The Insurance Risk in the SST and in Solvency II: Modeling and Parameter Estimators

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Introduction

- **SST and Solvency II**
  - common goal: to install a risk based solvency regulation
  - solvency capital required (SCR) should depend on the risks a company has on its book
- **SST**
  2004: standard SST model developed and first field test
  2008: all Swiss companies have to calculate the SST figures
  2011: SST SCR will be in force
- **Solvency II**
  2007: SII Framework Directive Proposal adopted by the EU Commission
  2008: 4th quantitative impact study
  2012: "original" schedule to put the regulation into force
  schedule under discussion
- **Subject of this presentation: non-life insurance risk**
  modeling and parameter estimators

The Insurance Risk

- **Non-Life Insurance Risk**
  non-life insurance risk = next years technical result

$$TR = P - K - C^{CY} - C^{PY}$$
$$\simeq \underbrace{E\left[P - K - E\left[C^{CY}\right]\right]}_{\text{expected technical result}} - \left(C^{CY} - E\left[C^{CY}\right]\right) - C^{PY}$$

where
$$P = \text{earned premium,}$$
$$K = \text{administrative costs,}$$
$$C^{CY} = \text{total claim amount current year (CY),}$$
$$C^{PY} = \text{total claim amount previous years (PY)}$$
$$= -CDR$$ (CDR = claims development result)

- segmented into lines of business (lob) *i=1,2,....,I ;*

Modeling in SST: Insurance Risk

- **CY claim amount**
  C^{CY} is split into $C^{CY,n}$ "normal claim" amount
  and $C^{CY,b}$ "big claim amount"

- **analytical insurance risk model**
  modeling of $(C^{CY,n}, C^{CY,b}, C^{PY})$
  describes adequately reality except for extraordinary situations

- **scenarios**
  complements analytical model to take into account extraordinary situations;
  to take into account extraordinary situations;
  by means of scenrios SC_k , *k=1,2,...,K,* characterised by face amounts c_k with occurrence probabilities p_k .

Modeling in SST: Insurance Risk

- **Risk measure in the SST**
  99% expected shortfall

- **SCR for insurance risk**

$$SCR_{ins} = ES_{99\%}\left[-TR\right].$$

Modeling in SST: normal claim amount CY

- **Model assumption**

  Conditional on $\Theta_i^T = \left(\Theta_{1i}, \Theta_{2i}\right)$,

  $C_i^{CY,n}$ is compound Poisson;

  Θ_{1i}, Θ_{2i} are random factors with expected value 1 indicating how much next year's "true underlying" claim frequency and the "true underlying" expected claim severity will deviate from their a priori expected values due to things like weather conditions, change in economic environment, change in legislation, etc.

  $\Theta_i^T = \left(\Theta_{1i}, \Theta_{2i}\right)$ is the "risk characteristics" of next year for lob *i*

Modeling in SST: normal claim amount CY

- $X_i = \frac{C_i^{CY,n}}{\tilde{P}_i}$, where $\tilde{P}_i = E\left[C_i^{CY,n}\right]$ pure risk premium;

- **variance structure**
  from model assumptions follows that

$$\sigma_i^2 := \mathbf{Var}\left(X_i\right) = \sigma_{i,param}^2 + \frac{\sigma_{i,fluct}^2}{\nu_i},$$

where $\sigma_{i,param}^2 \simeq \mathbf{Var}\left(\Theta_{1i}\right) + \mathbf{Var}\left(\Theta_{2i}\right),$

$$\sigma_{i,fluct}^2 = CoVa^2\left(Y_i^{(\upsilon)}\right) + 1.$$

and where

$CoVa\left(Y_i^{(\upsilon)}\right) =$ the coefficient of variation of the claim severities,

$\nu_i = w_i \lambda^{(i)} =$ a prori expected number of claims.

