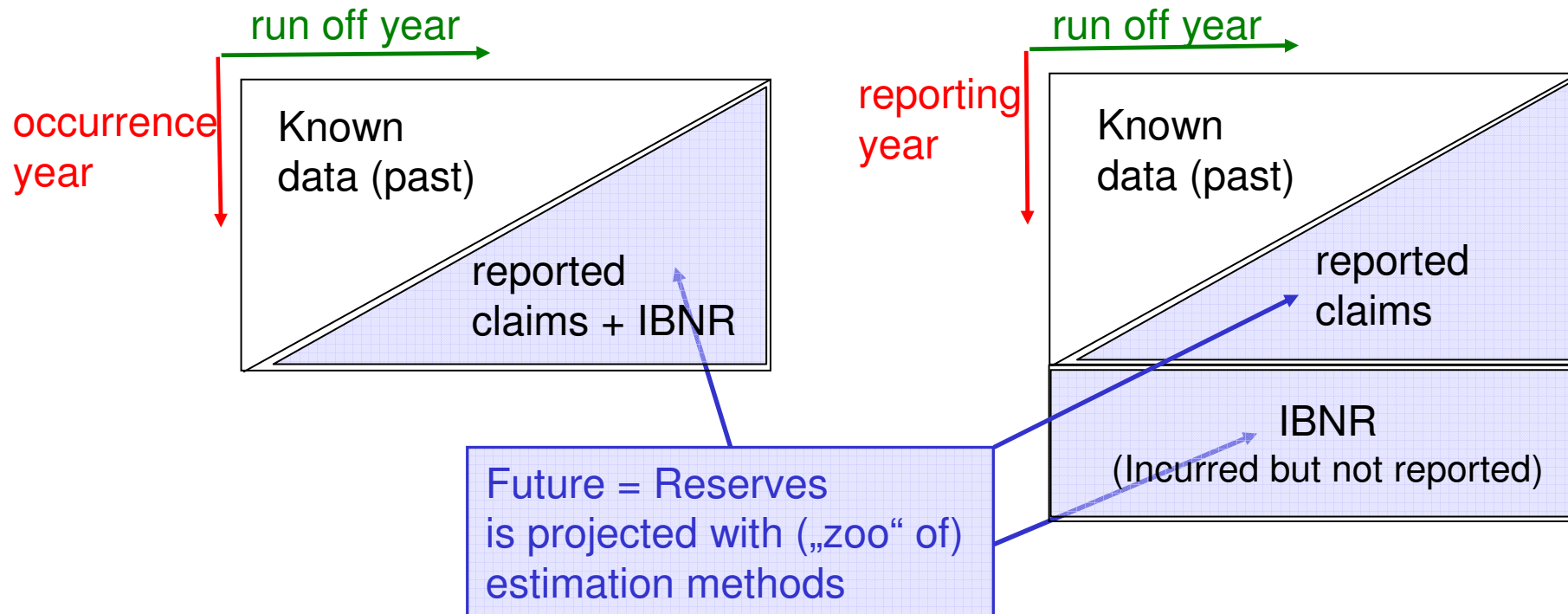
Future payments = reserve

# Claim Reserving – Classical Models

## Classical Models

- operate on a 2D data structure (cumulated data)
- Projection of complex process to a 2D “world”
- calculate only expectation of reserves (Var under restrictive assumptions)
- No information about the tail of the reserve distribution

