Equilibrium Pricing of Contingent Claims in Tradable Permit Markets

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The 39th ASTIN Colloquium – *Helsinki, Finland*

The full paper is available at **http://ssrn.com/abstract=1344103**.

Plan of Talk

Introduction

Kyoto Protocol

- The first five-year commitment period started on January 1, 2008.
- Ratifying countries are required to meet the cap of pollution emission amount.
- Each country also imposes the emission limitation to its industries.
- How to meet the regulation:
  - Reduce the emission amount by self-efforts.
  - Buy pollution rights (pollution permit).

Statistics of carbon market (1)

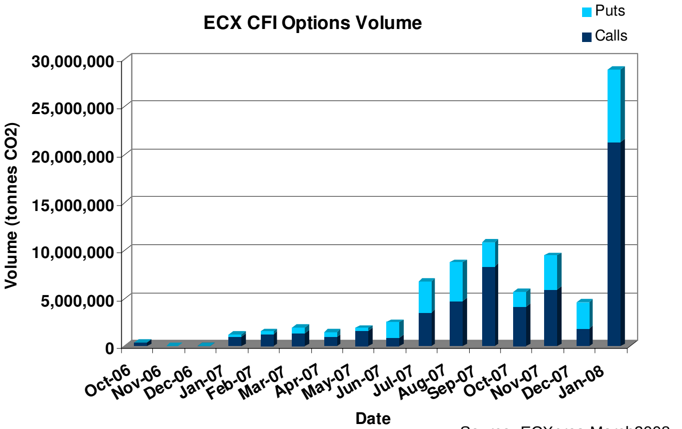
Table 3.1: Reported volumes and value 2005, 2006, forecast 2007

Reported and estimated volumes 2005 and 2006, together with forecasted volumes for 2007, in Mt CO2e and million €. 7 % discount rate employed for CDM and JI where price is at point of delivery. Prevailing carbon prices at time of writing for 2007 forecast.

	2005		2006		2007	
	Final figures		*Final figures*		*Forecast*	
	[Mt]	[€ million]	[Mt]	[€ million]	[Mt]	[€ million]
EU ETS total	362	7,218	1,017	18,143	1,750	18,503
- OTC + exch.	*262*	*5,400*	*817*	*14,575*	*1,550*	*15,903*
- Bilateral	*100*	*1,818*	*200*	*3,568*	*200*	*2600*
Other ETSs	7.8	52	31	300	50	500
CDM	397	1,985	523	3,349	456	3,260
CDM 2nd	4	50	40	571	96	1,061
JI	28	96	21	95	45	277
Sum	**799**	**9,401**	**1,632**	**22,458**	**2,397**	**23,601**

Source: Carbon 2007.

Statistics of carbon market (2)



Source: ECXpres-March2008.

Summary of the paper

- The purpose of the paper is to propose a pricing formula that evaluates any contingent claim in the pollution permit market.
- Literature review:
  - Cronshaw and Kruse (1996), Rubin (1996), Seifert et al. (2008) only focus on the spot prices.
  - Maeda (2004): only the forward price is examined.
  - Daskalakis *et al*. (2007), Chesney and Taschini (2008): the spot price (underlying) is exogenously given.
- To our best knowledge, this paper is the first to study the pricing of any contingent claim of permit markets in a general equilibrium framework.

Pricing Functional

- Let $\tilde{Y}$ be the payoff of any contingent claim.
- It is a well-known result in the finance literature that the price of $\tilde{Y}$, denoted by $\pi(\tilde{Y})$, can be expressed with some random variable $\tilde{\phi}$ as

$$\pi(\tilde{Y}) = \mathbb{E}[\tilde{\phi}\tilde{Y}].$$

- The r.v. $\tilde{\phi}$ is a so-called state price density.

The Model

The market

- 2-period economy: time 0 (present) and time 1 (future).
- Market participants (all price takers)
  - regulated emitters who need to meet the cap of emission.
  - unregulated financial traders who are not obliged to follow the regulation.
- At time 0, there are two markets:
  - Spot market: spot permits of time 0 are traded.
  - Contingent claims market: contingent claims written on the permits of time 1 are traded.
- The contingent claims market is complete in the sense that any contingent claim is tradable.

Glossary

$\mathcal{M}_E$	set of emitters
$\mathcal{M}_S$	set of financial traders
E_{it}	the net abatement target of emitter $i \in \mathcal{M}_E$ at time t
$c_{it}(X_{it})$	the cost function to reduce the amount of emission X_{it} by emitter i at time t,
S_t	spot permit price at time t,
R_{kt}	exogenous income of agent $k \in \mathcal{M}_E \cup \mathcal{M}_S$,
r	risk-free rate determined in the financial market.

