A PREDICTIVE EARTHQUAKE MODEL AND ALTERNATIVE RISK TRANSFER TECHNIQUES

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Abstract

Although, traditional reinsurance has successfully covered the economic consequences of the large natural disasters, there is still a great discussion for alternative risk transfer techniques. In this article, we investigate the data of the earthquakes in Greece. Actually, we explore all the basic characteristics of an earthquake as normally reported by seismologists and the potential relationship with the volume of the respective damages. Then, we design a stochastic model using the tools of extreme value theory and after some necessary calibration we use it as the basic framework for pricing special derivative products. Numerical results are provided for the potential Greek market.

Key-words: Extreme Value Theory, Catastrophe Bonds, Monte Carlo Simulation, Incomplete Market.

1. Introduction

In the insurance and re-insurance market, the notion of the securitization of *catastrophe risks* became very prominent after the hurricane Andrew in 1992 with losses of $19.6 billion and the Northridge earthquake in 1994 with losses of $14.9 billion; see ***R**einsurance **A**ssociation of **A**merica* (**RAA**) Nutter (2002). After that, several colossal insurance and re-insurance companies in USA, such as AIG, Hannover Re, St. Paul Re and USAA, were forced to complete the first experimental transactions to bring more risk-bearing capacity into the catastrophe (re-)insurance market. Consequently, new risk-linked securities, which were created and used in the mid-1990s in the aftermath of the two largest natural disasters on record in the U.S.A., were **Catastrophe (CAT) Bonds**, which is also named as an “Act of God bond” or “insurance-linked bond”.

According to Swiss Re Capital Markets data, the value of outstanding CAT bonds increased substantially from 1997 through 2004 (figure 1) about 615%. However, the value of $4.3 billion was small compared to industry catastrophe exposures. For instance, a 5 Saffir-Simpson scale hurricane striking densely populated regions of Florida alone could cause more than an estimated $50 billion in insured losses; USGAO (2005).

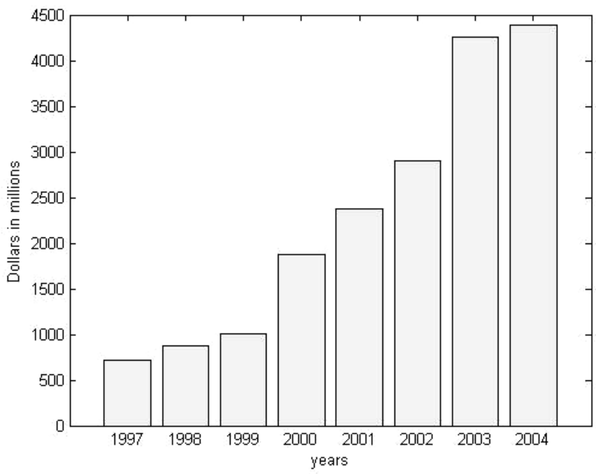


Figure 1: Catastrophe Bond Amount Outstanding, Year-end 1997 – 2004

(Source: USGAO analysis of Swiss Re Capital Markets data).

Under the terms of non indemnity-based catastrophe bonds, for the sponsoring insurance company to collect part or the investors’ entire principal when the catastrophe occurs, an independent third party must confirm that the objective catastrophic event was met, such as an earthquake reaching i.e. 7.0 in local magnitude as reported by the National Geological Survey. Moreover, the ***B**ond **M**arket **A**ssociation* (**BMA**) Miller (2002) commented that there are often compelling reasons for sponsors of the risk-linked securities to use non indemnity-based structures. For instance, they can more effectively shield the confidentiality of the sponsor’s underwriting criteria and provide for more streamlined deal structuring and deal execution, as well. Moreover, they may facilitate a more rapid payout in response to triggering events.

Thus, in this paper, we will use the special framework explained above to create a CAT bond for the earthquakes placed in the Greek boarder area. Thus, the creation of a CAT bond for Greece provides a secure mechanism for direct transfer of major catastrophic earthquakes' casualties to capital markets. This is one way to debilitate the homeowners' insurance market and keep earthquake insurance available at affordable prices.

A brief outline of the paper follows. Section 2 provides an analysis of some significant elements and notions from the fields of geophysics and seismology. Actually, we explore all the basic characteristics of an earthquake as normally reported by seismologists and the potential relationship with the volume of the respective damages. In this section, we provide some more details about where and what kind of catastrophes are expected from an earthquake in the broader area of Greece. In Section 3, a quick overview of the theory for pricing catastrophe bonds for incomplete markets is provided. Moreover, we build a completely parameterized, one-period, model for pricing process of CAT bonds. Section 4 relies extensively on multivariate Extreme Value Theory. We provide some preliminary results for the calculation of the distribution function of the annual maximum local magnitude of the earthquakes and their depths, respectively, in the boarder area of Greece for the period 1966-2008. The final section 5 provides a general overview of the results and reveals other potential directions for further research.