## Typical data structures for estimation:

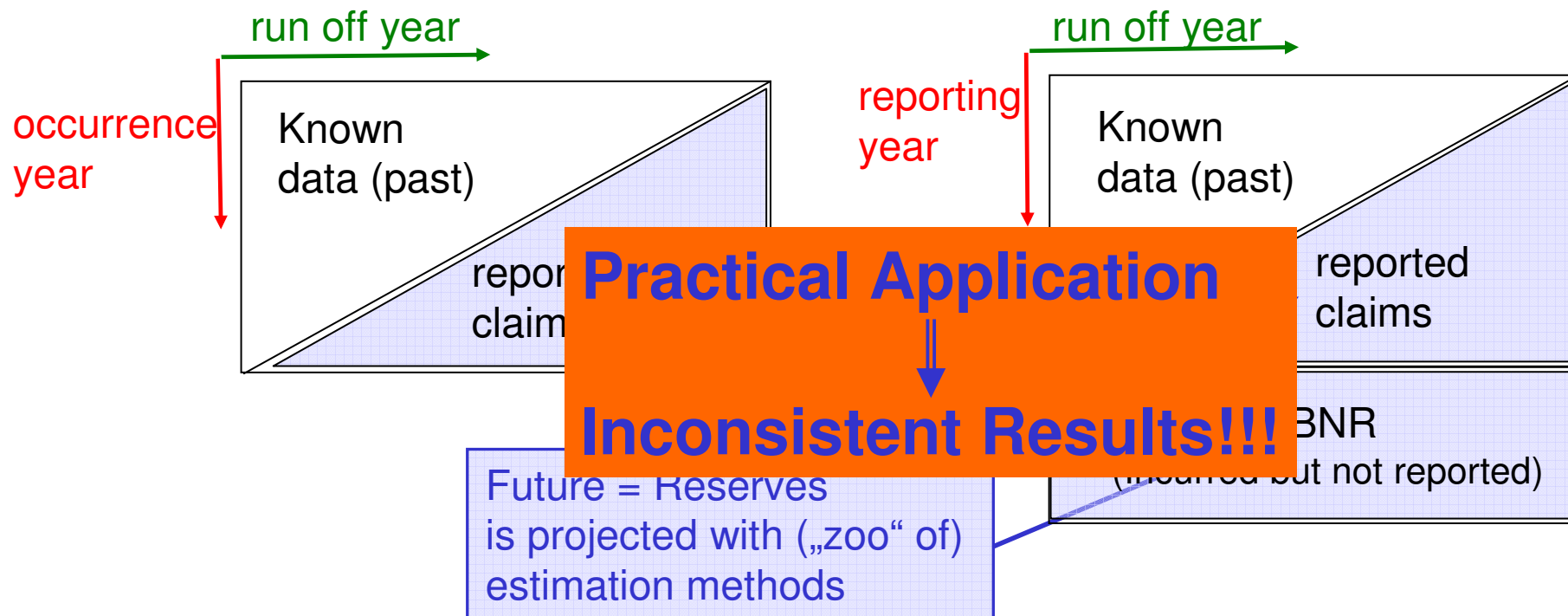


# Claim Reserving – Classical Models

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# The 3D Model

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## Idea

Given a single claim process

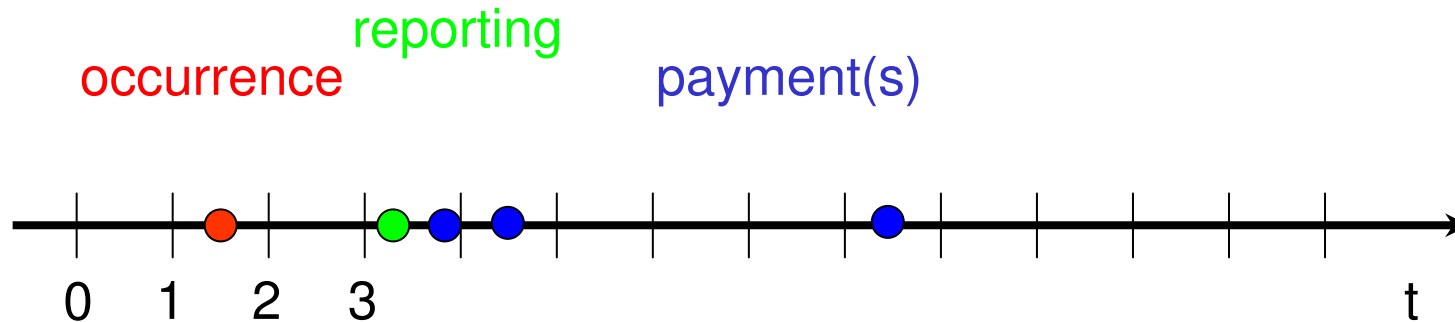
How develop

- Claim Portfolio
- Claim reserves

# The 3D Model – the Claim Process

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## Time Dynamics of (single) claim process



3 dimensional structure:

occurrence (i) // reporting (j) // payment(s) (k)

Examine a portfolio of claims → stochastic model

# The 3D Model

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## Modeled quantities:

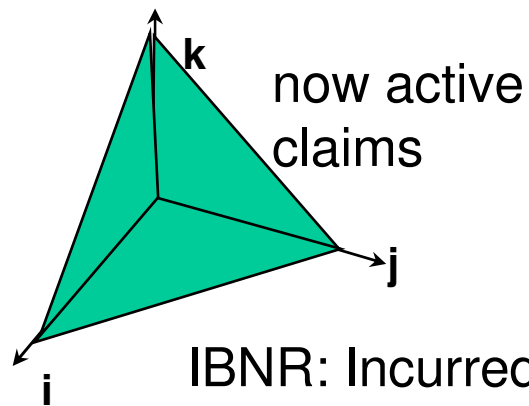
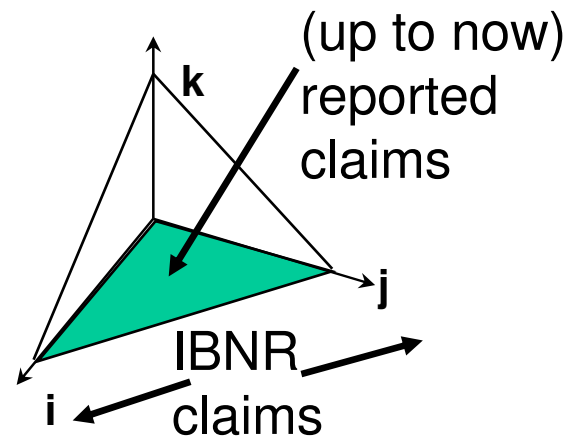
1) Number of active claims  $N_{ijk}$   
occurred in  $i$ th year  
reported  $j$  year after occurrence  
still active  $k$  years after reporting

2) Total claim payments  $Z_{ijk}$   
in  $k$ th year after reporting arising from all active claims with  
occurrence year  $i$  reported with  $j$  years delay.

