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**Various Faces of Risk Measures:
Internal Model's Perspective**

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Outline

We critically review the most widely used classes of **risk measures**:

- We consider several aspects (faces) of risk measures from the internal models' perspective;
 - Different classes
 - Robustness – model risk
 - Internal / External Use
- The use of tail conditional trimmed mean is proposed as an alternative for a robust risk estimator.

1. Background: Regulation

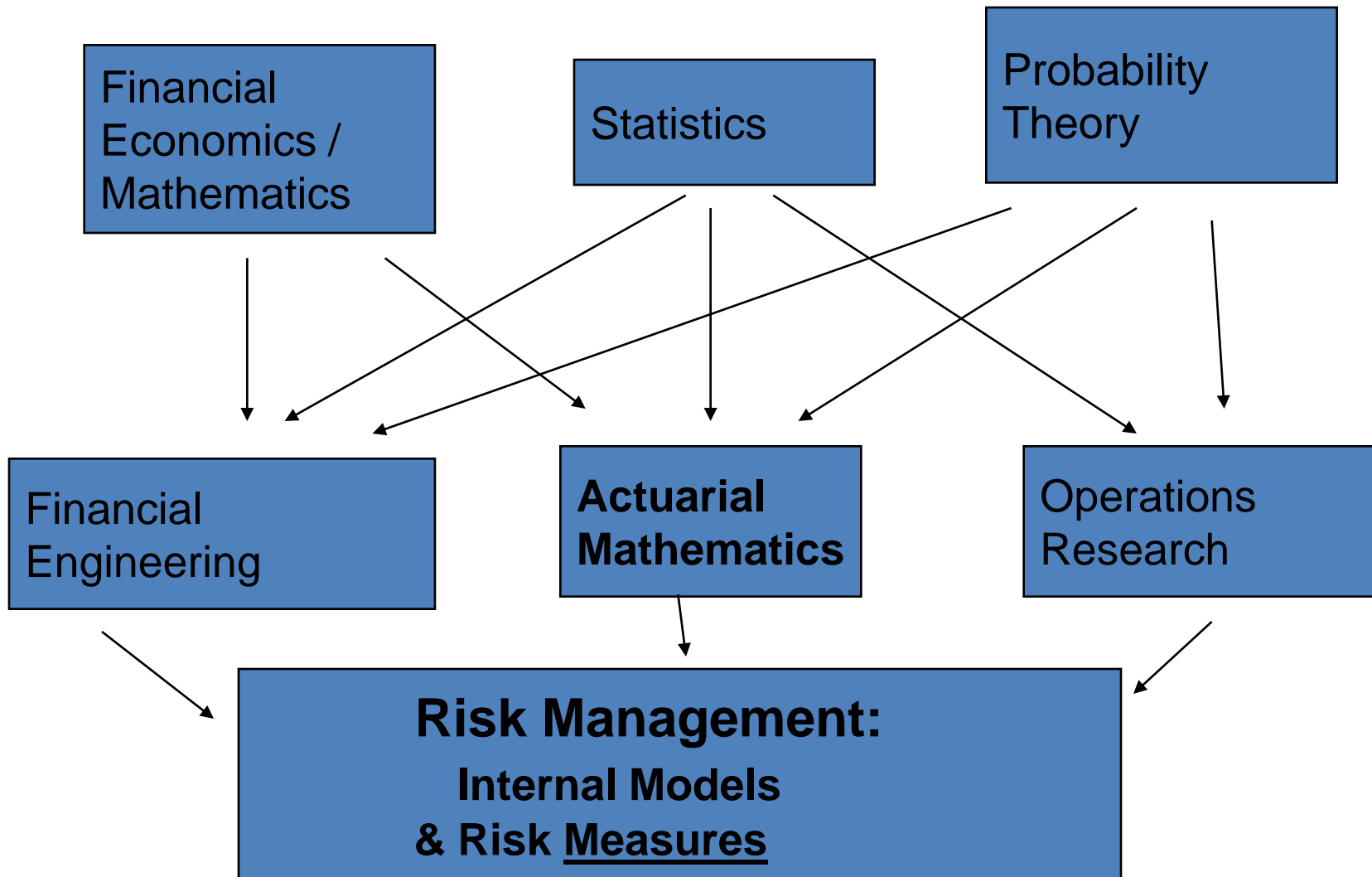
The main reasons for giving European insurance companies (in S2) an option to apply internal models for calculating the main solvency requirement are:

- to enhance better risk management in firms;
- to give a chance to derive a more accurate risk-oriented capital requirement than the standard Solvency Capital Requirement would be able to provide.

All this relies on **correct/sensible risk measurement!**

The **choice of risk measure** is a central decision.

2. Background: Theory



3. Background: Pentikäinen

Pentikäinen (1975) stated before the age of internal models:

a) Whether or not the applications of the theory of risk have been useful for practical management may have depend very much on how the risk theoretical treatments have been linked with the complexity of various other aspects involved with the actual decision making.

b) The strategy of 'practical men' can be a random product of old traditions, more or less reliable institutions.

A discussion on theoretical aspects and on the theoretical point of view, even if the direct numerical results are of little value, may anyway direct attention to the statement and restatement of problems and to a conscious analysis of the facts and possibilities.

4. Classes of Risk Measures

$\text{VaR}_\alpha[X] = Q[X]$, $\text{TCE}_\alpha[X] = \text{mean}[X|X > Q_\alpha[X]]$, $\alpha \in (0, 1)$.