Emitters

- To match the regulation, emitter i can reduce emission by her own effort and by buying the spot permits in the market.
- The final wealth of emitter i is described as

$$\begin{aligned} W_i(\omega) &= (1+r)[R_{i0} - c_{i0}(X_{i0}) - S_0 \cdot (E_{i0} - X_{i0}) - \pi(\tilde{H}_i)] \\ &+ R_{i1}(\omega) - c_{i1}(X_{i1}(\omega)) - S_1(\omega)\{E_{i1}(\omega) - X_{i1}(\omega)\} + H_i(\omega), \end{aligned}$$

where $\tilde{H}_i$ is the contingent claim that emitter i purchases.

- The preference of emitter i is represented by an CARA utility with absolute risk-aversion coefficient γ_i;

$$\mathbb{E}[-\exp\{-\gamma_i \tilde{W}_i\}].$$

Financial traders

- Financial traders have no incentive to enter the spot market, and only trade in the contingent claims market to hedge the risk of their exogenous income.
- The final wealth of financial trader j is written as

$$W_j(\omega) = (1+r)[R_{j0} - \pi(\tilde{H}_j)] + R_{j1}(\omega) + H_j(\omega), \quad j \in \mathcal{M}_S.$$

- The preference of financial trader j is also represented by an CARA utility;

$$\mathbb{E}[-\exp\{-\gamma_j \tilde{W}_j\}].$$

Equilibrium Prices without Banking

Market clearing

- Spot markets:

$$\sum_{i \in \mathcal{M}_E} (E_{i0} - X_{i0}) = 0,$$

$$\sum_{i \in \mathcal{M}_E} (E_{i1}(\omega) - X_{i1}(\omega)) = 0$$

- Contingent claims market:

$$\sum_{k \in \mathcal{M}_E \cup \mathcal{M}_S} H_k(\omega) = 0$$

for almost all ω.

Cost function

Assumption

The cost function $c_{it}(\cdot)$ is strictly increasing, continuously differentiable, strictly convex, and satisfies

$$c'_{it}(0) = 0, \quad c'_{it}(\infty) = \infty.$$

- The assumption guarantees the existence and uniqueness of the equilibrium.

Some notations

- Let $c_t := \sum_{i \in \mathcal{M}_E} c_{it}$. Then, we verify that c_t is a function of E_t in equilibrium, where $E_t := \sum_{i \in \mathcal{M}_E} E_{it}$, i.e., we have an expression

$$c_t = c_t(E_t).$$

- Define

$$R_t := \sum_{k \in \mathcal{M}_E \cup \mathcal{M}_S} R_{kt}, \quad \gamma := \frac{1}{\displaystyle\sum_{k \in \mathcal{M}_E \cup \mathcal{M}_S} \frac{1}{\gamma_k}}.$$

Pricing formula in the case of no banking

Proposition

The state price density ϕ *is given by*

$$\phi(\omega) = \frac{1}{1+r} \frac{\mathrm{e}^{\gamma\{c_1(E_1(\omega)) - R_1(\omega)\}}}{\mathbb{E}\left[\mathrm{e}^{\gamma\{c_1(\tilde{E}_1) - \tilde{R}_1\}}\right]}.$$

Proof.

Apply the result of Bühlmann (1980). □

Forward price

Corollary

The price of forward contract is derived as

$$F = \frac{\mathbb{E}\left[c_1'(\tilde{E}_1)\mathrm{e}^{\gamma\{c_1(\tilde{E}_1)-\tilde{R}_1\}}\right]}{\mathbb{E}\left[\mathrm{e}^{\gamma\{c_1(\tilde{E}_1)-\tilde{R}_1\}}\right]}.$$

Proof.

Verify that $S_1 = c_1'(E_1)$ in equilibrium. □

Normal risks (1)

- Suppose

$$\begin{pmatrix}\tilde{E}_1 & \tilde{R}_1\end{pmatrix} \sim \mathcal{N}\left[\begin{pmatrix}\mu_E & \mu_R\end{pmatrix}, \begin{pmatrix}\sigma_E^2 & \rho\sigma_E\sigma_R \\ \rho\sigma_E\sigma_R & \sigma_R^2\end{pmatrix}\right]$$

- The price whose payoff at time 1 is $g(S_1)$ is given by

$$\pi(g(\tilde{S}_1)) = \frac{\mathbb{E}\left[h(\tilde{Z}^*)\mathrm{e}^{\gamma c_1(\tilde{Z}^*)}\right]}{(1+r)\mathbb{E}\left[\mathrm{e}^{\gamma c_1(\tilde{Z}^*)}\right]},$$

where $h(\cdot) = g(c_1'(\cdot))$, and $\tilde{Z}^*$ is a normal with mean $\mu_E - \gamma\rho\sigma_E\sigma_R$ and variance σ_E^2.