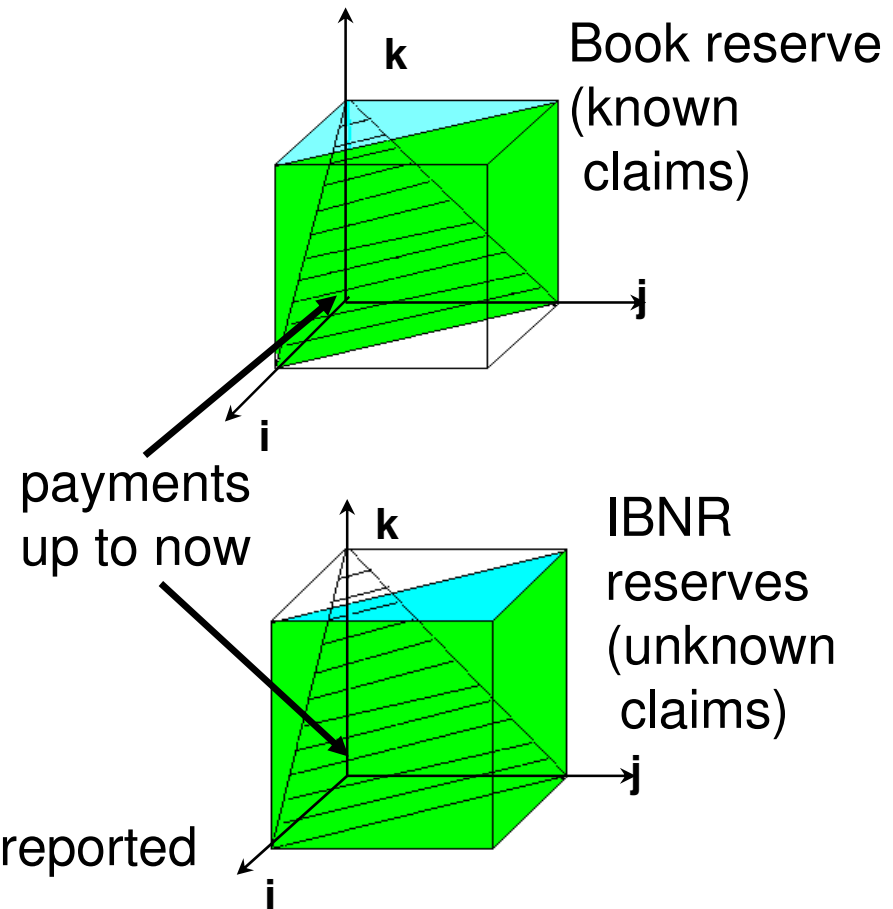
# The 3D Model

## Visualising the model

Number of active claims  $N_{ijk}$



Payments  $Z_{ijk}$



# Connection 3D and 2D Model

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Up to now reported claims:  $N_{ij0}$  with  $i + j \leq I_{\max}$

Number of IBNR Claims:  $\hat{N}_{IBNR} = \sum_{\{i, j | i+j > I_{\max}\}} \hat{N}_{ij0}$

IBNR Reserve:  $\hat{R}_{IBNR} = \sum_{\{i, j, k | i+j+k > I_{\max} \vee i+j < I_{\max}\}} \hat{Z}_{ijk}$

Total Reserve:  $\hat{R}_{total} = \hat{R}_{IBNR} + \sum_{\{i, j, k | i+j > I_{\max}\}} \hat{Z}_{ijk}$

The run off triangle “payments: occurrence versus run off year” :

$$S_{mn}^{(1)} = \sum_{\{j, k | j+k=n \vee m+j+k \leq I_{\max}\}} Z_{mjk}$$

The run off triangle “payments: reporting versus run off year” :

$$S_{mn}^{(2)} = \sum_{\{i, j | i+j=m \vee i+j+n \leq I_{\max}\}} Z_{ijn}$$

## The 3D Model

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### Modeling the number of active claims

$$N_i = \sum_j N_{ij0} \quad (\text{number of claims with occurrence year } =i)$$

$$N_i \propto \text{Poisson}(\bar{N}_i)$$

Claim numbers mult. nom. dist. Along reporting years

with parameters  $\lambda_j \quad j \in \{1, 2, \dots, I_{\max}\}$  and  $\sum_j \lambda_j = 1$

Closing the claims along years after reporting (Binomial process):

$$\eta_k \quad k \in \{1, 2, \dots, K_{\max}\} \text{ with } \eta_0 = 1 \text{ and } \eta_{k+1} < \eta_k$$

$$N_{ijk} \propto \text{BiNom}[N_{ijk-1}, \eta_k]$$

Expectation:  $E[N_{ijk}] = \bar{N}_i \cdot \lambda_j \cdot \eta_k$

# The 3D Model

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## Modeling the total claim payments: Collective Model