Modeling in SST: normal claim amount CY

- aggregation over lob

the variance of $X_\bullet = C_\bullet^{CY,n} / \tilde{P}_\bullet$ is calculated by assuming

a correlation matrix $\mathbf{R}_{CY} = \mathtt{Corr}\left(\mathbf{X}, \mathbf{X}^T\right)$ $\left(\mathbf{R}_{CY}\left(i, j\right) = \mathtt{Corr}\left(X_i, X_j\right)\right)$

=> $\sigma^2 := \mathtt{Var}\left(X_\bullet\right) = \frac{1}{\tilde{P}_\bullet^{\,2}}\left(\mathbf{W}_{CY}^T \cdot \mathbf{R}_{CY} \cdot \mathbf{W}_{CY}\right),$

where $\mathbf{W}_{CY} = \left(\tilde{P}_1\sigma_1, \tilde{P}_2\sigma_2, \ldots, \tilde{P}_I\sigma_I\right)^T.$

Modeling in SST: big claim amount CY

Model Assumptions

i) for each lob *i* the big claim amount $C_i^{CY,b}$ is compound Poisson-distribution with (essentially) Pareto-distributed claim sizes

ii) $C_i^{CY,b},\ i=1,2,\ldots,I$ are independent

=> $C_\bullet^{CY,b} = \sum_{i=1}^{I} C_i^{CY,b}$ is again compound Poisson with

$$\lambda = \lambda_\bullet^b = \sum_{i=1}^{I} \lambda_i^b\ , \quad F = \sum_{i=1}^{n} \frac{\lambda_i^b}{\lambda_\bullet^b} F_i .$$

Modeling in SST: normal and big claim amount CY

- lob and standard parameters normal and big claim amount CY

lob	description
1	motor liability
2	motor hull
3	property
4	general liability
5	workers compensation (UVG)
6	corporate accident without UVG
7	corporate health
8	individual health
9	marine
10	aviation
11	credit and surety
12	legal protection
13	others

big claims CY

lob	Pareto parameter
1	2.50
2	1.85
3	1.40
4	1.80
5	2.00
6	2.00
7	3.00
8	3.00
9	1.50
10	
11	0.75
12	
13	1.50

standard parameters for CY- risks

lob	σ_{par}	CoVa (claim size)
1	3.50%	7.00
2	3.50%	2.50
3	5.00%	5.00
4	3.50%	8.00
5	3.50%	7.50
6	4.75%	4.50
7	5.75%	2.50
8	5.75%	2.25
9	5.00%	6.50
10	5.00%	2.50
11	5.00%	5.00
12	4.50%	2.25
13	4.50%	5.00

Correlation matrix for CY- year risks

lob	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1	0.5	0	0.25	0.25	0.25	0	0	0	0	0	0	0
2	0.5	1	0.25	0	0	0	0	0	0	0	0	0	0
3	0	0.25	1	0.25	0	0	0	0	0	0	0	0	0
4	0.25	0	0.25	1	0	0	0	0	0	0	0	0	0
5	0.25	0	0	0	1	0.5	0.5	0	0	0	0	0	0
6	0.25	0	0	0	0.5	1	0.5	0	0	0	0	0	0
7	0	0	0	0	0.5	0.5	1	0.25	0	0	0	0	0
8	0	0	0	0	0	0	0.25	1	0	0	0	0	0
9	0	0	0	0	0	0	0	0	1	0	0	0	0
10	0	0	0	0	0	0	0	0	0	1	0	0	0
11	0	0	0	0	0	0	0	0	0	0	1	0	0
12	0	0	0	0	0	0	0	0	0	0	0	1	0
13	0	0	0	0	0	0	0	0	0	0	0	0	1

Modeling in SST: claim amount PY

- **Reserve risk (claim amount PY)**

  $L_i =$ outstanding claims liabilities at 1.1. for lob i,
  $R_i =$ best estimate of L_i per 1.1. = best estimate reserve,
  $\tilde{R}_i = PA_i^{PY} + R_i^{31.12.,PY}$ = best estimate of L_i per 31.12.,
  $Y_i = \dfrac{\tilde{R}_i}{R_i}$.

  note that $C_i^{PY} = \tilde{R}_i - R_i$.

