Strategic Planning, Risk Pricing and Firm Value

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Abstract

Strategic risk management attempts to evaluate which business units have the best profitability given their risk. This exercise is closely linked with risk pricing, although the pricing and risk management exercises have tended to use different methodologies. Linking to firm value is a unifying principle that can make pricing and risk management more consistent, but it is a somewhat difficult approach. Two easier but perhaps less fundamental alternatives are discussed: capital allocation based on pricing principles, and risk pricing from a firm-wide perspective.
Keywords. ERM; risk pricing; firm value; capital allocation.

1. INTRODUCTION

Capital allocation within insurance companies attempts to analyze which business units are generating better profits compared to their risk. Carried a step further, it can then be used to set profit targets for the units. Thus it can become a substitute for pricing analysis. Unfortunately, some practices in this area ignore basic principles of risk pricing. For instance, allocating then pricing based on tail risk gives no charge for less extreme risks which nonetheless contribute to overall firm volatility.

On the other hand, pricing in accord with financial models is viewed as making capital allocation unnecessary. For instance, Gründl and Schmeiser 2007 say "However, we could not find reasons for allocating equity capital back to lines of business for the purpose of pricing. This holds true for cases that do not integrate frictional costs and also when such costs are considered." And later, "We explained the main difficulties an insurance firm runs into when using capital allocation models for capital budgeting decisions such as expanding or contracting certain business segments. (This) … typically leads to wrong decisions by an insurance company."

Sherris 2006 marginally allocates a risk measure – the default put option – to lines of business in a pricing context, but does so only to subtract it from the price otherwise developed. He discusses capital allocation, but finds that it does not affect prices. Both these papers assume that pricing is done with transformed expected values within the arbitrage pricing framework. Thus they are essentially finding that capital allocation is not necessary if there is an available pricing mechanism.

Thus to a large extent, capital allocation and risk pricing are substitutes for each other. ERM professionals in the insurance industry are looking to capital allocation to price risk because in many insurance companies virtually no one, including actuaries, accountants, risk managers, and senior management, accepts standard risk-pricing approaches such as CAPM and arbitrage pricing theory. Some reasons for this reluctance are discussed below. This is in stark contrast to academic finance professionals, who routinely use these pricing methods and on the other hand tend to be skeptical about the role of capital allocation in pricing.

Standard risk-pricing models aim to provide a market-consistent valuation of risky transactions. Many insurers feel that they do not need models for market prices. They know the market prices already. What they want to know is whether or not the market prices for the various business segments give them adequate returns for the risks they are taking given the particular situation of their firm. Finance theorists tend to believe that the firm risk is the risk of the shareholders, who are diversified, so the proper market price for a risky proposition is the same for all firms.

The first issue that needs to be addressed is thus: does the proper price for risk transfer for a given risk vary from one firm to another? It is argued below that it does. In that case, the market price would be the one from the firms best situated to take that risk, i.e., the lowest such price. Given that, how does a firm decide if the market price is adequate for it? Standard pricing theories like the capital asset pricing model (CAPM) and arbitrage-free pricing (AFP) look for market prices in general, and so do not directly address that question. However capital allocation incorporates naïve pricing tools that do not necessarily bear much relationship to adequate risk pricing, so does not give an adequate answer either.

One, and probably the best, solution is to see if taking the risk at the market price enhances firm value. Approaches to firm-value modeling will be discussed. However the modeling assumptions being made can benefit from further research. An alternative for now is to customize pricing theories to the insurance market and then apply these to firms' particular situations by allocating capital in a manner consistent with risk-pricing theories. This is less satisfactory but more readily accomplished than full firm-value modeling. A further step down from theory to application is to base the capital allocation on traditional pricing techniques like standard deviation loading, but still in the context of overall firm risk. This will also be addressed.

The structure of the paper is to explore the issue of firm-specific risk in insurance companies in Section 2. Section 3 reviews firm-value models and their application to capital setting and risk management. Section 4 discusses the state of pricing theory and developments needed for application in a risk model. Section 5 deals with capital allocation using traditional and financial risk-pricing algorithms. Sections 6 concludes.

2. SPECIFIC RISK OF INSURERS

Specific risk is contrasted to the systematic risk of a firm, which can be represented as the covariance of the firm return with the market return. Specific risk is the remaining risk, which is not correlated with the market. Assuming that shareholders hold a diversified portfolio, the specific risk of any firm in the portfolio has a negligible contribution to the overall portfolio risk. This is the traditional argument that only the systematic risk of a firm is of concern to shareholders and is priced.

Modern financial thought does not completely accept this, and it is particularly problematic for insurers. One issue is that accounting does not give a totally transparent view of firm risk and opportunities, and this is even more of a problem in insurance than other industries, with the multiple accounting systems that apply and the difficulty of narrowly defining some of the accounts. Loss reserves in particular are difficult to meaningfully represent by a single number. With accounting opacity, it is difficult for an analyst to separate random fluctuations from trends, or signal from noise. As a consequence, randomness might generate an overreaction among analysts and in the share price.

Another key issue arises when bearing specific risk reduces expected earnings. One example is financing costs, e.g., see Froot, Scharfstein and Stein 1993. If a firm needs capital, it can usually arrange financing in the capital markets. However the cost of this capital to existing shareholders can sometimes be high. It is often cost-effective for firms that anticipate future capital needs to retain earnings instead of paying them out in dividends temporarily. Also hedging risk to reduce the chance of a capital shortfall can be cost effective in this situation. Paying a risk charge to reduce the chance of needing outside financing in the future could in fact increase expected earnings over time.

The risk aversion of parties other than shareholders can impact earnings as well. For instance suppliers and employees might demand a risk premium if future business is uncertain. For insurers the risk aversion of policyholders can create penalties for companies that carry high specific risk. Both growth and profitability can suffer from capital loss. There are numerous examples of strong companies that took a capital hit and subsequently both lost business and had to offer more competitive pricing. Share prices sometimes fall significantly more than the capital hit in such a situation. Even the risk of taking such a hit can impact the market view of an insurer's financial strength.