Number of single payments

$$Z_{ijk} = \sum_{l=1}^{v_{ijk}} X_l^{(ijk)}$$

Size of single payment

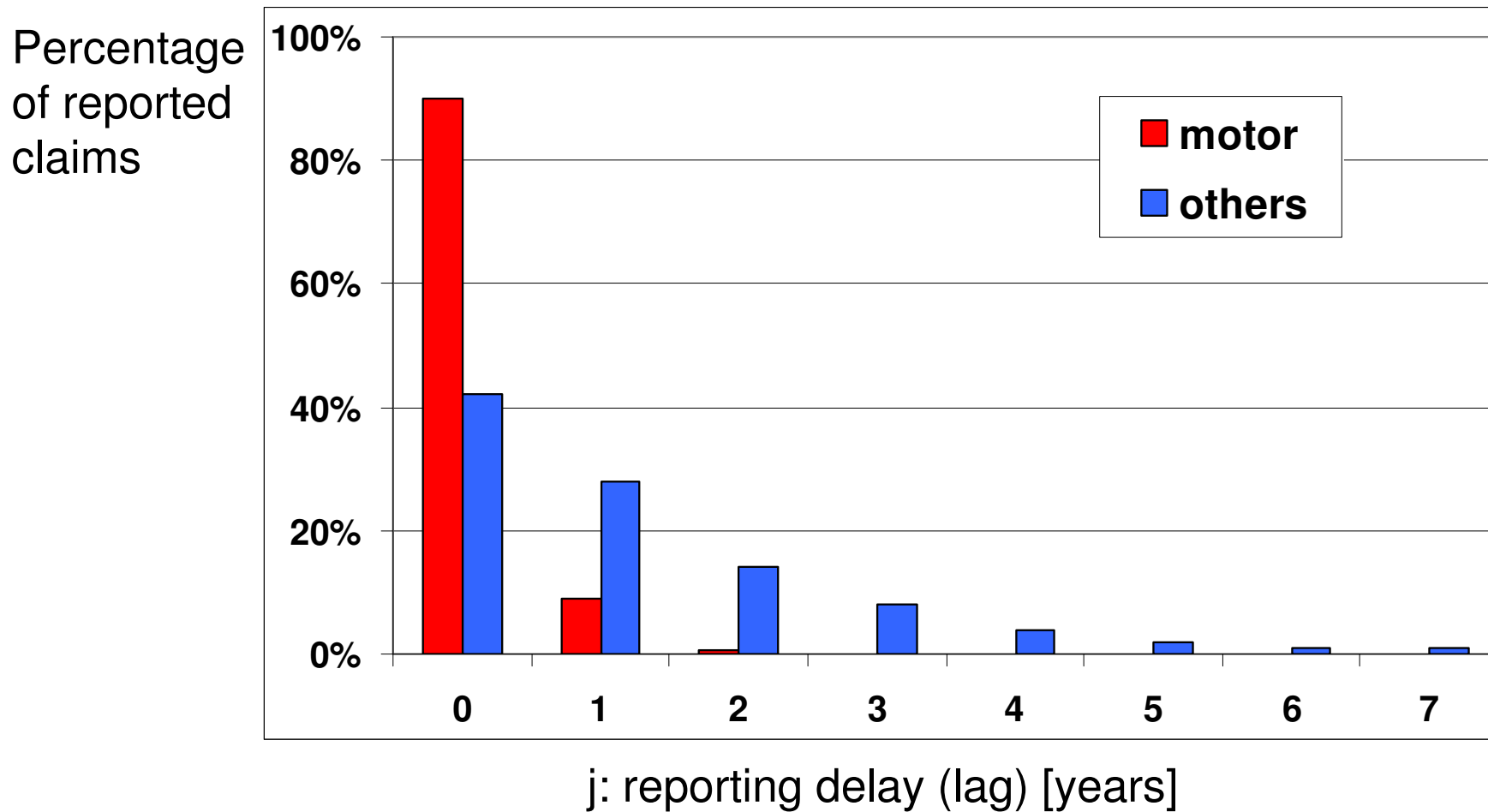
Probability of payment for an active claim

$$v_{ijk} \propto \text{Binom}[N_{ijk}, p_{ik}]$$
$$X_{ijk} \propto \Gamma[EW_{ijk}, Var_{ijk}]$$