- **Model Assumptions**
  it is assumed that

$$\tau_i^2 := \mathbf{Var}(Y_i) = \tau_{i,param}^2 + \frac{\tau_{i,fluct}^2}{R_i}$$

Modeling in SST: claim amount PY

- **current standard parameters for PY-risks**

lob	description
1	motor liability
2	motor hull
3	property
4	general liability
5	workers compensation (UVG)
6	corporate accident without UVG
7	corporate health
8	individual health
9	marine
10	aviation
11	credit and surety
12	legal protection
13	others

standard parameters for PY-risks

lob	T_{par}	T_{fluct}
1	3.5%	2.5%
2	3.5%	20.0%
3	3.0%	15.0%
4	3.5%	4.0%
5	2.0%	4.0%
6	3.0%	5.0%
7	3.0%	7.0%
8	5.0%	0.0%
9	5.0%	25.0%
10	5.0%	20.0%
11	5.0%	15.0%
12	5.0%	0.0%
13	5.0%	50.0%

Modeling in SST: claim amount PY

- **aggregation over lob**

  the variance of $Y_\bullet = \tilde{R}_\bullet / R_\bullet$ is calculated by assuming

  a correlation matrix $\mathbf{R}_{PY} = \mathbf{Corr}\left(\mathbf{Y}, \mathbf{Y}^T\right)$ $(\mathbf{R}_{PY}(i,j) = \mathbf{Corr}\left(Y_i, Y_j\right))$

- **current standard SST assumption**

  Y_i, $i=1,2,...,I$, are independent, i.e. $\mathbf{R_{PY}}$ **=** identity matrix.

  => $$\tau^2 := \mathbf{Var}\left(Y\right) = \frac{1}{R_\bullet^2} \sum_{i=1}^{I} R_i^2 \tau_i^2$$

- **Discussion on correlation assumption**

  current standard SST assumption is questionable;
  reason: calendar year effects affecting several lob simultaneously;
  an obvious example of is claims inflation.

Modeling in SST: combined normal claim amount CY + claim amount PY

- **Notations**

$$S_i = C_i^{CY,n} + \tilde{R}_i, \quad Z_i = \frac{C_i^{CY,n} + \tilde{R}_i}{\tilde{P}_i + R_i} = \frac{\tilde{P}_i X_i + R_i Y_i}{\tilde{P}_i + R_i}, \quad V_i = \tilde{P}_i + R_i,$$

$$S_\bullet = C_\bullet^{CY,n} + \tilde{R}_\bullet, \quad Z_\bullet = \frac{C_\bullet^{CY,n} + \tilde{R}_\bullet}{\tilde{P}_\bullet + R_\bullet} = \frac{\tilde{P}_\bullet X_\bullet + R_\bullet Y_\bullet}{\tilde{P}_\bullet + R_\bullet}, \quad V_\bullet = \tilde{P}_\bullet + R_\bullet \; .$$

- **Model assumption**

  It is assumed that $S_\bullet$ is lognormal distributed with

$$E[S_\bullet] = \tilde{P}_\bullet + R_\bullet, \quad \mathbf{Var}(S_\bullet) = \begin{pmatrix} \mathbf{W}_{CY} \\ \mathbf{W}_{PY} \end{pmatrix}^T \cdot \mathbf{R} \cdot \begin{pmatrix} \mathbf{W}_{CY} \\ \mathbf{W}_{PY} \end{pmatrix}.$$

Modeling in SST: combined normal claim amount CY + claim amount PY

- **Correlation matrices:**

$$\mathbf{R} = \mathtt{Corr}\left(\begin{pmatrix} \mathbf{X} \\ \mathbf{Y} \end{pmatrix}, \begin{pmatrix} \mathbf{X} \\ \mathbf{Y} \end{pmatrix}^T \right) = \begin{pmatrix} \mathbf{R}_{CY} & \mathbf{R}_{CY,PY} \\ \mathbf{R}_{CY,PY} & \mathbf{R}_{PY} \end{pmatrix},$$

where $\mathbf{R}_{CY,PY} = \mathtt{Corr}\left(\mathbf{X}, \mathbf{Y}^T\right)$

- **current standard SST assumption**
current year claims and previous year claims are uncorrelated, that is

$$\mathbf{R}_{CY,PY} = \mathbf{0}.$$

$$\Rightarrow \mathtt{Var}\left(Z_\bullet\right) = \frac{\tilde{P}_\bullet^2 \sigma^2 + R_\bullet^2 \tau^2}{\left(\tilde{P}_\bullet + R_\bullet\right)^2}$$