2. Earthquakes and earthquake damage

The first significant step of our analysis takes us into the realm of geophysics and seismology. Actually, we explore all the basic characteristics of an earthquake as normally reported by seismologists and the potential relationship with the volume of the respective damages. Thus, before talking about the modelling, the extreme value theory and the pricing process of a CAT bond, one needs to understand somewhat what earthquakes are, how they are measured, where they can cause serious damages, and what kind of damage they can create in the boarder area of Greece. There is a somewhat warped perception of the kind of damage to expect from an earthquake in Greece, so it is helpful to understand some of the facts.

In the science of seismology, every scientist very well knows the theory of the *litho-spheric plates* and their relative *motions*. In the case of Greek territory, McKenzie (1972, 1978) has presented a small-scale lithospheric model, which is presented in the following figure.

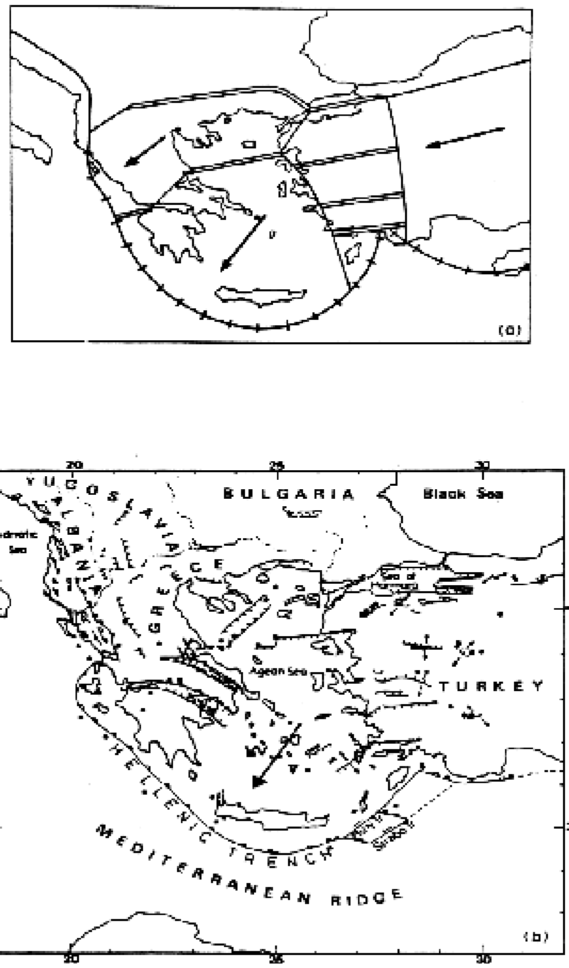


Figure 2: McKenzie's (1972, 1978) lithospheric plate model for the Greek territory

As these plates are in continuous motion due to applied forces from the surrounding larger plates, extensional or compressional fracture zones are formed. The kinematics of the Greek territory tectonic plates have been extensively studied by Papazachos and Kiratzi (1996) and are presented in the following figure 3.

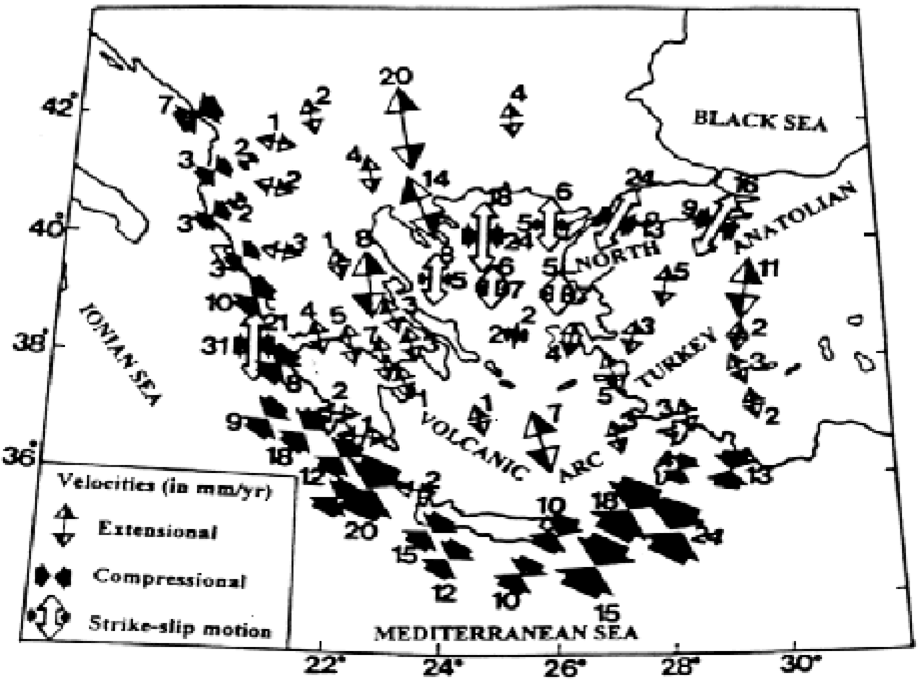


Figure 3: The kinematics of the Greek territory tectonic plates (Papazachos et al. 1996).