- **Insurance risk measures**
 - Insurance risk premium principles
- **Coherent risk measures**
 - Justify scenario analysis and diversification
 - Monotonicity, Positive homogeneity, Translation invariance and; $\text{VaR}[X] = Q[X]$, $\alpha \in (0, 1)$.
Subadditivity: $\rho[X + Y] \leq \rho[X] + \rho[Y]$ (TCE is, VaR is not)
- **Convex risk measures** (nonlinearities e.g. liquidity)
Relaxing the conditions of positive homogeneity and of subadditivity and to require the weaker property of
Convexity
 $\rho[aX + (1 - a)Y] \leq a\rho[X] + (1 - a)\rho[Y]$, $a \in [0, 1]$

5. Problems with Aggregation?

Pentikäinen in the 1960s: "*Simulation offers a flexible and powerful means of coping with even the most complicated model specifications*".

Example: Portfolio of 5 equity indices. Risk measures historical scenarios and simulated scenarios from various **copulas**. Measured **risk depends heavily on the dependence model** (Koskela, Koskinen and Ronkainen (2009): "On dependence modelling in financial risk management", *Festschrift in honour of Jaakko Astola on the occasion of his 60th birthday*, Tampereen yliopistopaino, Finland):

	<u>min(P_{t+1})</u>	<u>VaR_{t+1}(99%)</u>	<u>TCE_{t+1}(99%)</u>
Historical	-217.6	-78.3	-126.6
Gaussian C.	-83.7	-65.3	-73.5
t(50) C.	-95.4	-67.2	-75.0
t(10) C.	-140.5	-82.1	-105.7
t(2) C.	-1134.3	-210.0	-425.1

=> the choice of dependence model more important?

6. Robustness?

Robustness: results are not unduly affected by outliers or other small departures from model assumptions.

Robustness => Good risk forecasts

Experiences suggest that **a simple model** is more often robust than a complicated one.

Pentikäinen et al.(1969) wrote: *"Various alternatives, lines and methods of presentation are possible. Our aim has been for simplicity."*

7. Tail Estimation and Model Selection

- **The calibration target in Solvency II is once in 200 years, the available data set is usually too small**
 - => the **confidence intervals** for the estimated parameters are large;
- **The distinctions between different distributions is exceedingly difficult.** Particularly large samples are needed for clear discrimination;
- **Using heavy tailed distributions and/or copulas makes the data issue even more critical.** In the **tail modelling subjectivity is unavoidable** and this must be appropriately dealt with by the insurers and supervisors.

8. Tail Dependence vs. Outliers

- In practice with extensive data there will almost always be important unanticipated features. These outliers are the most important observations;
- Statistics derived from data sets after wrong decision:
a) eliminate or b) not - may be totally misleading;
- One possible approach is the use of **robust statistical methods**. For instance, Tail Conditional Trimmed Mean:

$$\text{TCTM}_{k,\alpha} [X] = \text{k-trimmed mean}[X | X > Q_\alpha[X]],$$

$k \in \mathbb{Z}^+$, $\alpha \in (0, 1)$. It discards k smallest observations.

=> a compromise: tail vs. robustness!

9. Forecasting risk

- Risk measures should be **forward-looking**
 - => forecast the future based on
 - a) past experience and b) available data;
- Giving a better historical fit, do not necessary produce better out-of-sample forecasts
 - => **Robustness (and related parsimony) again important;**
- Typically, the forecasting errors increase with the horizon
 - In Solvency II there is one-year forecasting horizon,
 - => the **forecasting error can be substantial;**
- In time series (conditional) context the most common model class for VaR forecasting is the GARCH class.

10. Forecasting error

The main sources of errors when **forecasting risks** are:

1. Model parameters have to be estimated - This constrains the number of parameters in risk model;
2. Exogenous variables may also have to be forecasted - This is quite common problem for internal models;
3. The wrong model may be identified - Tail and dependence modelling are especially demanding tasks faced in risk modelling; and
4. The underlying model may change - This is important for risk forecasting since economic environment may change rapidly.

11. In Defence of Value at Risk - Part A

- The VaR is not unproblematic to use:
 - a) it is not subadditive, b) its estimation is subject to large errors, b) the estimate is downward biased;
- However, these shortcomings do not imply that VaR is not a useful tool in risk management. **It has clear advantages:**

VaR is simple to use and robust!

In practice, the conflict between VaR and diversification may not be severe.

- On the other hand, the robustness of the coherent risk measures - like TCE - is questionable.

12. In Defence of Value at Risk - Part B

- The regulator may want the solvency capital requirement to be **as large as possible**, clearly holding too much capital is costly.

Optimal solvency capital requirement ρ under a **natural cost function** (Dhaene et al):

$$C(X, \rho[X]) = E[\rho(X - [X])_+] + \rho[X]\varepsilon, \quad 0 < \varepsilon < 1,$$

is $\rho = \mathbf{VaR}(1-\varepsilon)$ (in paper also under vpp)

- When applying a (too)subadditive risk measure in a merger, one could end up with a larger shortfall than the sum of the shortfalls of the stand-alone entities.

Conclusions

Final comments on internal models and risk measures:

- The financial institutions that are surviving this crisis best are not those who relied on models to do the management's job;
- While many risk measures may be suitable for internal risk management, **robustness is especially important consideration for a regulatory risk measure,**
- **Is axe better tool than saw?** It depends on the issue;
- **Pentikäinen et al (1969):** "The solvency margin is only one of the pillars of a comprehensive solvency system."