Modeling in SST: correlation CY and PY; convolution with big claims

- **Discussion on correlation assumption between CY and PY**
  current standard SST assumption is questionable;
  reason: calendar year effects affecting the CY-year claim amount as the previous years' claim amounts of several lob simultaneously;
  an example of such a calendar year effect is claims inflation;

- **Convolution with big claim**
  The distribution of $\tilde{T} = C_{\bullet}^{CY,n} + C_{\bullet}^{CY,b} + C_{\bullet}^{PY}$ can be calculated by convoluting the lognormal distribution of $C_{\bullet}^{CY,n} + C_{\bullet}^{PY}$ with the compound Poisson distribution of $C_{\bullet}^{CY,b}$

  => distribution $\tilde{F}$ before scenarios

Modeling in SST: scenarios

- **Model Assumptions:**
  Scenarios SC_k^{ins} , $k=1,2,\ldots,K$, are characterized by face amounts c_k and occurrence probabilities p_k.
  It is assumed that only one of the scenarios can occur within the next year (mutual exclusion of scenarios).

  Remark:
  The "exclusion assumption" is not such a big restriction as it seems, since one is free in defining the scenarios. One can always define new scenarios combining two already existing scenarios.

- **Distribution after scenarios**
  distribution function of $T = C_{\bullet}^{CY,n} + C_{\bullet}^{CY,b} + C_{\bullet}^{PY} + SC_{\bullet}^{ins}$:

$$F(x) = \sum_{k=0}^{K} p_k \tilde{F}(x - c_k), \text{ where } p_0 = 1 - \sum_{k=1}^{K} p_k \text{ and } c_0 = 0.$$

Modeling in SII: Insurance risk

- **General**

  to compare with SST: only one region, company is working in;
  SCR for non-life insurance risk is named SCR_{nl} in solvency II (SII).

  SII also considers *CY-risk* (named *premium risk*) and *PY-risk* called *reserve risk*.
  For *CY-risk* : no distinction is made between normal and big claims.

  In addition: *CAT-risks,* mainly thought for natural peril risks.
  Characterized by face amounts similar to the scenario risks in the SST.

  SII provides formulas how to calculate the *SCR* and not models. Models presented here = models leading to the formulas in SII to calculate the *SCR .*

Modeling in SII: Insurance risk

- **Notation**

$$X_i = \frac{C_i^{CY}}{P_i} \text{ (loss ratio CY)}, \quad Y_i = \frac{\tilde{R}_i}{R_i},$$

$$\sigma_i^2 = \mathbf{Var}(X_i), \qquad \tau_i^2 = \mathbf{Var}(Y_i),$$

where

$$P_i = \text{premium}$$
$$R_i = \text{reserve per 1.1.}$$
$$\tilde{R}_i = \text{"a posteriori reserves" per 31.12. of } L_i.$$

Modeling in SII: premium (CY) risk

- **calculation premium risk per lob**

$$\sigma_i = \sqrt{\alpha_i \cdot \sigma^2_{i,ind} + \left(1-\alpha_i\right) \cdot \sigma^2_{i,M}},$$

where $\alpha_i =$ credibility weight,

$\sigma_{i,M} =$ standard "market" parameter,

$$\sigma^2_{i,ind} = \frac{1}{n_i - 1} \sum_{j=1}^{n_i} \frac{P_{ij}}{P_{i\bullet}} (X_{ij} - \overline{X_i})^2 \text{ with } \overline{X_i} = \sum_{j=1}^{n_i} \frac{P_{ij}}{P_{i\bullet}} X_{ij}.$$

- **Model assumption CY-risk (premium risk)**
  Neither $\sigma^2_{i,M}$ nor the credibility weight depend on the size of the company
  => model assumption: $\mathbf{Var}\left(X_i\right) = \sigma_i^2$.