There have been some empirical studies in this area. Mayers and Smith 1982 and 1990 list reasons why firm-specific risk is important for insurers. Staking and Babbel 1995 report an increase in insurer market value from using risk management to avoid financial distress. Sommer 1996 finds that the profit load insureds are willing to pay decreases as the ratio of insurer capital to assets declines, and also decreases as the volatility of that ratio increases. Phillips, Cummins and Allen 1998 estimate the price discount that insureds demand for accepting a higher expected cost of insurer default. They find the discount is about 10 times the economic value of the default put value for long-tailed lines and 20 times for short-tailed lines. Grace, Klein and Kleindorfer 2004 find that insurer security issues impact insureds' buying decisions for homeowners insurance. Epermanis and Harrington 2006, who look at financial strength as measured by rating agencies, find that growth rates are higher for higher rated insurers, and that the growth rate of a company moves up and down with ratings.

All in all, there is strong evidence that firm-specific risk is important for insurers to manage and price, and that hedging this risk even if at a cost can add to value. Another implication is that different insurers with different capital structures and different business portfolios may require different premium levels for the same policy. This emphasizes that risk pricing and the value of risk transfer do not reduce to finding the market value of the risk. The impact on the firm has to be considered.

3. FIRM VALUE, CAPITAL, AND RISK

The ideal way to evaluate risk-pricing and strategic decisions is to look at their impact on firm value, but this requires a detailed model of how firm value responds to risk. One conclusion from the empirical policyholder-risk-aversion studies is that the value of an insurer increases non-linearly with capital. For a weakly capitalized insurer, improved market reception can increase value steeply as capital is added. With strong capital these effects are minimal but the frictional costs of holding capital makes the value increase less than the capital increase. The opposite effects occur from losing capital: the drop in firm value can be much greater than the capital lost. There are many examples of this in the financial histories of insurers.

If the impact of risk on the market value of an insurer can be modeled, this model becomes a general framework for evaluating risk and opportunity. Froot 2003 uses the term "M-curve" to describe the response of market value to capital. He shows that besides addressing capital targets, the M-curve is a general tool for risk-management. If losing \$X in insured losses reduces firm value by more than gaining \$X increases it, then reducing risk by costly reinsurance can increase expected firm value. On the other hand, increasing investment risk, even with higher expected returns, can sometimes reduce value due to policyholder risk aversion.

It is difficult to construct the M-curve directly from empirical studies, so it would be valuable to be able to calculate it from more basic models. Modeling firm value involves capturing the impacts of both capital and risk. Broadly speaking, more capital makes the firm more valuable, as does less risk. These effects are reflected very differently in different models, however. Traditional actuarial analysis featured the probability of ruin, with more capital increasing the expected time to ruin. This was put into a firm-value context beginning with de Finetti 1957, who expressed firm value as the expected present value of future dividends to shareholders, which is very much in the tradition of analysis of life insurance policies. This firm value is not the same as market cap, as it is supposed to reflect a somewhat stable underlying value. Market cap is based on the price those interested in buying or selling shares immediately would pay or take for those shares, which are typically a small fraction of outstanding shares. When a company is bought as a whole it is usually at a significant multiple of market cap.

In de Finetti's approach, more capital increases the likelihood of the firm being around in the future to pay more dividends so adds to value. However a dividend paid earlier has a higher present value than one paid later, which creates a tension between keeping capital and paying it out. This suggests that there is an optimal capital level. Taking more risk can reduce the probability of survival as well, but if it comes with the possibility of more return, it might increase firm value. In some cases, reducing risk, even by costly risk transfer, can increase firm value by increasing the likelihood of extended survival.

De Finetti was assuming the company starts with a fixed capital and survives unless the capital is used up. In the model of Modigliani and Miller 1958, the company is assumed to be able to raise capital at any time at a fixed borrowing-lending rate. Risk and value are measured by CAPM. It is not too surprising that they find that costly risk transfer is never worthwhile. Thus in 1957-8 the actuarial and financial worlds were off on quite different paradigms.

The direction of the actuarial approach is well illustrated by Gerber and Shiu 2006. They use a compound Poisson process with a sum-of-exponentials severity distribution, which has been shown to be a good approximation to any severity if you use enough exponentials. They then optimize firm value using the approach of Bellman 1954. They find that the optimal dividend strategy is a barrier strategy : keep all earnings until capital reaches a level the model determines, and pay out anything beyond that in dividends. (They do not consider share repurchasing, but for these purposes, that can be viewed as equivalent to paying dividends.) Reinsurance is also considered in the actuarial approach. Bather 1969 includes proportional reinsurance, and Asmussen, Højgaard and Taksar 2000 look at excess. The pricing assumed for excess cover makes a difference in these models, however.

Major 2007 reports on efforts to combine the actuarial and finance approaches, using the Bellman equation, to find the capital and reinsurance choices that optimize firm value while taking into account the possibility of refinancing, albeit expensively, and insurance buyer risk preferences. He uses the policyholder risk-aversion results cited above to model profitability as a function of capital. Optimal capital and reinsurance strategies are found, and the M-curve is an output from which other risk management work can then be undertaken. Further understanding of the relationship between profitability and financial stability – including more empirical quantification of the impacts of policyholder risk aversion – could improve this aspect of the model. Also the cost of distress financing has to be input. The insights of Myers and Majluf 1984 provide a starting point for that aspect, but better quantification of the costs of financing for impaired companies would improve this modeling. This is relevant for non-impaired companies as well, for the probability of becoming impaired and having to bear such costs has to be part of the calculus. Also the discount rate is an issue. The cost of capital implied by CAPM may be a starting point but probably not the final answer.

Thus risk management through firm-value modeling is progressing, but empirical and analytical work on the underlying modeling assumptions can improve this approach.

4. APPLYING RISK PRICING THEORY

The insurance industry has been reluctant to embrace financial risk pricing, such as CAPM and AFP. One problem with CAPM is that the distributional assumptions are inconsistent with the heavy-tailed distributions in some branches of insurance. There is also a strong emphasis in practice on pricing specific risk with no reference to the market risk. However ERM is breaking down the resistance to pricing in a diversified context, at least within a single insurance enterprise. Thus it is not as much as a conceptual leap to pricing from the investor point of view as it once was.

The distributional problem is perhaps the strongest. However this is not unique to the insurance industry. Rubinstein 1973 already pointed out that investor preferences related to higher moments should and do influence security prices. Kozik and Larson 2001 provide a more detailed history of the consideration of higher moments in asset pricing, and how this can be applied in insurance pricing. Basically investors prefer negative odd moments and low even moments for portfolio returns.