It can be clearly understood that the larger seismic events are expected to occur where the larger tectonic activities take place. Thus, the above map (see, figure 3) can provide us with a general overview for the *future* seismic activities in the boarder Greek territory. Consequently, for the creation of a CAT bond relative to earthquakes, the mapping of fracture zones-faults is of a great importance and high priority. These zones are the places where earlier large earthquakes took place and the future earthquakes might be occurred. The problem that arises is what faults-fracture zones must be mapped and whether it is always possible or not.

Actually, the classical geological mapping from the surface observations can not detect all the existent faults-fracture zones. This is especially true when no surface trace has been produced by any deep subsurface tectonic activity. Furthermore, it is extremely difficult to assign a seismological significance to a detected surface fault without any other deep tectonic knowledge. For instance, on the 13th of May 1995, a devastating earthquake (Local Magnitude 6.1R, $40^0.18$, $21^0.71$, see also Table 1) occurred at *Grevena* area, northern Greece that caused many damages at the nearby towns and villages (some deaths were also reported). What also is very interesting on this event is the fact of the total absence of any statistical indication that an earthquake could take place in an area being characterized of a very low seismic risk (zone 1 –safe– out of 4). Moreover, the total absence of any known large tectonic features that could justify any possible large seismic event should also concern us. Unfortunately, the very same scenario was replicated on the 7th of September 1999 on Athens, the capital city of Greece. Although Athens can be characterized as zone 2 (partially safe) as far as it concerns the seismic risk, the absence of knowledge of deep tectonics has labeled the capital of Greece that is has been built over “safe” ground. Unfortunately, for the people (over a hundred) that died, apart from the very large damages in buildings and properties, during this earthquake, this proved of being totally false. Therefore, into the pricing process of an earthquake CAT bond, the question that always arises has to do where the nest large earthquakes can take place and what is the level of damages that can occur. The following elements may be the answer to this.

It is well known that large earthquakes occur when a fracture zone/fault releases its stress load and rupture occurs along a large part of it. The longest the fault/fracture zone is being activated, the largest the magnitude of the earthquake is. The problem therefore is formulated as follows: Is it possible to map the deep fracture zones where the large earthquakes occur? The study of the large earthquakes in Greece, statistically treated, may reveal vaguely the large deep tectonic features that are prerequisites for its occurrence. The same problem, of the detection of deep tectonics of an area, faced from the view of applied geophysics, is rather simple, as it will be explained later on.

The key point of this section is that large earthquakes must coincide in location to large tectonic faulting/fracturing systems of the lithosphere. For having a clear view on this topic, in terms of large seismic events of the Greek territory, the following map (figure 4) has been compiled, where the location of the larger earthquakes, that occurred during the period of 1966-2008, are presented.

Moreover, we have produced 5 (five) bigger ordered circles (numbered from 1 to 5) based on the length of the circle line) where the most significant earthquakes took place. Note that we have also included into those circles the earthquakes, which appeared to be the most catastrophic. Thus, we have included the capital of Greece, Athens (circle 1), the area of Thessalonica (see circle 2), the southeast part of Aegean, i.e. Dodecanese -

Rhodos island (circle 3), the Ionian - Zakynthos and Kefalonia islands (circle 4) and the region of Achaia, i.e. Patra (circle 5). Actually, these five circles include the most populated towns in Greece, where more than 70% of the total population are living.