- **Model assumption PY-risk (reserve risk)**
  model assumption: $\mathbf{Var}\left(y_i\right) = \tau_i^2$.

Modeling in SII: premium + reserve risk

- **premium + reserve risk per lob**

$$Z_i = \frac{1}{V_i}\left(P_i X_i + R_i Y_i\right), \text{ where } V_i = P_i + R_i.$$

$$\text{Assumption: } \mathbf{Corr}\left(X_i, Y_i\right) = \rho_{CY,PY} = 50\%$$

$$\Rightarrow \varphi_i := \sqrt{\mathbf{Var}\left(Z_i\right)} = \sqrt{\frac{\left(P_i\sigma_i\right)^2 + 2\rho_{CY,PY} P_i \sigma_i R_i \tau_i + \left(R_i \tau_i\right)^2}{V_i^2}}.$$

- **correlation and aggregation**

$$\text{assumption: } \mathbf{Corr}\left(Z_i, Z_j\right) = \rho_{ij}, \ \rho_{ij} \text{ given standard parameters}$$

$$Z_\bullet = \sum_{i=1}^{I} \frac{V_i}{V_\bullet} Z_i \Rightarrow \varphi^2 = \mathbf{Var}\left(Z_\bullet\right) = \sum_{i,j=1}^{I} \frac{V_i V_j \varphi_i \varphi_j}{V_\bullet^2} \rho_{ij},$$

Modeling in SII: premium + reserve risk

- **implications and discussion of correlation assumptions**

  $\mathbf{Corr}(Z_i, Z_j) = \rho_{ij}, \; \mathbf{Corr}(X_i, Y_i) = \rho_{CY,PY} = 50\%.$

  must hold for any company

  => $\mathbf{Corr}(X_i, X_j) = \mathbf{Corr}(Y_i, Y_j) = \mathbf{Corr}(Z_i, Z_j) = \rho_{ij},$

  – correlation between lob result from calendar year effects affecting several lob simultaneously. To assume the same correlation matrix for **X** and for **Y** is questionable, since the calendar year effect for CY- and PY-risks might not be the same or might have a different impact.

  – $\mathbf{Corr}(X_i, Y_j)$ for $i \neq j$ depend on the volumes and difficult to interpret

Modeling in SII: formula to calculate SCR

- **lob and parameters**

correlation matrix for Z-variables (combined CY and PY risk)

lob	1	2	3	4	5	6	7	8	9	10	11	12
1	1	0.50	0.50	0.25	0.50	0.25	0.50	0.25	0.50	0.25	0.25	0.25
2	0.50	1	0.25	0.25	0.25	0.25	0.50	0.50	0.50	0.25	0.25	0.50
3	0.50	0.25	1	0.25	0.25	0.25	0.25	0.50	0.50	0.25	0.25	0.50
4	0.25	0.25	0.25	1	0.25	0.25	0.25	0.50	0.50	0.50	0.25	0.50
5	0.50	0.25	0.25	0.25	1	0.50	0.50	0.25	0.50	0.25	0.50	0.25
6	0.25	0.25	0.25	0.25	0.50	1	0.50	0.25	0.50	0.25	0.25	0.25
7	0.50	0.50	0.25	0.25	0.50	0.50	1	0.25	0.50	0.25	0.50	0.25
8	0.25	0.50	0.50	0.50	0.25	0.25	0.25	1	0.50	0.50	0.25	0.25
9	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	1	0.25	0.25	0.50
10	0.25	0.25	0.25	0.50	0.25	0.25	0.25	0.50	0.25	1	0.25	0.25
11	0.25	0.25	0.25	0.25	0.50	0.25	0.50	0.25	0.25	0.25	1	0.25
12	0.25	0.50	0.50	0.50	0.25	0.25	0.25	0.25	0.50	0.25	0.25	1