Fama and French 1992 report another problem with CAPM: other factors besides covariance are priced. In particular they find a higher return for larger companies, and for companies with low market-to-book ratios. There are different ways to interpret their result. One strong contender is the efficient market view: portfolios of these securities get higher returns because they have higher risk, after controlling for covariance. Efficient market theory is perhaps a faith-based matter these days, but there is some support for it here, mainly because these effects persist even though they are well-known. If they were just due to pricing mistakes, the market would have corrected for them already. However the implication is that the Fama-French factors (FFF) are surrogates for risk measures. There has therefore been some attempt at replacing them with more direct risk quantification.

Hung 2007 shows that including the 3rd and 4th co-moments gives better fits than does CAPM plus FFF, even though those factors still improve the fit further. Chung, Johnson and Schill 2006 find that the FFF become insignificant if enough higher co-moments are included. However this might require as many as 10 higher moments, which seems intricate. That gives some impetus to looking for other measures of risk besides moments.

One such possibility was suggested by Wang 2002, following Buhlmann 1980. He proposes a sort of co-moment generating function risk measure $E[Xe^{Y/c}]/Ee^{Y/c}$, which he calls the exponential tilting of X with respect to Y. (This is only weakly related to the exponential tilting method of parameter estimation discussed in Kitamura and Stutzer 1997.) If X and Y are bivariate normal and Y is the market portfolio, then a value of c can be found that makes this risk measure the CAPM price – see Landsman and Sherris 2007. Thus exponential tilting can be regarded as an extension of CAPM to non-normal risks, including with a non-normal market, without explicit reference to moments. Empirical work is needed to see how well this works, however.

AFP's pricing by probability transforms has not fared much better than CAPM in insurance practice. Even though this appears to be risk-specific pricing, there are other areas of resistance. Often the comment is heard that arbitrage pricing, being additive, does not reflect the benefit of diversification. Also there is not much secondary trading in the insurance market, so continuous hedging strategies are not available. However there is enough competition in the insurance business to force non-diversified companies to match the prices of their diversified competitors. Also the theory of pricing in incomplete markets requires pricing by transformed expected values even if hedging is not possible. Thus the objections to using this approach are a bit weak. The main problem is calibrating the pricing for different business units consistently with overall company risk, not market risk.

The interaction of AFP with CAPM is another issue. In a complete market the arbitrage-free price is unique and has an implied hedging scheme, e.g., as in the Black-Scholes formula, so covariance with the market does not have to be addressed. In an incomplete market, pricing with a transformed mean is not unique, there is no complete hedging, and just being arbitrage-free does not guarantee that risk is properly priced. This would appear to require recognition of the risk of diversified investors and dependency with market risk in the transformed-mean pricing.

One thing that helps in that context is that CAPM can be expressed as a transformed mean and so is a type of arbitrage-free price. To see this, let r be the risk-free rate, and suppose that Y has density f(y) with $EY = (1+r)Y_0$. The transformed distribution will be g(y) = f(y)h(y), and since this must integrate to unity, $Eh(Y) = 1$. Take $h(y) = 1 + b(E[M|y] - EM)$, where M is the market portfolio and b is a constant small enough that h is not negative. It is easy to see that $Eh(Y) = 1$. The transformed mean is $E^*(Y) = E[Yh(Y)] = EY + b\{E[YE(M|Y)] - [EM][EY]\} = EY + b\{E[YM] - [EM][EY]\} = EY + b\mathrm{Cov}[Y,M]$. Let the returns for Y and M be $R = Y/Y_0-1$ and $R_M = M/M_0-1$. Then $E^*(Y) = (1+r)Y_0 + b\mathrm{Cov}[RY_0, R_M M_0] = (1+r)Y_0 + bY_0M_0\mathrm{Cov}[R,R_M]$ and so $E^*(R) = r + bM_0\mathrm{Cov}[R,R_M]$. To get CAPM, bM_0 should be replaced by setting $b = (R_M - r)/(M_0\sigma_M^2)$ to get the expected return under transformed pricing of $E^*(R) = r + \mathrm{Cov}[R,R_M](R_M - r)/\sigma_M^2$. Also $E^*(Y) = EY + \mathrm{Cov}[Y,M][M-(1+r)M_0]/(M_0\sigma_M)^2$.

Comparable transforms can be made for higher co-moments as well. The n^{th} co-moment $s_n(Y, M) = E\{[Y - EY][M - EM]^{n-1}\}$. Note that this is not commutative, that is it is not usually the same as $s_n(M, Y)$ for $n \neq 2$. Higher co-moments can also be represented as probability transforms. For instance, expanding the co-3rd moment shows that it is $E[YM^2] - EM^2EY - 2E[YM]EM + 2EY(EM)^2$. Then similar arguments as for covariance show that setting $h(y) = 1 + b\{E[M^2|y] - EM^2 + 2[EM]^2 - 2E[M|y]EM\}$ gives the loaded co-3rd moment as a transformed mean. Perhaps other transforms can retain the connection with the market risk as well as distort the probabilities in other ways.

Thus AFP has the potential to represent the risk needs of a diversified investor. However this requires certain types of probability transforms. Hopefully there is a way to do this without using 10 co-moments, perhaps with exponential tilting. Further research is needed to flesh out these ideas.

Recognizing jump risk may also be necessary in order to get a suitable pricing formula for insurance risk. Arbitrage-free pricing formulas for processes including jumps are fairly widespread. See for example Kou and Wang 2004, Jang 2007 or Jang and Krvavych 2004. Ramezani and Zeng 2007 find that US stock indices can be best modeled with a mean of about 10 jumps per year in addition to a diffusion process. Dunham and Friesen 2007 find that a model with about 50 jumps per year works better for equity futures, and that jumps make up a large part of the price risk. Also a market with jump risk makes perfect hedging impossible. For all these reasons, diversified shareholders probably care about jump risk, and pricing models should price for it. Perhaps an approach would be to use co-jumps, i.e., the jumps in a process when the market has a jump. This is another area for development of pricing tools.

Defining co-jump risk is most readily done relative to a particular model of pricing movements. One possibility would be a compound-Poisson model, or other such compound frequency-severity model. The minimum-entropy martingale transform discussed below for a compound-Poisson process applies to the jump risk component by simply increasing the expected number of jumps. Another possibility would be a Levy process. In general Levy processes allow infinitely many small jumps in a finite period, but only finitely many of these are greater than ε no matter how small ε is. Thus it should be reasonable to ignore the small jumps and consider a model that is a continuous process plus a compound-Poisson process. The jumps would be at the Poisson events, and the co-jump risk could simply be a measure of the risk jumps when the market jumps. For something like earthquake insurance, the insurance losses could have a strong link with the market losses, so a reasonable first approximation to the co-jump would be the risk's own jump. Thus just pricing jump risk as idiosyncratic risk may be reasonable as a starting point.