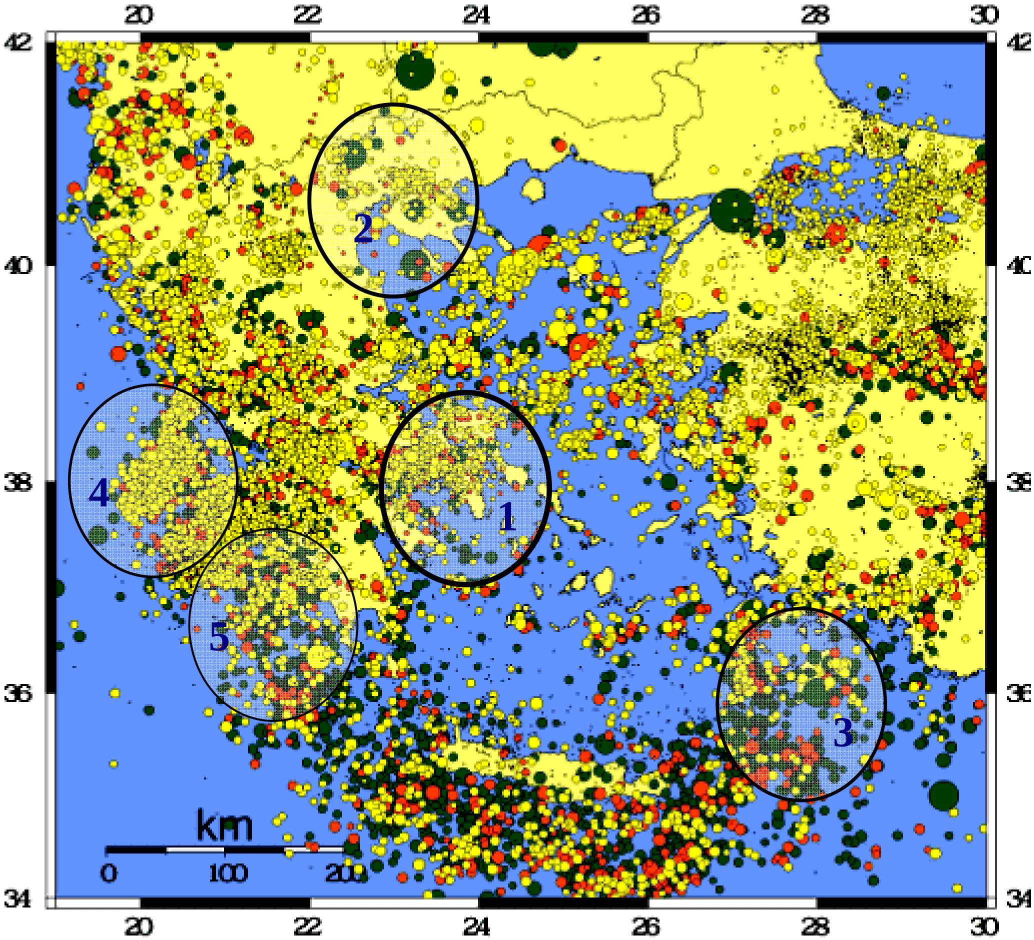
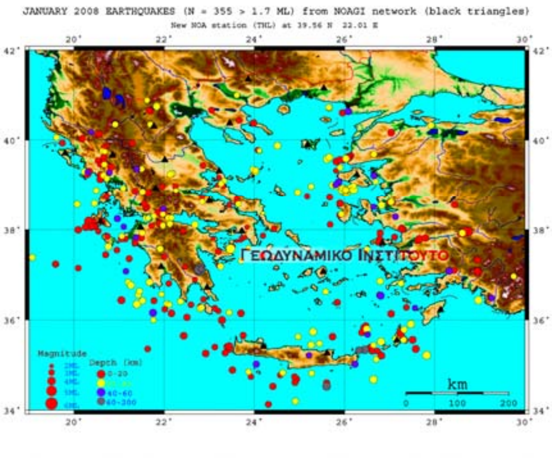
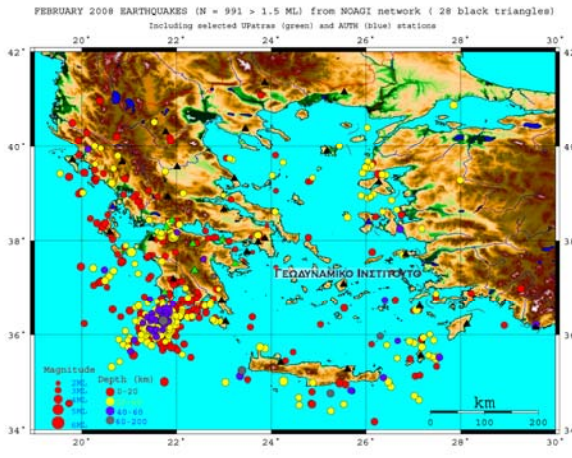
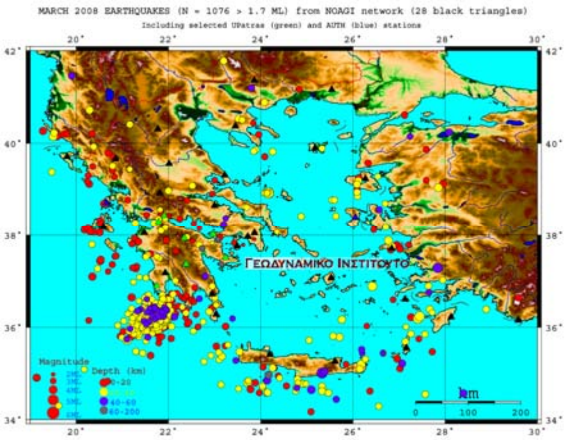