lob	description
1	motor, third party liability
2	motor, other classes
3	marine, aviation & transport (MAT
4	fire and other damage to property
5	third-party liability
6	credit and suretyship
7	legal expenses
8	assistance
9	miscellaneous non-life insurance
10	NP reins property
11	NP reins casualty
12	NP reins MAT

standard parameters

lob	CY-risk (premium) σ	PY-risk (reserve) τ	max historical years m_n
1	9%	12%	15
2	9%	7%	5
3	13%	10%	10
4	10%	10%	5
5	13%	15%	15
6	15%	15%	15
7	5%	10%	5
8	8%	10%	5
9	11%	10%	10
10	15%	15%	5
11	15%	15%	15
12	15%	15%	10

credibility weights α_i for σ^2

historical years available

m_n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
5	0	0	0.64	0.72	0.79	-	-	-	-	-	-	-	-	-	-
10	0	0	0	0	0.64	0.69	0.72	0.74	0.76	0.79	-	-	-	-	-
15	0	0	0	0	0	0	0.64	0.67	0.69	0.71	0.73	0.75	0.76	0.78	0.79

Modeling in SII: premium + reserve risk

- **formula for SCR premium + reserve risk**

$$SCR_{pr+res} = V_\bullet \left(\frac{\exp\left(\Phi^{-1}(0.995)\cdot\sqrt{\log(\varphi^2+1)}\right)}{\sqrt{\varphi^2+1}} - 1 \right)$$

$$= V_\bullet VaR_{0.995}^{mean}(\Psi)$$

where Ψ = logormal distributed r.v. with $E[\Psi]=1$ and $\mathbf{Var}(\Psi)=\varphi^2$,

$VaR_{0.995}^{mean}(\Psi) = VaR_{0.995}(\Psi - E(\Psi))$.

$\Phi(x)$ = standard normal distribution.

Modeling in SII: premium + reserve risk

- **model assumption behind this formula**
  $S_\bullet - E[S_\bullet]$ has the same distribution as $V_\bullet(\Psi - 1)$, where
  Ψ has a lognormal distribution with $E[\Psi] = 1$ and $\mathbf{Var}(\Psi) = \varphi^2$.

- **remarks and discussion**

  $S_\bullet - E[S_\bullet] = V_\bullet(Z_\bullet - E[Z_\bullet])$ is aproximated by $V_\bullet(\Psi - 1)$.

  but contrary to the SST: $E[Z_\bullet] \neq 1$ (usually smaller than 1).

  => $S_\bullet$ is modeled by a lognormal distribution with mean $E[S_\bullet]$, but with a variance which is different from $Var[S_\bullet]$

Modeling in SII: premium + reserve risk

Comparison of 99.5% VaR of $Z_{\bullet} - E[Z_{\bullet}]$ and $\Psi - 1$ for $E[Z_{\bullet}] = 85\%$.

φ =	$VaR_\alpha^{mean}(\Psi)$	$VaR_\alpha^{mean}(Z)$	ratio
	(a)	(b)	(c)=(b)/(a)
1%	0.070	0.070	1.004
2%	0.144	0.145	1.007
3%	0.224	0.226	1.011
4%	0.309	0.313	1.015
5%	0.399	0.407	1.019
6%	0.496	0.508	1.023
7%	0.599	0.615	1.027
8%	0.709	0.731	1.032
9%	0.826	0.856	1.036
10%	0.951	0.989	1.040
11%	1.084	1.132	1.045
12%	1.225	1.285	1.049
13%	1.376	1.449	1.053
14%	1.536	1.625	1.058
15%	1.706	1.813	1.063
16%	1.887	2.014	1.067
17%	2.080	2.229	1.072
18%	2.284	2.459	1.076
19%	2.501	2.704	1.081
20%	2.732	2.966	1.086

Modeling in SII: cat risks and total insurance risk

- **SCR for CAT risks**

$$SCR_{CAT} = \sqrt{\sum_{k=1}^{K} c_k^2}.$$

- **total SCR for nl-insurance risk**

$$SCR_{nl} = \sqrt{SCR_{CY+PY}^2 + SCR_{CAT}^2}.$$

- **model assumptions behind these formulas**

  The cat risks $CAT_k,\ k = 1,2,\ldots,l$ are independent and normally distributed with $VaR_{0.995}\left(CAT_k\right) = c_k$.