There is something a bit artificial about a diffusion process for prices, since trades are not continuous in time. Thus all trade prices follow jumps. But if the small jumps in short timeframes follow specific patterns, a continuous model can be imagined for the price levels between trades. It is the deviation of actual prices from these patterns that makes adding jumps a better model. Adding a jump process is a way to reconcile many small price movements, which define a specific volatility, with some occasional larger movements, which have a different volatility. There might be an ambiguity between the two volatilities which could produce competing models – 50 jumps vs. 10 jumps expected. Also, while traded prices are discrete, bid and ask prices exist over continuous ranges. These can be thought of as a sort of cloud of probability around the price between trades. One potential model is a quantum-type approach, where the price process does not have actual values between trades – only the cloud of probability exists. This is perhaps easier to accept conceptually for security prices than it is for particles. If such a model would have practical implications is not clear.

The standard pricing models, perhaps with some more development as outlined above, can be regarded in a comparative framework as giving the relative risk-price adequacy among business units, which is what strategic planners are looking for. However since these models usually have a free parameter or two, it can be a problem to create a consistent comparison. One way to do this is addressed in the next section. Basically the models are calibrated for the entire firm, then brought down to individual business units through the methods of capital allocation.

5. CAPITAL ALLOCATION IN A RISK PRICING CONTEXT

As discussed above, capital allocation applied to comparing risk-adjusted returns and setting target returns is a risk-pricing exercise, but it is frequently carried out without reference to risk-pricing methodology. There are numerous ways that this can lead to misleading conclusions. The risk-measures allocated might be only loosely related to the cost of carrying the risk, for example. Also the allocation methods could be allocating too much capital to low-risk lines and too little to high-risk lines. Some allocation methods and risk measures will be reviewed in this context, then approaches that are more likely to give realistic risk appraisals are presented.

At this point some notation is needed: Y is a random variable with distribution F(y), with $Y = \sum X_j$, the sum of business units (which could even be individual policies). $\rho(Y)$ is a risk measure on Y (i.e., a functional mapping F to a real number) and r is the allocation, i.e., $\rho(Y) = \sum r(X_j)$.

A central requirement on an allocation method is that it be marginal. That is, the allocation is in proportion to the impact of the business unit on the company risk measure. This can be either last-in marginal allocation where the impact of the business unit is measured by $\rho(Y)-\rho(Y-X_j)$, the company risk measure with and without the unit, or Aumann allocation, where the impact is averaged over every coalition of business units that unit can be in, or incremental marginal allocation where the impact is $[\rho(Y)-\rho(Y-\varepsilon X_j)]/\varepsilon$, the change in the company risk measure from ceding away a small proportional part of the unit, grossed up to the size of the whole unit. In the limit $\varepsilon\to 0$ this is the derivative of the company risk measure with respect to the volume of the business unit. The derivative is taken by subtracting a small portion instead of adding one because it is easier to imagine ceding a small proportional bit of the business unit than adding one. See Mildenhall 2004 for a discussion of the importance of using a proportional (i.e., homogeneous) change.

One reason that marginal allocation is a priority in this context is the financial principle of marginal pricing. Prices should be proportional to the marginal cost of production. When the production cost is basically the risk cost, as it is in insurance, the price for bearing risk should be based on the marginal risk of the business being priced.

If the incremental marginal impacts add up to the whole risk measure, the allocation is sometimes called a marginal decomposition of the company risk measure. By Euler's Theorem, this happens when the risk measure is homogenous degree 1, i.e., for a positive constant k, $\rho(kY) = k\rho(Y)$. Marginal decomposition is also called Euler allocation. It was introduced to the actuarial literature in Patrik, Bernegger and Rüegg 1999. Several examples are derived in Venter, Major, and Kreps 2006, who also show how to apply Euler's Theorem to sums of random variables.

As an example, the standard deviation has a marginal decomposition equal to the covariance of the unit with the company, divided by the standard deviation of the company. Showing this is the marginal decomposition involves taking the derivative of the risk measure of the firm wrt X_i , that is, $r(X_i) = \lim_{\varepsilon \to 0} \frac{\rho(Y) - \rho(Y - \varepsilon X_i)}{\varepsilon}$. Often the limit is most easily accomplished by l'Hôpital's rule, which gives $r(X_i) = -\rho'(Y-\varepsilon X_i)|_0$, where the prime denotes the derivative wrt ε. For standard deviation, the derivative of $-\mathrm{Std}(Y-\varepsilon X_i)$ is $-[\mathrm{Var}Y-2\varepsilon\mathrm{Cov}(X_i,Y)+\varepsilon^2\mathrm{Var}(X_i)]^{1/2}$ which gives $r(X_i) = \mathrm{Cov}(X_i,Y)/\ \mathrm{Std}(Y)$ at $\varepsilon = 0$.

Another reasonable requirement on capital allocation is that it is suitable, as defined by Tasche 2000. This is really a joint requirement on the risk measure and the allocation method. To define it, suppose that you allocate capital by the allocation of a risk measure, compute the return on capital so allocated, and then slightly proportionally increase the size of a business unit that has a higher-than-average return on capital. Assuming also that the firm capital increases by the increase in its overall risk measure, if the allocation is suitable this exercise will increase the return on capital for the firm. Venter, Major and Kreps 2006 show that marginal decomposition always produces a suitable allocation, and it appears that it is the only method that guarantees suitability. If there is a marginal allocation possible, but another allocation is used, a unit that has a below average return on marginal capital but an above average return on actual allocated capital will violate suitability. Suitability seems to be a fundamental property that an allocation should provide, and adds further support to requiring that allocation be a marginal decomposition.

Other marginal decompositions are the natural allocations of VaR and TVaR, sometimes called co-VaR and co-TVaR. For instance, TVaR at the 99% level is $\rho(Y) = E[Y|\ Y > F^{-1}(0.99)]$ and co-TVaR is $r(X_j) = E[X_j\ |\ Y > F^{-1}(0.99)]$. VaR just substitutes "=" for ">" in the condition.