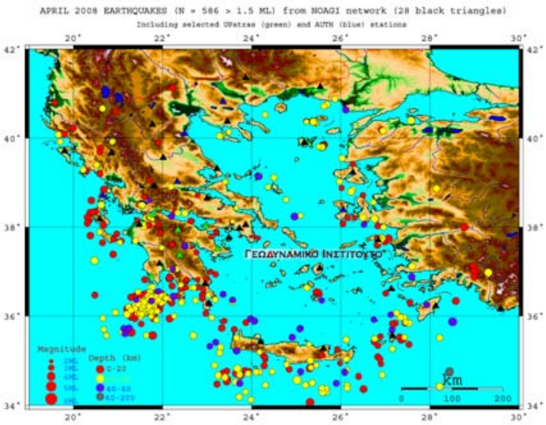
Figure 4: Past Earthquakes in the region of Greece
(Source: Institute of Geodynamics, National Observatory of Athens (IG-NOA))

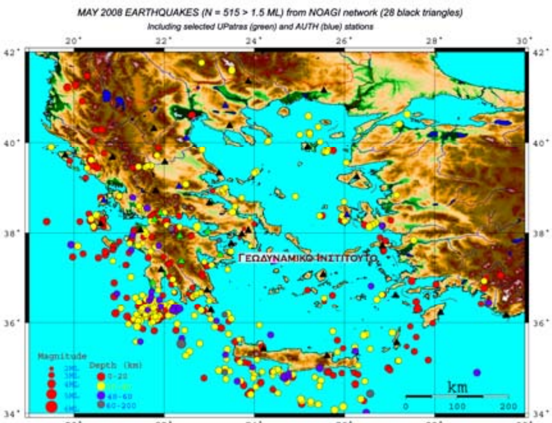
In the next figure, the most important earthquakes in the boarder area of Greece, during the year 2008 are presented. This figure provides a clear view of the seismic activities in the boarder Greek territory during the 12th months of the year 2008, where one-year model can be constructed, see 3rd section.

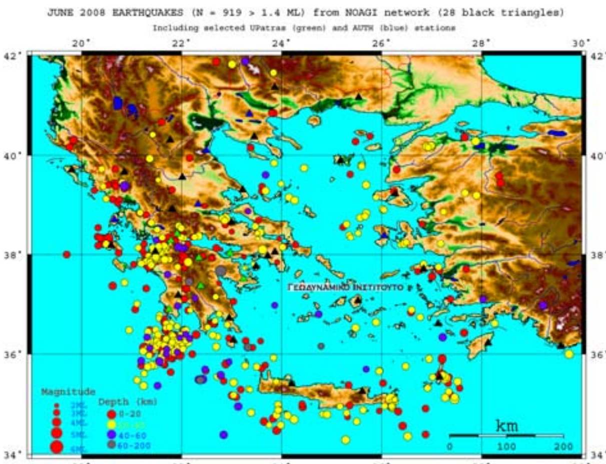

JANUARY 2008 EARTHQUAKES (N = 355 > 1.7 ML) from NOAGI network (black triangles)
New NOA station (THL) at 39.56 N 22.01 E
ΓΕΩΔΥΝΑΜΙΚΟ ΙΝΣΤΙΤΟΥΤΟ
Magnitude
Depth (km)
km


FEBRUARY 2008 EARTHQUAKES (N = 991 > 1.5 ML) from NOAGI network (28 black triangles)
Including selected UPatras (green) and AUTH (blue) stations
ΓΕΩΔΥΝΑΜΙΚΟ ΙΝΣΤΙΤΟΥΤΟ
Magnitude
Depth (km)
km


MARCH 2008 EARTHQUAKES (N = 1076 > 1.7 ML) from NOAGI network (28 black triangles)
Including selected UPatras (green) and AUTH (blue) stations
ΓΕΩΔΥΝΑΜΙΚΟ ΙΝΣΤΙΤΟΥΤΟ
Magnitude
Depth (km)
km


APRIL 2008 EARTHQUAKES (N = 586 > 1.5 ML) from NOAGI network (28 black triangles)
Including selected UPatras (green) and AUTH (blue) stations
ΓΕΩΔΥΝΑΜΙΚΟ ΙΝΣΤΙΤΟΥΤΟ
Magnitude
Depth (km)
km


MAY 2008 EARTHQUAKES (N = 515 > 1.5 ML) from NOAGI network (28 black triangles)
Including selected UPatras (green) and AUTH (blue) stations
ΓΕΩΔΥΝΑΜΙΚΟ ΙΝΣΤΙΤΟΥΤΟ
Magnitude
Depth (km)
km


JUNE 2008 EARTHQUAKES (N = 919 > 1.4 ML) from NOAGI network (28 black triangles)
Including selected UPatras (green) and AUTH (blue) stations
ΓΕΩΔΥΝΑΜΙΚΟ ΙΝΣΤΙΤΟΥΤΟ
Magnitude
Depth (km)
km