  Same assumption for aggregating the cat risks and the other insurance risks.

Modeling : Summary

- STT and SII "parametrized" models;
  SII: factor model; STT distribution based model;

- risk measure: STT 99% expected shortfall, SII 99.5% VaR

- variance assumptions CY- und PY-risks (for r.v. X and Y):
  STT: parameter risk and random fluctuation risk, where the latter is inversely proportional to the weight (size of the company);
  SII: CY- and PY-risks not dependent on the size of the company

- CY risk: STT distinguishes between "normal claims" and "big claims". No such distinction in SII.

- Correlation Assumptions (current state):
  SST: no correlations between lob for the reserve risks and no correlations between CY- und PY-risks;
  SII: same correlation between lob for CY- and PY-risks;
  SST as well as SII assumptions not fully satisfactory.

Modeling : Summary

- SST: scenarios for extraordinary situations
  can be taken into account in a natural way in the distribution calculation;
  SII: CAT-risks modeled similar to scenarios in the SST;
  however aggregation of cat-risks and with CY/PY-risks questionable

- SST: final product is a distribution, from which the SCR is calculated;
  SII: final product is one figure, the SCR.

- Results (AXA-Winterthur)
  with current standard parameters: SCR_{ins} higher in SII than in SST;
  split between CY- und PY-risks:
  SII: ca 25% CY and 75% PY
  SST: ca 27% CY and 73% PY

Parameter Estimators: SII parameters

- straightforward estimators

$$\hat{\sigma}_i^2 = \frac{1}{n_i - 1} \sum_{j=1}^{n_i} \frac{P_{ij}}{P_{i\bullet}} (X_{ij} - \overline{X}_i)^2,$$

$$\hat{\tau}_i^2 = \frac{1}{n_i - 1} \sum_{j=1}^{n_i} \frac{R_{ij}}{R_{i\bullet}} (Y_{ij} - \overline{Y}_i)^2,$$

Remarks:
$\hat{\sigma}_i^2$ can overestimate the risk in case of "strong" business cycles in the observation period;
$\hat{\tau}_i^2$ often underestimates the reserve risks because of "smoothing" effects in the reserves

Parameter Estimators: SST parameters

- **Random fluctuation risk CY**

$$\sigma^2_{i,fluct} = CoVa^2\left(Y_i^{(\nu)}\right) + 1.$$

$$\widehat{CoVa}_{ij} = \sqrt{\frac{\frac{1}{N_{ij}-1}\sum_{\upsilon=1}^{N_{ij}}\left(Y_{ij}^{(\nu)} - \overline{Y}_i\right)^2}{\overline{Y}_i^2}}$$

in long-tail lob: above estimator underestimates the CoVa in recent accident years

Development triangle CoVa claim amounts in motor liability

AY/DY	0	1	2	3	4	5	6	7	8	9
1998	6.1	7.1	8.1	8.5	9.1	9.7	10.1	10.2	10.2	10.9
1999	6.1	7.6	8.1	9.0	10.0	10.3	10.6	10.6	11.2	11.2
2000	5.8	6.9	8.4	8.9	9.4	9.5	9.5	10.3	10.3	10.3
2001	5.8	7.4	8.9	9.5	9.6	9.6	10.4	10.7	10.7	10.7
2002	5.7	7.8	9.0	9.0	9.2	9.9	10.3	10.6	10.6	10.6
2003	6.6	8.5	8.8	9.1	9.8	10.2	10.6	10.9	10.9	10.9
2004	6.6	8.5	9.2	10.1	10.7	11.1	11.5	11.9	11.9	11.9
2005	5.2	7.2	8.7	9.2	9.7	10.1	10.5	10.8	10.8	10.8
2006	5.4	7.5	8.5	9.0	9.5	9.9	10.3	10.6	10.6	10.6
2007	5.7	7.4	8.4	8.8	9.4	9.7	10.1	10.4	10.4	10.4
									mean	10.8

Parameter Estimators: SST parameters

- **parameter risk CY**
  specific lob; each year *j* characterized by $\Theta_j = \left(\Theta_{j1}, \Theta_{j2}\right)^T$;

  r.v. belonging to different years are independent and $\Theta_1, \Theta_1, \ldots, \Theta_J$ are i.i.d.