A less trivial example is risk-adjusted TVaR, or RTVaR, defined by Furman and Landsman 2006 (although they call it something else). With parameters α and c this is:

$$\rho(Y) = E[Y|\ Y > F^{-1}(\alpha)] + c\,\mathrm{Cov}(Y,Y|F(Y)>\alpha)\,/\mathrm{Stdev}[Y|F(Y)>\alpha]$$

$$= \mathrm{TVaR}_\alpha + c\,\mathrm{Stdev}[Y|F(Y)>\alpha].$$

The marginal decomposition, which we could call co-RTVaR, is:

$$r(X_j) = \text{co-TVaR}_\alpha(X_j) + c\,\text{Cov}(X_j,Y|F(Y)>\alpha)\,/\text{Stdev}[Y|F(Y)>\alpha].$$

Even if an allocation is marginal, it might not meet other reasonable pricing criteria like pricing all risk elements and risk price increasing more than linearly with potential loss. For instance, tail risk measures like VaR and TVaR ignore smaller losses which do contribute to the overall risk. If the tail probability level is chosen low enough to include all scenarios worse than the mean, TVaR does not ignore the small hits. However TVaR is linear in losses above the threshold, which is not consistent with how risk is usually viewed. A number of insurers have found that allocating by TVaR over a low threshold attributes less risk than seems reasonable to units exposed to large losses, which is a consequence of the linearity property.

A reasonable alternative is to use RTVaR over a low threshold. RTVaR increases more than linearly and the low threshold would include the smaller losses. This also meets the more general standard of being plausible as a risk-pricing metric, as standard-deviation loading is a traditional risk-pricing method. A similar effect can be produced by taking the risk measure to be a weighted sum of TVaR's at a few probabilities, from quite low to quite high. For instance, 60%, 90%, 98% and 99.6% could be used. This would include the impact of smaller losses, but emphasize the large losses. There is a good deal of flexibility here, which could be used to calibrate risk loads to market prices.

Standard-deviation loading is weak for extreme right-tail risks, like high layers of catastrophe reinsurance programs. It has been adapted to work by using higher percentages of standard deviation for higher layers, and imposing minimum rates on line, but in the end this shows that the risk prices are not all proportional to their standard deviations. Yet these ad hoc approaches do address some of the major problems that come from allocating TVaR and from not using marginal allocation.

A less ad hoc approach may be to incorporate elements of AFP into the capital allocation exercise. AFP pricing sets prices as the expected values of the risk using transformed probabilities. Essentially the less favorable outcomes are given more probability, and the more favorable outcomes less probability, so that the transformed mean is higher than the actual mean. The difference can be considered to be a risk load.

For this the discussion will not be limited to so-called coherent risk measures. Artzner etal. 1999 define the concept of a coherent risk measure as one meeting a few mathematical requirements, the most controversial and most often failing being subadditivity: the risk measure of a sum of independent random variables should not be greater than the sum of their risk measures. This is a useful criterion if the question being addressed is measuring the diversification benefit of combining business units, and the analyst wants to guarantee in advance that the answer will not be negative. Otherwise it is not a necessary requirement. Since marginal allocation does not use the risk measures of individual units, but rather considers their contributions to the risk of the whole, subadditivity is not needed in this context.

There are some well-known links between capital allocation by risk measure and probability transforms. Wang 1996 defines a distortion measure as one that can be specified by a distribution function G(x) on the unit interval (formally the requirement on G is: G(0)=0, G(1) = 1 and G is non-decreasing) so that the risk measure $\rho(Y)$ is $\int_0^\infty G[S(y)]dy$, where $S(y) = 1 - F(y)$ is the survival function of Y. Actually G[S(y)] is itself a survival function since it is non-increasing and starts at 1 and goes to 0. Thus the role of G is to transform the probabilities of Y to another probability distribution. Since the mean of a distribution is the integral of its survival function, a distortion risk measure is a transformed mean. The density of the transformed distribution is the negative of the derivative of its survival function so $f^*(y)=G'[S(y)]f(y)$ and $\rho(Y)=E^*[Y]=\int_{-\infty}^{\infty} YG'[S(y)]f(y)dy=E[YG'[S(Y)]]$.

Examples of such distortion measures are $G(p) = p^a$ (proportional hazards transform) and the Wang transform $G(p) = 1 - T_a[\Phi^{-1}(1-p) + \lambda]$, where T_a is the t-distribution function with *a* degrees of freedom, and Φ is the standard normal distribution. The original form of the Wang transform used Φ instead of T_a. $VaR_{0.99}$ and $TVaR_{0.99}$ are also distortion measures. They both have G(p) = 1 if p > 0.01. G[S(y)] is then 1 for F(y) < 0.99, so the part of the integral from 0 to $F^{-1}(0.99)$ is $F^{-1}(0.99)$. VaR has G(p) = 0 otherwise, whereas TVaR has G(p) = p/0.01 otherwise.

Another example is the Esscher transform. With parameter ω, let $c = S^{-1}(1/\omega)$ and $k = E[e^{Y/c}]^{-1}$. The Esscher transform of the density is $f^*(y) = f(y)ke^{y/c}$. Thus the transform is $E^*[Y]= E[Yke^{Y/c}]$. To show that this is a distortion, a G is needed so that $E^*[Y]=\int_{-\infty}^{\infty} YG'[S(y)]f(y)dy=E[YG'[S(Y)]]$. Define $G(p) = k\int_0^p \exp[S^{-1}(q)/c]dq$. Then $G'[S(y)] = ke^{y/c}$ so $E^*[Y]=E[YG'[S(Y)]]$.

Also the Esscher transform scales. That is, if Z = bY, then with the same ω, $c_Z = bc_Y$. Then $k_Z = k_Y$ so $E^*[bY] = E[bYke^{Y/c}] = bE^*[Y]$.

Distortion measures tie in to AFP and so are plausible as pricing tools. However some distortion measures satisfy the criteria of using the entire distribution of losses and being non-linear in large losses, and some fail to. Balbás, Garrido and Mayoral 2009 define types of distortion measures that narrow down the field. First they define a complete risk measure as one that uses the entire probability distribution of Y in a non-trivial way. This can be formally defined by requiring that G(p) is not constant on any interval, and so is an increasing function on the unit interval. No tail measures satisfy this definition. The motivation is that if the risk measure is to be used to express preferences among random variables, this cannot be done using the tail alone.

They define an adapted risk measure as one meeting two more requirements. If a risk measure is going to be used in pricing, typically you would not want it to be less than the mean of Y. For a distortion measure this requires that $G(p) \geq p$. However another practical requirement is that in the tail the relative risk load is unbounded. This would be needed to get behavior similar to a minimum rate on line, where the ratio of risk load to expected losses can be very large. For distortion measures this can be expressed as $G'' < 0$ and G' goes to infinity at $p = 0$. The Wang and Esscher transforms are examples.

