The Prediction Error of Bornhuetter/Ferguson

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The B/F reserve estimate for accident year i:

$$\hat{R}_i^{BF} = \hat{U}_i \cdot \hat{w}_{n+1-i}$$

= initial estimate of the ultimate claims amount U_i *

* estimated %age of claims outstanding ($0 < \hat{w}_k < 1$) .

Main property: $\hat{R}_i^{BF}$ does not at all depend from current claims CC_i ,

whereas the chain ladder reserve

$\hat{R}_i^{CL} = CC_i \cdot \left(\hat{f}_{n+1-i} - 1\right)$ = current claims amount * (age-to-ult-factor – 1)

is volatile because of its strong dependency from CC_i .

A stochastic model underlying the B/F method

(BF1) The increments S_{i1}, S_{i2}, … of each acc. year i are independent.

(BF2) $E(S_{ik}) = x_i y_k$ with $y_1 + y_2 + \ldots = 1$. (Note: y_k may be < 0)

(BF3) $Var(S_{ik} / x_i) = s_k^2 / x_i$.

Justification of (BF2): $R_i = S_{i,n+2-i} + S_{n+3-i} + \ldots$

$E(R_i) = x_i (y_{n+2-i} + y_{n+3-i} + \ldots)$ has the structure of $\hat{R}_i^{BF}$.

Justification of (BF3): $U_i = S_{i1} + S_{i2} + \ldots$

$E(U_i) = x_i$ shows that x_i can be used as measure of volume.

Consequences of the model

(1) $\hat{U}_i = \hat{x}_i$ has to be an estimate for $x_i = E(U_i)$ and not for $E(U_i \mid CC_i)$.

It is wrong to use $CC_i + \hat{R}_i^{BF}$ obtained last year as initial estimate $\hat{U}_i$!

Thus, $\hat{U}_i < CC_i$ is possible, even at paid data (accidental large claim) !

(2) The development pattern $y_1, y_2, \ldots$

should be estimated using $y_k \approx S_{ik} / \hat{x}_i = S_{ik} / \hat{U}_i$

and not using the chain ladder pattern!

Moreover, this makes B/F a really stand-alone reserving method!

The calculation of the prediction error $\text{msep}(\hat{R}_i^{BF})$

and of its components $\text{Var}(R_i)$ = process error

and $\text{Var}(\hat{R}_i^{BF})$ = estimation error

is now merely a matter of technique.

Required additional input: $\hat{s}_k^2$, $\text{s.e.}(\hat{U}_i)$, $\text{s.e.}(\hat{w}_k)$.

- $\hat{s}_k^2$ and $\text{s.e.}(\hat{w}_k)$ can be estimated from the data $\{ S_{ik} \}$,
- $\text{s.e.}(\hat{U}_i)$ is to be assessed by those who deliver $\hat{U}_i$,

  e.g. by formulating a range for $\hat{U}_i$ and dividing its length by 4.

Numerical Example (last 3 acc. years of paid excess data)

AY		BF	CL		BF	CL
2002	Reserves	139	133	Prediction	22	42
2003		149	72	std. error	22	54
2004		155	353		23	265
2002	Estimation	15	17	Process	15	38
2003	std. error	16	13	std. error	15	52
2004		17	122		16	235