# MC Simulation – model parameters

Lag distribution

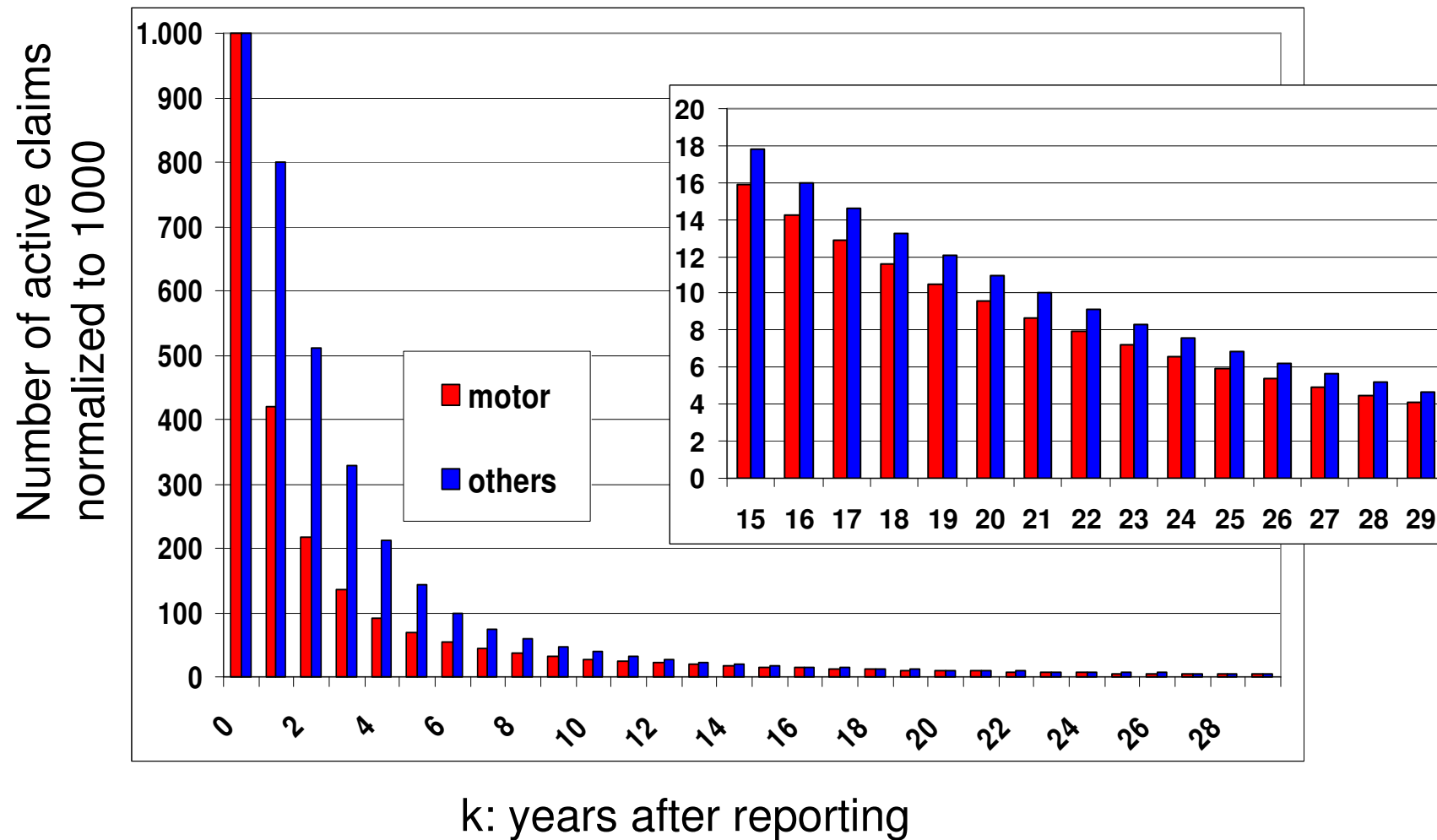
Third party liability / person injuries



# MC Simulation – model parameters

Closing of open claims

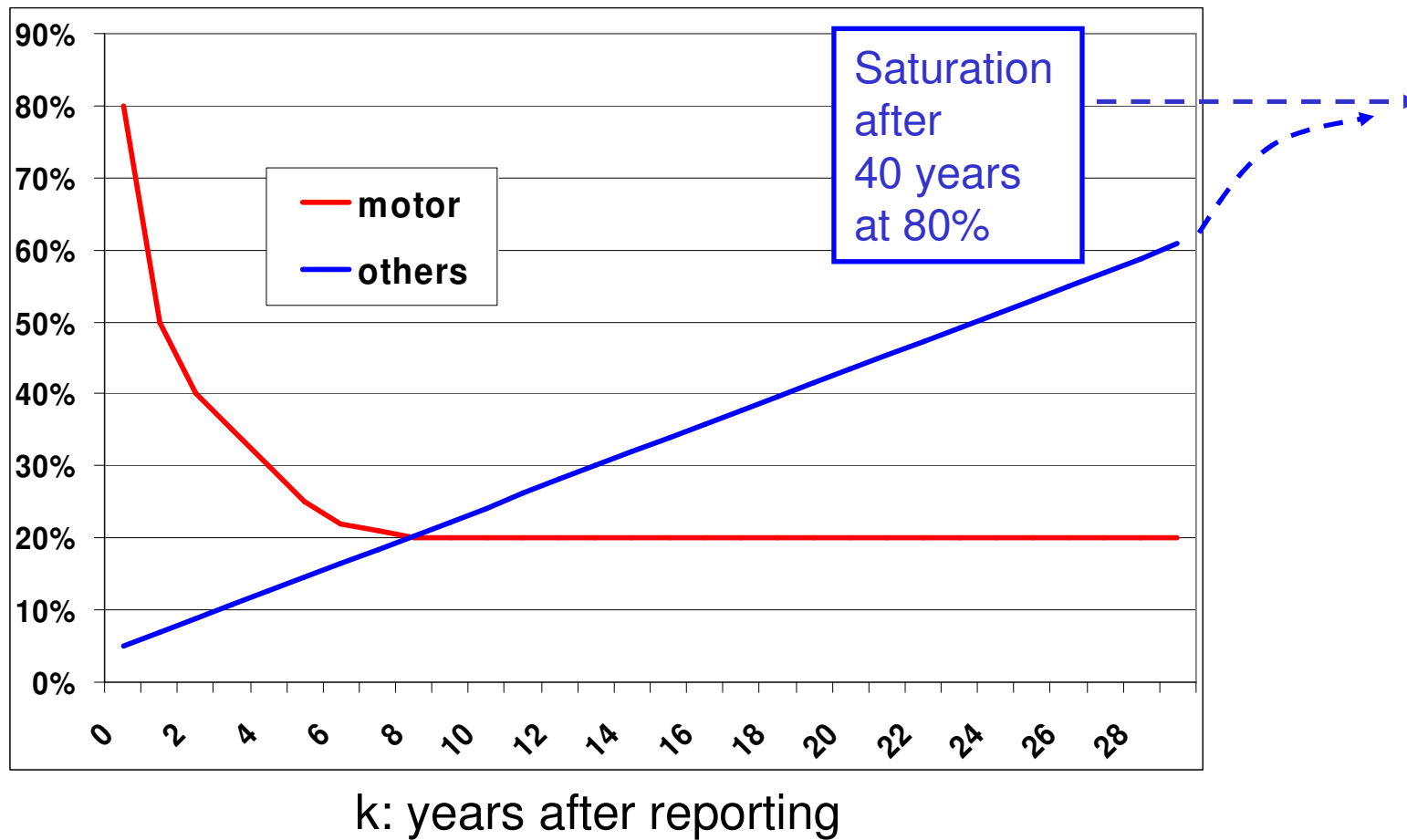