Transformed means are arbitrage-free if the transformation is made on the probabilities of events. However this may not be the case if the transform is on the probabilities of outcomes of deals – see Ruhm 2003, whose title "Distribution-Based Pricing Formulas Are Not Arbitrage-Free," says it. Also see the discussion of Venter 2004. Distortion measures can be applied either way, so may or may not be arbitrage-free.

As an example, consider a random variable Y which is a loss severity variable with $S(y) = (1+y)^{-2}$ for $y < 99$ and a point mass of one basis point (0.01%) at $y = 99$. This has mean 0.99, so the maximum loss is 100 times the mean. Suppose there are two tranches of loss: Tranche A pays if the loss is below 0.5, and Tranche B pays the entire loss otherwise. This is an awkward situation for probability transforms, as they move probability away from smaller losses to larger losses. Thus they can give prices for Tranche A below its expected value. However this depends on whether the transform is made on the probability distribution for Y or separately on the distributions of the tranches.

For instance, consider the original form of the Wang transform with $\lambda = 0.1$. For loss severity, the Wang transform can be expressed as $S^*(y) = \Phi[\Phi^{-1}[S(y)] + \lambda]$. For calculations it is convenient to use $f^*(y)$, which can be estimated numerically. The expected values of Y, Tranche A and Tranche B are 0.990, 0.111, and 0.879. Transforming the probabilities of Y and each tranche separately gives prices of 1.219, 0.119, and 1.132, respectively. The prices of the tranches sum to 1.250, which is greater than the price for Y as a whole. This is an example of the subadditivity of the Wang transform. On the other hand, if the transformed probabilities for Y are used to compute prices as the expected values of the tranches, they become 0.107 and 1.112. These do add up to the price for Y, as is required by arbitrage-free pricing, but now the price for Tranche A is below its expected value.

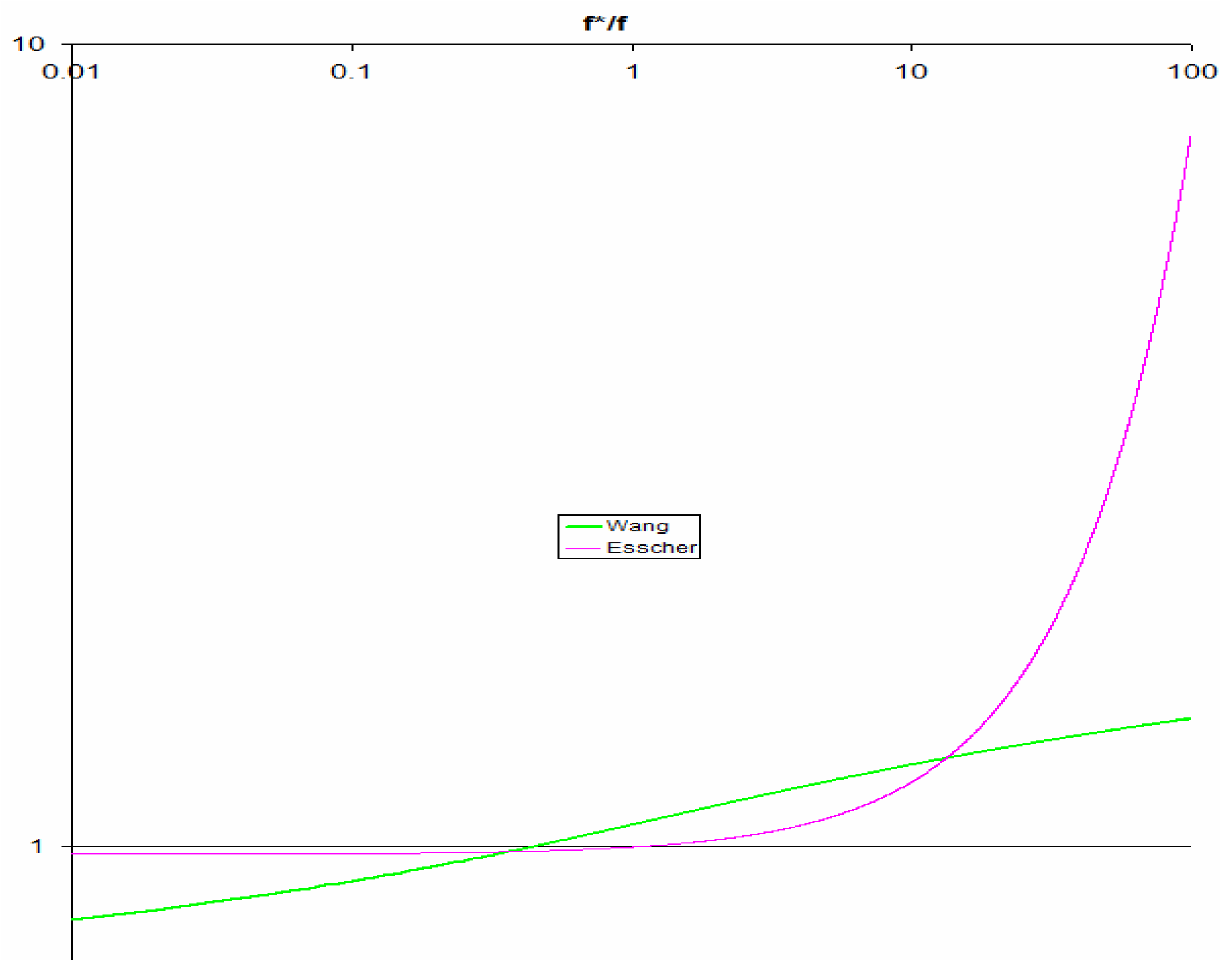

Figure 1 Ratios of Transformed to Actual Density

The situation is similar for the Esscher transform. Taking $\omega = 2401$ gives $c = 48$ and $E^*[Y] = 1.219$. The prices for the tranches are 0.109 and 1.110 with this transform. Despite the similarity in results, the transformed probabilities are quite different. Figure 1 graphs the ratio f^*/f for both on a double log scale. The lowest the ratio gets is about 80% for the Wang transform and 98% for the Esscher transform, but the Esscher ratio gets much higher for the largest losses, which have low probabilities to begin with.