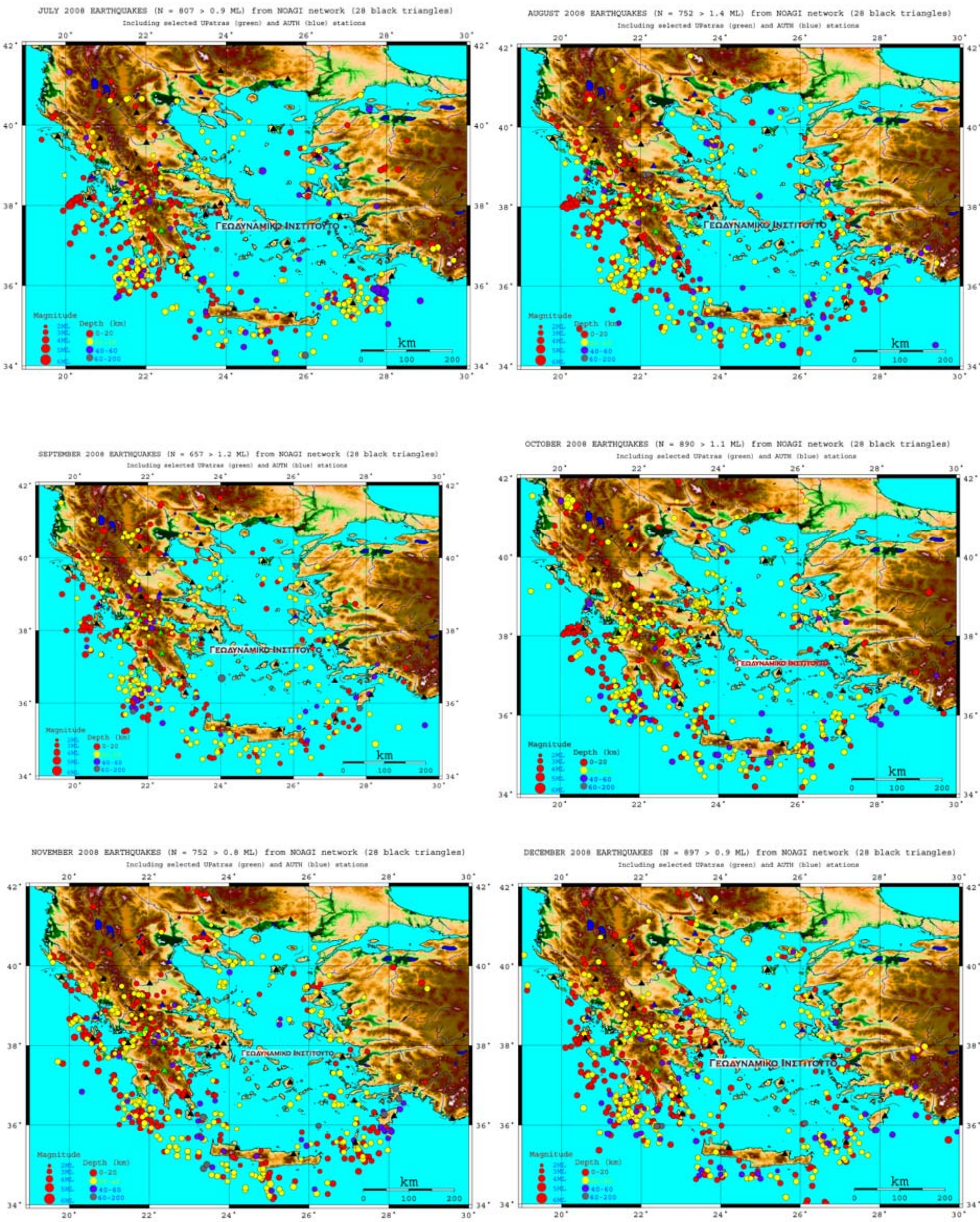


Figure 5: Earthquakes in the region of Greece for the year 2008

(Source: Institute of Geodynamics, National Observatory of Athens (IG-NOA))

It was stated earlier that large earthquakes have their origin in deep and large faults/fracture zones of the earth. In the following figure 6, a comparison is made between the location of the large earthquakes, determined by seismological methods, and the location of the deep faults/fracture zones determined by the transformation of the gravity field into its horizontal gradient.

This fault zone is presumably the actual fault zone that was activated and generated the corresponding earthquake. Consequently, the "most probably true" location of the corresponding earthquake is not the one suggested by seismological methods, but the one that has resulted by the projection of the "seismological" location of it, to the nearest fault/fracture zone.