  => $$E\left[X_j\right]=1, \quad \mathbf{Var}\left(X_j\right)=\sigma^2_{param}+\frac{\sigma^2_{fluct}}{\nu_j} \simeq \sigma^2_{param}+\frac{\hat{\sigma}^2_{fluct}}{\tilde{P}_j},$$

  fulfill the assumptions of the Bü-Straub credibility model

  => estimator
  $$\hat{\sigma}^2_{param} = c \cdot \left\{ \frac{J}{J-1} \sum_{j=1}^{J} \frac{w_j}{w_\bullet} \left(X_j - \overline{X}\right)^2 - \frac{J\,\hat{\sigma}^2_{fluct}}{n_\bullet} \right\},$$

  where $$c = \frac{I-1}{I} \left\{ \sum_{i=1}^{I} \frac{w_{i\bullet}}{w_{\bullet\bullet}} \left(1 - \frac{w_{i\bullet}}{w_{\bullet\bullet}}\right) \right\}^{-1}, \quad \hat{\sigma}^2_{fluct} = \widehat{CoVa}^2\left(Y^{(\upsilon)}\right)+1,$$

  $n_\bullet$ = observed number of claims.

Parameter Estimators: SST parameters

- **parameter risk CY (continued)**
  since

$$\sigma^2_{param} \simeq \mathbf{Var}(\Theta_1) + \mathbf{Var}(\Theta_2)$$

  one can, alternatively to the estimator given before, estimate the two components separately based on the observed claim frequencies and the observed claim sizes.

  Here again one can use a credibility procedure.

  more details: see paper

Parameter Estimators: SST parameters

- **Estimation of the Pareto parameters for big claim CY**
  ML-estimator (adjusted for unbiasedness)

$$\hat{\vartheta} = \left(\frac{1}{n-1} \sum_{\nu=1}^{n} \ln\left(\frac{Y_\nu^b}{c} \right) \right)^{-1}$$

with

$$E\left[\hat{\vartheta}\right] = \vartheta, \quad CoVa\left(\hat{\vartheta}\right) = \frac{1}{\sqrt{n-2}}.$$

Number of observed big claims often rather small; combine individual estimate with market wide estimate; ML-estimators fulfill Bü-Straub cred. assumptions

=> credibility estimator $\hat{\vartheta}^{cred} = \alpha\,\hat{\vartheta} + (1-\alpha)\,\vartheta_0$

where $\alpha = \frac{n-2}{n-1+\kappa}$, ϑ_0 = standard value from the SST, $\kappa = CoVa(\Theta)^{-2}$.

Example: $CoVa(\Theta) = 25\%$, n=16 => give a credibility weight of 32% to your individual estimate

Parameter Estimators: SST parameters

- **reserve risk**
  reserve risk should be valuated with reserving techniques;
  well known: Mack's mse of the ultimate for chain ladder reserving method;

  for solvency purposes one needs the one-year reserve risk;
  the formula can be found in Bühlmann and alias (2009);

  In Solvency we are interested in the one in a century adverse reserve events. What scenarios come to our mind: for instance a hyper-inflation or a big change in legislation. These are "calendar-year" events not observed in the triangles and not captured by standard reserving methods.

  => the reserve risk resulting from standard reserving methods are not sufficient for solvency purposes and should be supplemented by reserve scenarios.

Parameter Estimators: SST parameters

- **reserve risk (continued)**
  For small and medium sized companies the observed figures in a development triangle might fluctuate a lot. It would be helpful if one could combine industry wide patterns with the one evaluated with the data of the individual company.

  For chain ladder a credibility method was developed of how one could combine the information gained from the two sources: individual data and industry wide information. The idea is to estimate the age-to-age factors by credibility techniques.

  For more information see Gisler-Wüthrich (2008).

References

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