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# MC Simulation – model parameters

Third party liability / person injuries

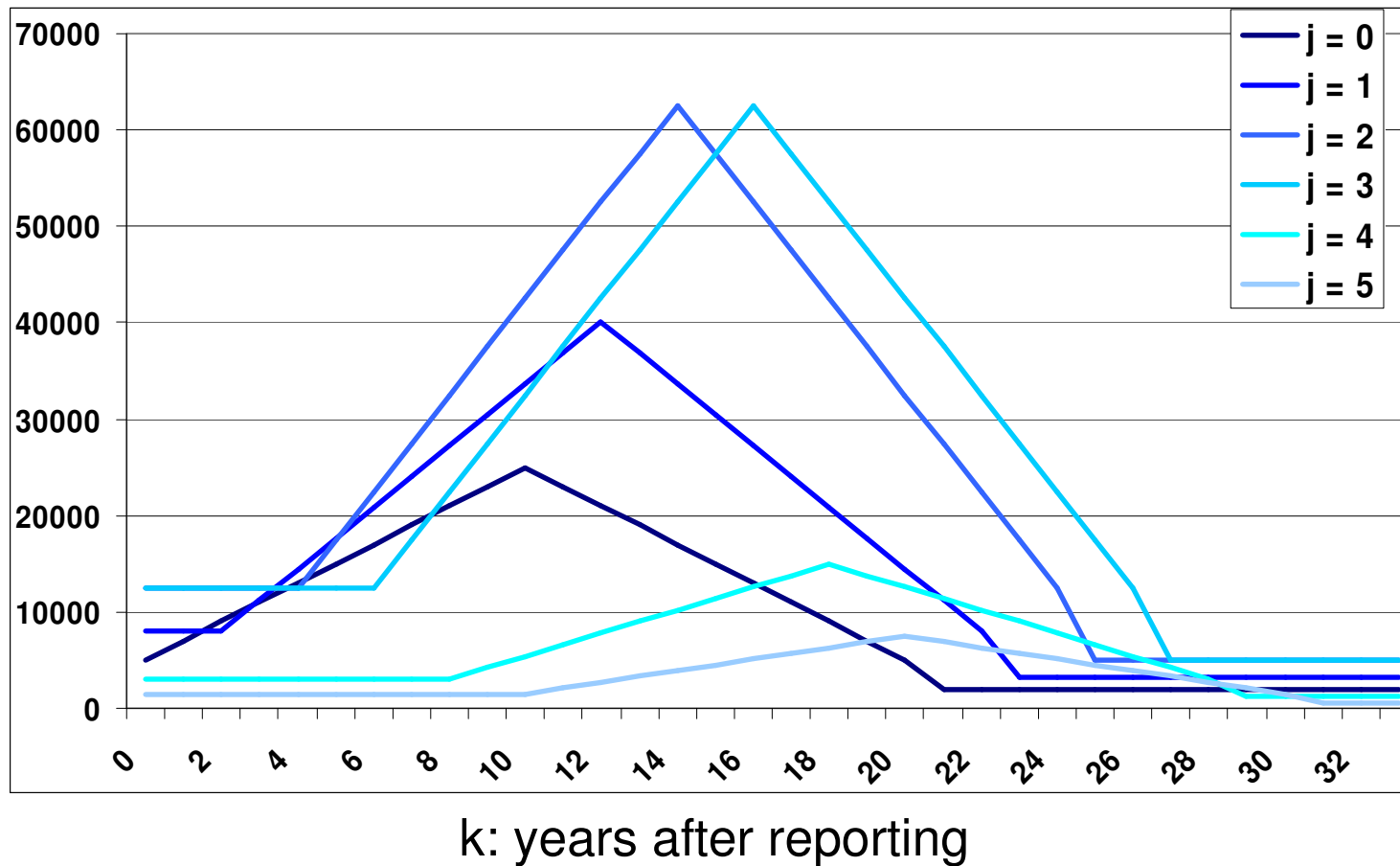
Probability of payment for an active claim