If Y is a compound frequency-severity process, a combined transform can be defined that simultaneously applies to the frequency and severity probabilities. A popular transform in the finance literature is the minimum entropy martingale measure, which is the martingale that minimizes a particular information distance measure from the original process. Møller 2004 shows how to apply this to an insurance profit process consisting of a fixed premium flow minus compound Poisson losses. The result is just the Esscher transform for severity with a related increase in expected frequency:

$$\lambda^* = \lambda/k \text{ , } f^*(y) = f(y)e^{y/c}/Ee^{Y/c}.$$

Venter, Barnett and Owen 2004 find that applying this transform to ground-up losses provides a reasonable representation of catastrophe reinsurance prices, including high risk loads that resemble minimum rates on line for high layers.

The minimum entropy transform of the mean is just the Esscher transform of severity times $1/k = E[e^{Y/c}]$. But no severity probability decreases by more than a factor of k, so the minimum entropy transform is arbitrage-free and results in positive risk loads. For example, taking ω about 2745 results in a transformed mean for Y of 1.219, with transformed tranche means of 0.112 and 1.107, which are both above their actual means. In this case k is about 0.979. This transform can be computed by just applying the Esscher transform to severity then dividing the mean by k. Probably the same thing could be done for the Wang transform, using the lowest ratio of f^*/f as an estimate of k.

In a simulation model, the events represented are the simulated scenarios. Thus to apply a distortion measure in an arbitrage-free manner, the probabilities of the scenarios would be modified, and the transformed probabilities then applied to the results of every business unit in the scenario. Those business unit results are the sums of numerous claims, so this method does not directly provide frequency or severity transforms for the units, but does give enough to compute transformed means for each unit. These would provide a marginal decomposition of the firm transformed mean.

A complication is that the events are the losses before the application of reinsurance, or at least before the application of any reinsurance that could be changed in the analysis being undertaken. The probabilities for the scenarios without reinsurance would be retained and applied to the same scenarios after application of reinsurance. This is what is required by AFP but also avoids a common problem with capital allocation: the worst scenarios after reinsurance are often not the worst scenarios before reinsurance. Thus capital allocation before and after often uses different scenarios or different weights on scenarios. This can result in the purchase of reinsurance by one line decreasing the overall capital need but increasing the imputed capital allocated to a line not buying reinsurance.

Since there is at least one parameter in any of the transforms, the parameters would have to be calibrated to provide the target profit-load for the whole firm. This is usually a number that a company will know. However if the target profit is to be applied after reinsurance, the calibration would have to be done on the parameters of the pre-reinsurance scenarios in a way that would produce the target profit-load when applied after reinsurance.

This would be fairly simple in practice. Say the Wang transform is selected. Wang's 2004 results suggest that taking a around 5 or 6 is consistent with some market prices, so the main issue is the choice of λ. The scenarios would be sorted by firm loss to get the observed survival function. Then the transformed mean can be calculated as a function of λ, and the λ that gives the target return for the firm can be found. This would then give the transformed probability for each scenario, and these then give the transformed unit means. The minimal entropy transform might be the best one to use in practice, since it avoids negative risk loads and heavily penalizes the tail.

Transformed distributions can be applied to other statistics than the mean. TVaR under transformed probabilities is called weighted TVaR, or WTVaR. This is no longer linear in large losses, so if it is used over a low probability threshold, it also meets the pricing criteria. This is similar to the use of RTVaR discussed above but may work better for tail losses.

6. CONCLUSIONS

Capital allocation is an attempt to do risk pricing while avoiding the rigors of the pricing project. But to work well it has to face up to the problems of risk pricing and incorporate them. This can be done to some degree within the typical ERM setting if specific risk measures and risk attribution methods are employed. An alternative is direct risk pricing. However, risk pricing needs further development for this application, such as addressing higher moments and jump risk. Also its emphasis on market price for risk needs to be further tailored to get to the adequacy of market prices given a firm's unique risk profile. The impact on firm value is the bottom line for strategic and pricing decisions. Better understanding of the relationship between financial strength and profitability is the key to advancing firm-value models.

Marginal decomposition of a transformed mean on the company level is the transformed mean of the business unit, where the transform uses the transformed probabilities of the aggregate firm variable. This is theoretically stronger than allocating other risk measures, but not as directly applicable as modeling firm value. It is probably in-between the two in degree of difficulty as well.

References

[1] Artzner, P., Delbaen, F., Eber, J.-M., Heath, D. 1999. Coherent measures of risk. *Math. Fin.* 9(3):203–228.

[2] Asmussen, S., B. Højgaard and M. Taksar. 2000. "Optimal Risk Control and Dividend Distribution Policies: Example of Excess-of Loss Reinsurance for an Insurance Corporation." *Finance and Stochastics* 4(3):299–324.

[3] Balbás, Alejandro, José Garrido and Silvia Mayoral. 2009. "Properties of Distortion Risk Measures," *Methodology and Computing in Applied Probability*, to appear.

[4] Bather, J.A. 1969. "Diffusion Models in Stochastic Control Theory." *J. Royal Statistical Society A.*, 132: 335–352.

[5] Bellman, Richard E. 1954. The Theory of Dynamic Programming. Santa Monica, California: The RAND Corporation.

[6] Buhlmann, Hans. 1980. "An Economic Premium Principle," *ASTIN Bulletin* 11: 52–60.

[7] Chung, Y. Peter, Herb Johnson and Michael J. Schill. 2006. "Asset Pricing when Returns Are Non-normal," *Journal of Business* 79(March):923-940.

[8] De Finetti, Bennetto. 1957. "Su un' impostazione alternativa della teoria colletiva del rischio." *Transactions of the XVth International Congress of Actuaries* 2:433-443.

[9] Dunham Lee M. and Geoffrey C. Friesen. 2007. "An Empirical Examination of Jump Risk in U.S. Equity and Bond Markets," *NAAJ* 11(4):76-91.

[10] Epermanis, Karen and Scott E. Harrington. 2006. "Market Discipline in Property/Casualty Insurance: Evidence from Premium Growth Surrounding Changes in Financial Strength Ratings," *Journal of Money, Credit, and Banking* 38(6) 1515:1544.

[11] Fama, Eugene and Kenneth French. 1992. "The Cross-Section of Expected Stock Returns,".*Journal of Finance*, 47(June):427-465.

[12] Froot, Kenneth. 2003. "Risk Management, Capital Budgeting and Capital Structure Policy for Insurers and Reinsurers." *NBER Working Papers 10184*, National Bureau of Economic Research, Inc.

[13] Froot, Kenneth., D. Scharfstein and J. Stein. 1993. "Risk Management: Coordinating Corporate Investment and Financing Policies." *Journal of Finance* 48:1629–1658.