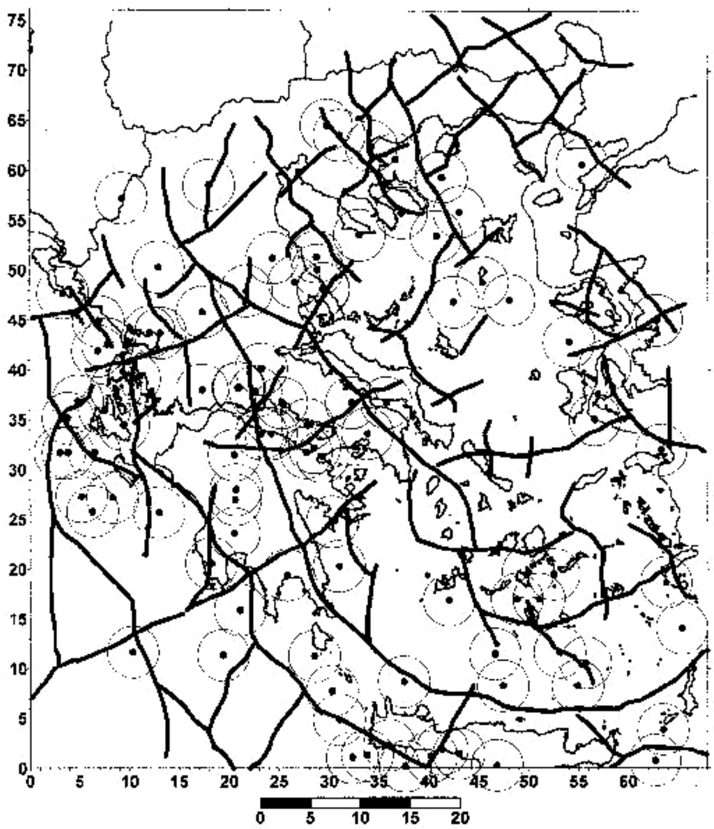


Figure 6: Location of large seismic events in Greece in relation to the calculated fracture zones - faults calculated from the transformation of the corresponding gravity field.

3. Modeling Catastrophe Bonds

3.1. Pricing Catastrophe Bonds and Incomplete Markets.

There are several important articles that focus on the pricing of catastrophe-linked securities. For instance, in the research work of Briys (1997) a simple formula for non-catastrophic insurance-linked bonds is derived in an arbitrage-free framework.

Although, the general problem of pricing a security may be well manipulated within the framework of a **complete** market using the no-arbitrage theory, the payoffs from insurance-based securities, whose cash flows may depend on earthquakes or other catastrophic events, can not be closely approximated by an appropriate portfolio of the traditional assets that already traded in the market such as stocks and corporate bonds, Cox and Pedersen (2000). Consequently, the pricing of a CAT bond requires an **incomplete** market framework.

In the case of incomplete markets, there is no “universal” theory to date that successfully addresses all aspects of pricing, such as specification of hedging strategies and robustness of prices, Young (2004). For that reason, various alternative pricing mechanisms have been developed that are tied to the specific nature of each market. Fortunately, the fact that catastrophe risk is uncorrelated with movements in underlying economic variables renders the incomplete market theory somewhat simpler than the case of significant correlation, Cox and Pedersen (2000). We use this approach and the theory of **equilibrium pricing** to develop a simple one-period and one more complicated multi-period model for pricing catastrophe bonds, see also Zimbidis et al. (2007).

Actually, the valuation is performed in two stages. The first stage with respect to the estimation of risk dynamics is presented in section 3, i.e. the distribution function of the annual maximum earthquakes of the boarder area of Greece. The statistical analysis of extremes is a key factor to many of the risk management problems related to Insurance, Reinsurance and generally speaking in Finance. In this paper, we develop a model using the tools of Extreme Value Theory. The second stage requires the selection or estimation of the interest rate dynamics.

3.2. One period (basic) Model

In this subsection, we proceed with the construction of the simple one-period model where the interest rate dynamics are restricted to constant values of different rates. First, we define the necessary symbols and the respective notation keeping in mind the discrete framework of our analysis, i.e.

K : is the face amount of the CAT bond.

r : is the risk free rate continuously compounding (up to maturity date).

e : is an extra premium rate for bearing earthquake risk.

R : is the basic element for the determination of the coupon payment rate for the one year period as long as a specified catastrophic event does not occur.

M_i : is the maximum magnitude level of the earthquake in the boarder area of Greece into the circle i, $i = 1, 2, \ldots, 5$. M_i is a random variable following the distribution obtained in the 3rd Section (see Table 2). Moreover, M_i is measured in local magnitude (LM) scale.

D_i : is the depth (Km) of the earthquake into the circle i, $i = 1, 2, \ldots, 5$.

P : is the price of the CAT bond.