# MC Simulation – model parameters

Third party liability - others / person injuries

Expectation value of single payment  
for claims reported  $j$  years after occurrence



# MC Simulation - Results

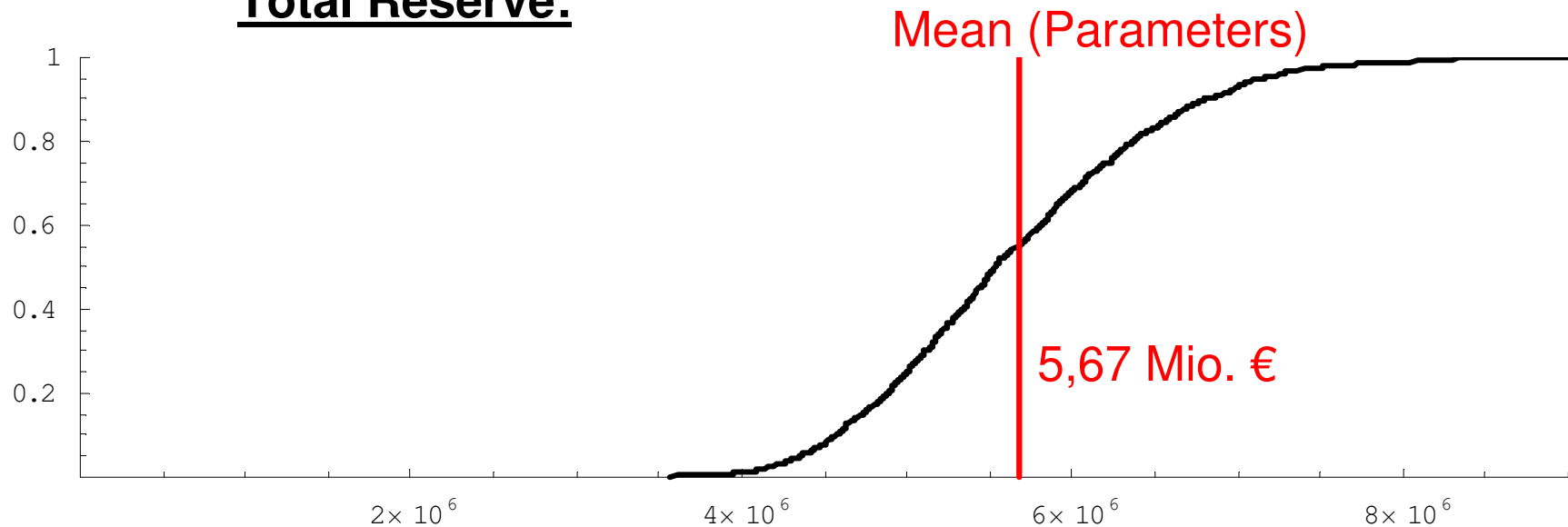
Parameter Set:

Third party liability - others / person injuries  
(as seen)