[14] Furman, E., and Zinoivy Landsman. 2006. "Tail Variance Premium with Applications for Elliptical Portfolio of Risks." *ASTIN Bulletin* 36(2): 433–462.

[15] Gerber, Hans U., and Elias S. W. Shiu. 2006. "On Optimal Dividend Strategies in the Compound Poisson Model." *North American Actuarial Journal* 10(2):76–93.

[16] Grace, Martin. F., Robert W. Klein and Paul R. Kleindorfer. 2004. "The Demand for Homeowners Insurance with Bundled Catastrophe Coveages," *The Journal of Risk and Insurance,* 71(3):351-379.

[17] Gründl, Helmut and Hato Schmeiser. 2007. "Capital Allocation for Insurance Companies—What Good Is It?" *The Journal of Risk and Insurance,*74(2):301-317.

[18] Hung, Chi-Hsiou. 2007. "Momentum, Size and Value Factors versus Systematic Co-moments in Stock Returns," WP-105, Apr. 07 ISSN: 1749-3641 (Online).

[19] Jang, Ji-Wook. 2007. "Jump Diffusion Processes and their Applications in Insurance and Finance," *Insurance: Mathematics & Economics*, 41(1):62-70.

[20] Jang, Ji-Wook and Yurily Krvavych. 2004. "Arbitrage-free premium calculation for extreme losses using the shot noise process and the Esscher transform," *Insurance: Mathematics & Economics*, 35(1):97-111.

[21] Kitamura, Y. and Stutzer, M. 1997. "An information-theoretic alternative to generalized method of moments estimation," *Econometrica* 65:861–874.

[22] Kou, Stephen G. and Hui Wang. 2004. "Option Pricing Under a Double Exponential Jump Diffusion Model," *Management Science* 50(9):1178-1192.

[23] Kozik, Thomas J. and Aaron M. Larson. 2001. "The n-Moment Insurance CAPM," *PCAS LXXXVIII*:39-63.

[24] Landsman, Zinoviy and Michael Sherris. 2007. "An Actuarial Premium Pricing Model for Nonnormal Insurance and Financial Risks in Incomplete Markets," *NAAJ* 11(1):119-135.

[25] Major, John. 2007. "Measuring the Market Value of Risk Management: An Introduction to FLAVORED Models." In *Enterprise Risk Management for Property & Liability Insurance Companies.* Brehm, P.J., Perry, G.R., Venter, G.G., and Witcraft, S.E., eds. New York: Guy Carpenter & Company.

[26] Mayers, David and Clifford W. Smith. 1982. "On the corporate demand for insurance," *Journal of Business* 55: 281-296.

[27] Mayers, David and Clifford W. Smith. 1990. "On the Corporate Demand for Insurance: Evidence from the Reinsurance Market," *Journal of Business*, 63(1):19-40.

[28] Merton, Robert and Andre Perold. 1993. "Theory of Risk Capital in Financial Firms," *Journal of Applied Corporate Finance*, Fall.

[29] Mildenhall, Stephen J. 2004. "A Note on the Myers and Read Capital Allocation Formula," *North American Actuarial Journal* 8(2):32-44.

[30] Modigliani, F. and M. H. Miller. 1958. "The cost of capital, corporation finance and the theory of investment," *American Economic Review* 48:261-297.

[31] Møller, Thomas. 2004. "Stochastic orders in dynamic reinsurance markets," *Finance and Stochastics* 8(4):479-499.

[32] Myers, Stewart C., and Nicholas S. Majluf. 1984. "Corporate Financing and Investment Decisions when Firms Have Information That Investors Do Not Have." *Journal of Financial Economics,* 13:187–221.

[33] Myers, Stewart C., and James A. Read, Jr. 2001. "Capital Allocation for Insurance Companies" *The Journal of Risk and Insurance* 68(4):545–580.

[34] Patrik, Gary, Stefan Bernegger and Marcel Beat Rüegg. 1999. "The Use of Risk Adjusted Capital to Support Business Decision-Making," *CAS Forum*, Spring.

[35] Phillips, Richard D., David Cummins and Franklin Allen. 1998. "Financial Pricing of Insurance in the Multiple-Line Insurance Company," *The Journal of Risk and Insurance*, 65(4):597-636.

[36] Ramezani, Cyrus A. and Yong Zeng. 2007. "Maximum likelihood estimation of the double exponential jump-diffusion process," *Annals of Finance* 3:487–507.

[37] Rubinstein, Mark E., 1973 "A Comparative Statics Analysis of Risk Premiums," *Journal of Business* 46:605-615.

[38] Ruhm, David L., 2003 " Distribution-Based Pricing Formulas Are Not Arbitrage-Free," *PCAS* XC:97-129.

[39] Sherris, Michael. 2006. "Solvency, Capital Allocation, and Fair Rate of Return in Insurance," *The Journal of Risk and Insurance*, 73(1):71-96.

[40] Sommer, David W. 1996. "The Impact of Firm Risk on Property-Liability Insurance Prices," *The Journal of Risk and Insurance*, 63(3):501-514.

[41] Staking and Babbel. 1995. "The Relationship between Capital Structure, Interest Rate Sensitivity, and Market Value in the Property-Liability Insurance Industry," *The Journal of Risk and Insurance*, 62(4):670-718.

[42] Tasche, Dirk. 2000. "Risk Contributions and Performance Measurement." Report of the Lehrstuhl fur Mathematische Statistik, T.U. Munchen.

[43] Venter, Gary. 2004. "Discussion of Distribution-Based Pricing Formulas Are Not Arbitrage-Free," *PCAS* XCI:25-33.

[44] Venter, Gary, Jack Barnett and Michael Owen. 2004. "Market Value of Risk Transfer: Catastrophe Reinsurance Case," *International AFIR Colloquium*.

[45] Venter, Gary, John Major and Rodney Kreps. 2006. "Marginal Decomposition of Risk Measures." *ASTIN Bulletin* 36(2):375–413.

[46] Wang, Shaun S. 1996. "Premium calculation by transforming the layer premium density," *ASTIN Bulletin* 26(1):71–92.

[47] Wang, Shaun S. 2002. "A Set of New Methods and Tools for Enterprise Risk Capital Management and Portfolio Optimization," *CAS Forum* (summer):43–78.

[48] Wang, Shaun S. 2004. "Cat Bond Pricing Using Probability Transforms," *Geneva Papers: Etudes et Dossiers, special issue on Insurance and the State of the Art in Cat Bond Pricing*, 278(January):19-29.

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