

SCENARIO ANALYSIS FOR A MULTI-PERIOD DIFFUSION MODEL OF RISK

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ABSTRACT. This paper extends and develops the results in the paper [9]. Dealing with a simplistic diffusion multi-period model of insurance operations, it illustrates the adaptive control approach when the object of control is harmonization of the solvency and equity requirements. With regard to [9], the main novelty is the incomplete knowledge of the forthcoming risk which is quite often the case in insurance. Represented by a scenario of nature, it introduces new and inevitable randomness in the model and induces qualitative difference beside the case of completely known risk.

1. Introduction

In the papers [7]–[9] the insurance process is viewed as a series of successive insurance periods called years. Each year starts with a manager’s adaptive, or sensitive to the financial results in the previous year, control decision. The insurance operations are represented by a probability mechanism. The manager’s decision concerns tariffs, reserves and other operational characteristics of the probability mechanism of insurance. By the nature of insurance, that control decision typically remains in force throughout the whole insurance year, i.e., until development of the next year-end financial report and subsequent control intervention.

The adaptive control approach in insurance modelling is inspired by many scholars including K. Borch who claimed back in 1967 that “general formulation of the actuary’s problem leads directly to the general theory of *optimal control processes* or *adaptive control processes*” and “the theory of control processes seems to be “tailor-made” for the problems which actuaries have struggled to formulate for more than a century” (see [2], p. 451).

The object of control set forth in [7]–[9] was harmonization of the solvency and equity requirements. Solvent controls mean that a prescribed probability of non-ruin must be guaranteed uniformly on the past-years financial results, whichever external particulars within certain limits might be. Equity requires premiums well-balanced with claims, loaded with an amount necessary to provide adequate security for the insureds, rather than benefit those who seek unearned profit. It means that the insureds ought to pay premiums which are sensibly concentrated around the long-run mean value of their losses. In that sense the customers will not be overcharged, but only in the long run (i.e., in the average throughout several insurance years), while in the separate insurance years the premiums may be above or below average. Insurers, spreading the cost of random losses among the policyholders, and over time, act as a buffer against claim fluctuations in consecutive years.

Key words and phrases. Multi-period insurance process, Diffusion annual mechanisms, Volatile scenario, Solvency, Equity, Adaptive control strategies.