Normal risks (2): comparative statics

Corollary

Let

$$\Delta_1 = \mathbb{E}_\phi[h'(\tilde{Z}^*)] - \gamma\mathbb{C}_\phi[h(\tilde{Z}^*), c_1'(\tilde{Z}^*)],$$

where $\mathbb{E}_\phi$ *and* $\mathbb{C}_\phi$ *are the expectation and covariance operators under the risk-neutral measure, respectively. Then,*

$$\frac{\partial \pi(g)}{\partial \mu_E} = \frac{1}{1+r}\Delta_1, \quad \frac{\partial \pi(g)}{\partial \rho} = -\frac{\gamma\sigma_E\sigma_R}{1+r}\Delta_1.$$

Quadratic cost function

- Suppose furthermore that the cost function of each emitter is quadratic as $c_{it}(x) = \frac{\hat{c}_{it}}{2}x^2$. Then, the social cost function is given by

$$c_t(E_t) = \frac{1}{2}\hat{c}_t E_t^2,$$

where $\hat{c}_t = \frac{1}{\sum_{i \in \mathcal{M}_E} \frac{1}{\hat{c}_{it}}}$.

- The forward price is derived as

$$F = \frac{\hat{c}_1 \left(\mu_E - \gamma\rho\sigma_E\sigma_R\right)}{1 - \gamma\hat{c}_1\sigma_E^2} = \frac{\hat{c}_1\mu_Z}{1-\alpha},$$

where $\mu_Z := \mu_E - \gamma\rho\sigma_E\sigma_R$ and $\alpha := \gamma\hat{c}_1\sigma_E^2$.

- This is the case that Maeda (2004) considered.

The Effect of Banking

Banking (and borrowing)

- Banking: additional reduction amount in the current period can be used in later periods.
- When banking and borrowing are allowed, the current and future spot markets are connected.
- The market clearing condition of the permit:

$$\sum_{t=0}^{1} \sum_{i \in \mathcal{M}_E} (E_{it} - X_{it}) = 0$$

or equivalently

$$\sum_{i \in \mathcal{M}_E} (E_{i0} + B_{i0} - X_{i0}) = 0 \text{ and } \sum_{i \in \mathcal{M}_E} (E_{i1}(\omega) - B_{i0} - X_{i1}(\omega)) = 0.$$

where B_{i0} denotes the banking of emitter i at time 0.

No arbitrage condition

- No arbitrage condition implies

$$(1+r)\underbrace{c_0'(E_0+B_0)}_{=\text{Spot price}} = \underbrace{\frac{\mathbb{E}\left[c_1'(\tilde{E}_1-B_0)\mathrm{e}^{\gamma\{c_1(\tilde{E}_1-B_0)+\tilde{R}_1\}}\right]}{\mathbb{E}\left[\mathrm{e}^{\gamma\{c_1(\tilde{E}_1-B_0)+\tilde{R}_1\}}\right]}}_{=\text{Forward price}}, \qquad \text{(NAC)}$$

  where $B_0 = \sum_{i\in\mathcal{M}_E} B_{i0}$.
- The aggregated amount of banking in equilibrium is determined by (NAC).

Normal risks (1)

- Consider the case of Example 1 and suppose that the system of banking and borrowing is introduced.
- The aggregated amount of banking is derived as

$$B_0 = \frac{\frac{\hat{c}_1(\mu_E - \gamma\rho\sigma_E\sigma_R)}{1-\gamma\hat{c}_1\sigma_E^2} - (1+r)\hat{c}_0 E_0}{(1+r)\hat{c}_0 + \frac{\hat{c}_1}{1-\gamma\hat{c}_1\sigma_E^2}}.$$

- The forward price is given by

$$F_{\mathrm{WB}} = \frac{\hat{c}_1\left(E_0 + \mu_E - \gamma\rho\sigma_E\sigma_R\right)}{1 - \gamma\hat{c}_1\sigma_E^2 + \frac{\hat{c}_1}{(1+r)\hat{c}_0}} = \frac{\hat{c}_1(E_0 + \mu_Z)}{1 - \alpha + \frac{\hat{c}_1}{(1+r)\hat{c}_0}}.$$

