A three dimensional stochastic Model for Claim Reserving

Magda Schiegl

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Claim Reserving – P&C Insurance

Claim reserves must cover all liabilities arising from insurance contracts written in the presence and the past.

Claim reserves are calculated for homogeneous portfolios of insurance contracts

Claim reserves are needed for holistic risk management
→ Realistic modelling of the claim process!
→ Information about the tail of the reserve distribution necessary
Claim Reserving – Classical Models

One example for typical data structure in the framework of classical model:

The run – off – triangle

<table>
<thead>
<tr>
<th>Occurr. year</th>
<th>Run – off [years]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>2003</td>
<td>1.101</td>
</tr>
<tr>
<td>2004</td>
<td>1.113</td>
</tr>
<tr>
<td>2005</td>
<td>1.265</td>
</tr>
<tr>
<td>2006</td>
<td>1.490</td>
</tr>
<tr>
<td>2007</td>
<td>1.725</td>
</tr>
<tr>
<td>2008</td>
<td>1.889</td>
</tr>
</tbody>
</table>

- cumulated payments
- basis for reserve estimation methods

Future payments = reserve
Claim Reserving – Classical Models

Classical Models
operate on a 2D data structure (cumulated data)
→ Projection of complex process to a 2D “world”
→ calculate only expectation of reserves (Var under restrictive assumptions)
→ No information about the tail of the reserve distribution

Typical data structures for estimation:

- Known data (past)
  - run off year
  - occurrence year
- Reported claims + IBNR
- IBNR (Incurred but not reported)
- Future = Reserves is projected with („zoo“ of) estimation methods
Claim Reserving – Classical Models

Classical Models
operate on a 2D data structure (cumulated data)
→ Projection of complex process to a 2D “world”
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Typical data structures for estimation:

- Known data (past)
  - Reporting year
  - Run off year
  - Claim reserving

- Future = Reserves is projected with ("zoo" of) estimation methods

Practical Application
Inconsistent Results!!!
The 3D Model

Idea

Given a single claim process

How develop
  • Claim Portfolio
  • Claim reserves
The 3D Model – the Claim Process

Time Dynamics of (single) claim process

3 dimensional structure:
occurrence (i) // reporting (j) // payment(s) (k)

Examine a portfolio of claims $\rightarrow$ stochastic model
The 3D Model

Modeled quantities:

1) Number of active claims \( N_{ijk} \)
   - occurred in \( i \)th year
   - reported \( j \) year after occurrence
   - still active \( k \) years after reporting

2) Total claim payments \( Z_{ijk} \)
   - in \( k \)th year after reporting arising from all active claims with
   - occurrence year \( i \) reported with \( j \) years delay.
The 3D Model

Visualising the model

Number of active claims $N_{ijk}$

Payments $Z_{ijk}$

- (up to now) reported claims
- IBNR claims
- now active claims
- IBNR: Incurred but not reported
- payments up to now
- Book reserve (known claims)
- IBNR reserves (unknown claims)
Connection 3D and 2D Model

Up to now reported claims: $N_{ij0}$ with $i + j \leq I_{\text{max}}$

Number of IBNR Claims: $\hat{N}_{IBNR} = \sum_{\{i, j \mid i + j > I_{\text{max}}\}} \hat{N}_{ij0}$

IBNR Reserve: $\hat{R}_{IBNR} = \sum_{\{i, j, k \mid i + j + k > I_{\text{max}} \vee i + j < I_{\text{max}}\}} \hat{Z}_{ijk}$

Total Reserve: $\hat{R}_{\text{total}} = \hat{R}_{IBNR} + \sum_{\{i, j, k \mid i + j > I_{\text{max}}\}} \hat{Z}_{ijk}$

The run off triangle “payments: occurrence versus run off year”: $S^{(1)}_{mn} = \sum_{\{j, k \mid j + k = n + m \vee j + k \leq I_{\text{max}}\}} Z_{mjk}$

The run off triangle “payments: reporting versus run off year”: $S^{(2)}_{mn} = \sum_{\{i, j \mid i + j = m \vee i + j + n \leq I_{\text{max}}\}} Z_{ijn}$
The 3D Model

Modeling the number of active claims

\[ N_i = \sum_j N_{ij0} \]  \hspace{1cm} \text{(number of claims with occurrence year =i)}

\[ N_i \sim \text{Poisson}(\overline{N}_i) \]

Claim numbers mult. nom. dist. Along reporting years

with parameters \( \lambda_j \quad j \in \{1,2,\ldots,I_{\text{max}}\} \) and \( \sum_j \lambda_j = 1 \)

Closing the claims along years after reporting (Binomial process):

\[ \eta_k \quad k \in \{1,2,\ldots,K_{\text{max}}\} \text{ with } \eta_0 = 1 \text{ and } \eta_{k+1} < \eta_k \]

\[ N_{ijk} \sim \text{Binom}[N_{ijk-1}, \eta_k] \]

Expectation: \( E[N_{ijk}] = \overline{N}_i \cdot \lambda_j \cdot \eta_k \)
The 3D Model

Modeling the total claim payments: Collective Model

\[ Z_{ijk} = \sum_{l=1}^{v_{ijk}} X_l^{(ijk)} \]

- Number of single payments
- Size of single payment
- Probability of payment for an active claim

\[ \nu_{ijk} \sim Binom[N_{ijk}, p_{ik}] \]
\[ X_{ijk} \sim \Gamma[EW_{ijk}, Var_{ijk}] \]
MC Simulation – model parameters

Lag distribution

Percentage of reported claims

Third party liability / person injuries

j: reporting delay (lag) [years]

0 1 2 3 4 5 6 7

motor
others

0% 20% 40% 60% 80% 100%
MC Simulation – model parameters

Closing of open claims                     Third party liability / person injuries

Number of active claims normalized to 1000

k: years after reporting

motor
others
Probability of payment for an active claim

MC Simulation – model parameters

Saturation after 40 years at 80%

k: years after reporting
Expectation value of single payment for claims reported $j$ years after occurrence

MC Simulation – model parameters
MC Simulation - Results

Parameter Set:
Third party liability - others / person injuries (as seen)
For 15 occurrence years with mean claim number per occ. year:

\[ \bar{N}_i = 150 \cdot (1.03)^{i-1} \quad \text{for} \quad i = 1, 2, \ldots, 15 \]

Total Reserve:

Mean (Parameters)
5.67 Mio. €
MC Simulation - Results

Number of IBNR Claims:

Mean (Parameters)

262 claims

IBNR Reserve:

Mean (Parameters)

1,20 Mio. €
Overview and Outlook

• 3D model is extension of classical 2D models

• Calibration of 3D model via standard methods of data analysis

• Expectations and variance of reserves are analytic functions of model parameters

• MC Simulation → reserves distribution → all measures of risk accessible

• Microscopic parameters (claim process) → “global” macroscopic reserve of portfolio

Outlook:

Develop Method to decide on the basis of claim data if 2D model is sufficient or not.

Parameter Estimation: robust estimators, Bayes inference to include expert opinion.

Include a reserve process in the 3D model.
Thank you for your attention!

I look forward to your questions and our discussion!