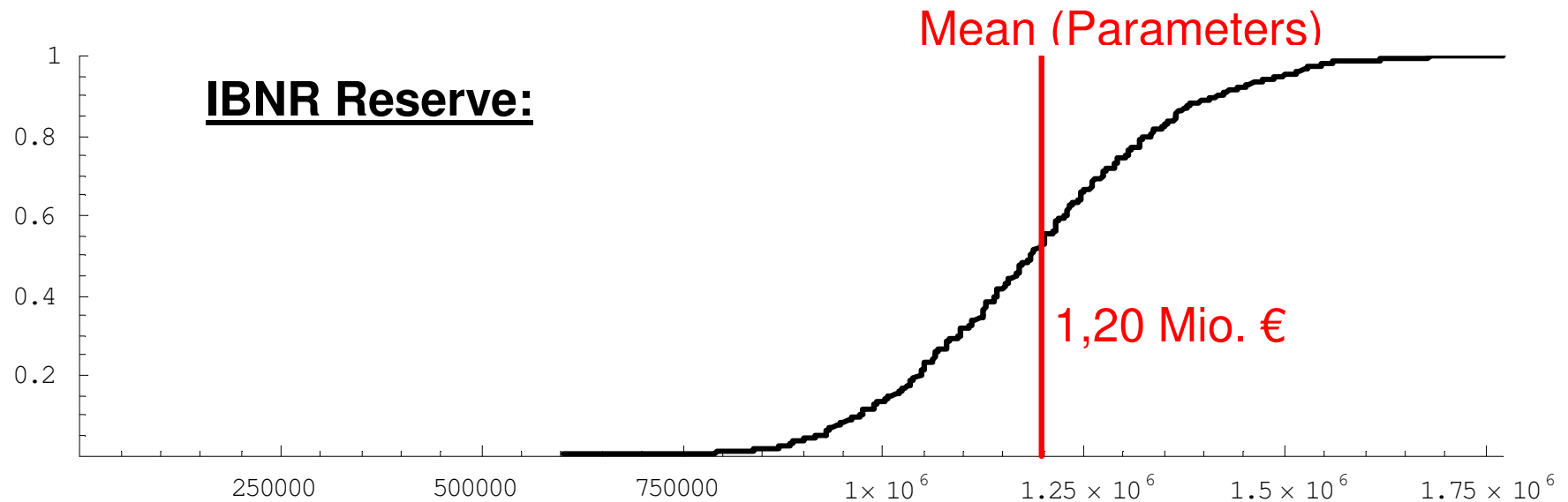
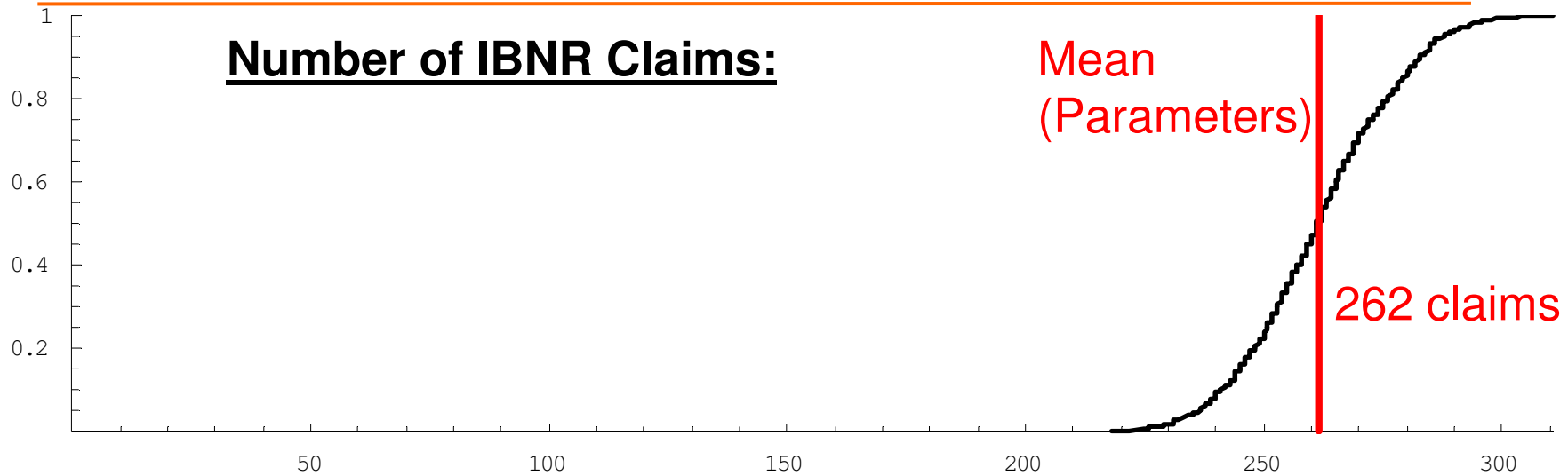
For 15 occurrence years with mean claim number per occ. year:

$$\bar{N}_i = 150 \cdot (1.03)^{(i-1)} \quad \text{for } i = 1, 2, \dots, 15$$

## Total Reserve:



# MC Simulation - Results



# Overview and Outlook

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- 3D model is extension of classical 2D models
- Calibration of 3D model via standard methods of data analysis
- Expectations and variance of reserves are analytic functions of model parameters
- MC Simulation → reserves distribution → all measures of risk accessible
- Microscopic parameters (claim process) → “global” macroscopic reserve of portfolio

## **Outlook:**

Develop Method to decide on the basis of claim data if 2D model is sufficient or not.

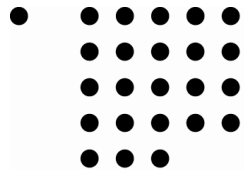
Parameter Estimation: robust estimators, Bayes inference to include expert opinion.

Include a reserve process in the 3D model.



Thank you for your attention !

I look forward to your questions and our discussion!



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