Normal risks (2)

- Comparing F and F_{WB}, we have

$$\frac{\partial F}{\partial \mu_E} \geq \frac{\partial F_{\mathrm{WB}}}{\partial \mu_E} > 0, \quad \frac{\partial F}{\partial \rho} \leq \frac{\partial F_{\mathrm{WB}}}{\partial \rho} < 0$$

- The introduction of banking and borrowing lowers the sensitivity of exogenous parameters on the forward price (smoothing effect).
- The sensitivity of the forward price is partly absorbed by the change of the current spot price.

Discussions

Price spike: cost function and spot price

- Suppose the normality of $\tilde{E}_1$ and $\tilde{R}_1$, and that the social abatement cost function is given by

$$c_t(x) = \hat{c}_L \times \begin{cases} 0 & x \leq 0, \\ \frac{1}{2}x^2 & x \leq \bar{X}_B, \\ \frac{\kappa}{2}x^2 - (\kappa-1)\bar{X}_B x + \frac{\kappa-1}{2}\bar{X}_B^2 & x > \bar{X}_B, \end{cases}$$

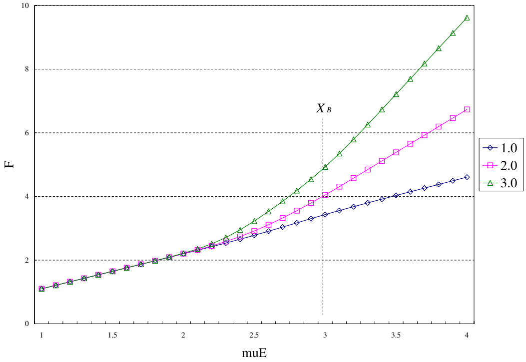
  where $\kappa \geq 1$.
- The spot price of time 1 is derived as

$$S_1 = \hat{c}_L \begin{cases} 0 & \tilde{E}_1 \leq 0, \\ \tilde{E}_1 & \tilde{E}_1 \leq \bar{X}_B, \\ \kappa\tilde{E}_1 - (\kappa-1)\bar{X}_B\tilde{E}_1 & \tilde{E}_1 > \bar{X}_B. \end{cases}$$

- The spot price kinks at $x = \bar{X}_B$, and the parameter κ measures the magnitude of the kink.

Price spike: no banking case

Forward price with different κ:



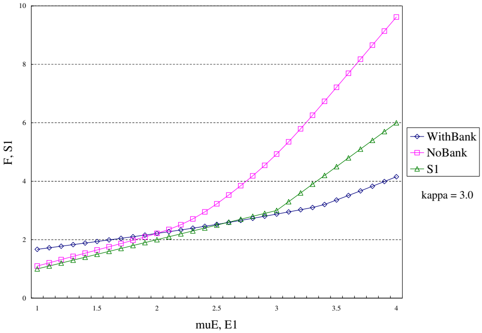
Parameter values:

γ	$\hat{c}_L$	σ_E	ρ	$\bar{X}_B$
1.0	1.0	0.3	0.0	3.0

The spike level is lower in the forward price.

Price spike: introduction of banking

Forward price with different κ:



Parameter values:

γ	$\hat{c}_L$	σ_E	ρ	$\bar{X}_B$
1.0	1.0	0.3	0.0	3.0

The price spike is significantly mitigated by the introduction of banking and borrowing.

Comparative statics

- By simply differentiating the forward price formula, we have

$\sigma_E \nearrow$	$\Rightarrow$	indeterminate
$\gamma \searrow$	$\Rightarrow$	indeterminate
$\rho \searrow$	$\Rightarrow$	$F \nearrow$
$\sigma_R \nearrow$	$\Rightarrow$	$F \searrow$

- The impact on the risk-premium and the correlation with the exogenous income should be considered.

Conclusions

Conclusions

In this paper,

- we have proposed a formula that prices any contingent claim in the tradable permit market.
- we have analyzed the equilibrium price in the permit market with the case of normal distributions.
- we have shown that the prices in permit market have some specific properties that are not observed in ordinary financial markets.

Summary

Key determinants of the pricing:

(i)	Social cost function to meet the abatement target
(ii)	Uncertainty of the required abatement level
(iii)	Risk-aversion of market participants
(iv)	Correlation of the abatement level with exogenous income.

- System design:
  - Banking and borrowing,
  - Limitation of market participants.

Thank you for your attention