C : is the cash value of the CAT bond at the maturity date depending upon the value of M according to the structure described in the following expression (1).

$$C = \begin{cases} K \cdot \left(1 + f_i\left(R\right)\right), & if \;\; M_i \in \left[0, 5.4\right], \; and \;\; \left\{D_i \le 20\right\} \;\; or \;\; \left\{D_i > 20\right\} \\ K \cdot \left(1 + g_i\left(R\right)\right), & if \;\; M_i \in (5.4, 5.8], \; and \;\; \left\{D_i \le 15\right\} \;\; or \;\; \left\{D_i > 15\right\} \\ K \cdot \left(1 + h_i\left(R\right)\right), & if \;\; M_i \in (5.8, 6.2], \; and \;\; \left\{D_i \le 10\right\} \;\; or \;\; \left\{D_i > 10\right\} \\ K, & if \;\; M_i \in (6.2, 6.6], \; and \;\; \left\{D_i \le 10\right\} \;\; or \;\; \left\{D_i > 10\right\} \\ \phi_i\left(K\right), & if \;\; M_i \in (6.6, 7.0], \; and \;\; \left\{D_i \le 10\right\} \;\; or \;\; \left\{D_i > 10\right\} \\ \gamma_i\left(K\right), & if \;\; M_i \in (7.0, 7.4], \; and \;\; \left\{D_i \le 10\right\} \;\; or \;\; \left\{D_i > 10\right\} \\ \eta_i\left(K\right) & if \;\; M_i \in (7.4, \infty] \end{cases}$$

(1)

Figure 7: The diagram for the one period model

Assuming that

$$f_i(R)=\begin{cases}2.7R, & for\ i=1,2\\ 2.9R, & for\ i=3,4,5\end{cases}\ when\ \{D_i\le 20\},\quad f_i(R)=\begin{cases}2.9R, & for\ i=1,2\\ 3R, & for\ i=3,4,5\end{cases}\ when\ \{D_i>20\}$$

$$g_i(R)=\begin{cases}1.6R, & for\ i=1,2\\ 1.8R, & for\ i=3,4,5\end{cases}\ when\ \{D_i\le 15\},\quad g_i(R)=\begin{cases}1.8R, & for\ i=1,2\\ 2R, & for\ i=3,4,5\end{cases}\ when\ \{D_i>15\}$$

$$h_i(R)=\begin{cases}0.5R, & for\ i=1\\ 0.7R, & for\ i=2\\ 0.8R, & for\ i=3\\ 0.9R, & for\ i=4\\ R, & for\ i=5\end{cases}\ when\ \{D_i\le 10\},\quad h_i(R)=\begin{cases}0.6R, & for\ i=1\\ 0.8R, & for\ i=2\\ 0.9R, & for\ i=3\\ R, & for\ i=4\\ 1.1R, & for\ i=5\end{cases}\ when\ \{D_i>10\},$$

$$\phi_i(K)=\begin{cases}0.75K, & for\ i=1\\ 0.80K, & for\ i=2\\ 0.85K, & for\ i=3\\ 0.90K, & for\ i=4\\ 0.95K, & for\ i=5\end{cases}\ when\ \{D_i\le 10\},\quad \phi_i(K)=\begin{cases}0.80K, & for\ i=1\\ 0.85K, & for\ i=2\\ 0.90K, & for\ i=3\\ 0.95K, & for\ i=4\\ 0.98K, & for\ i=5\end{cases}\ when\ \{D_i>10\}$$

$$\gamma_i(K)=\begin{cases}0.50K, & for\ i=1\\ 0.55K, & for\ i=2\\ 0.60K, & for\ i=3\\ 0.65K, & for\ i=4\\ 0.70K, & for\ i=5\end{cases}\ when\ \{D_i\le 10\},\quad \gamma_i(K)=\begin{cases}0.55K, & for\ i=1\\ 0.60K, & for\ i=2\\ 0.65K, & for\ i=3\\ 0.70K, & for\ i=4\\ 0.75K, & for\ i=5\end{cases}\ when\ \{D_i>10\}$$

and

$$\eta_i(K) = \begin{cases} 0, & for\ i = 1 \\ 0.20K, & for\ i = 2 \\ 0.30K, & for\ i = 3 \\ 0.40K, & for\ i = 4 \\ 0.50K, & for\ i = 5 \end{cases}.$$

Moreover, we assume that K, r, R and e are constants. And according to standard equivalence principle the price of the CAT bond is obtained as

$$P = E_Q\left(e^{-(r+e)} \cdot C\right) \tag{2}$$

where Q is the probability measure corresponding to the distribution for M_i.

It is obvious that a catastrophe might or might not occur prior to the scheduled maturity date, at time $T = 1$. As we can see from the cash-flow stream to the bondholders, in expression (1), the CAT bond with the face amount of € K is scheduled to make coupons payments of €$K \cdot f_i(R)$, €$K \cdot g_i(R)$, €$K \cdot h_i(R)$ at the end of each period if magnitude level of earthquake is between $(0, 5.4]$, $(5.4, 5.8]$, $(5.8, 6.2]$, respectively or zero coupon payment if the magnitude of the earthquake exceeds the level of 6.2. While the CAT bond is scheduled to repay all, a $\phi_i(K)$, a $\gamma_i(K)$, and $\eta_i(K)$ (or nothing from the capital at maturity day) if the maximum magnitude level of earthquakes during that period, let us say one-year time period, is between $(0, 6.6]$, $(6.6, 7.0]$, $(7.0, 7.4]$ and $(7.4, \infty)$, respectively. Note that in this fully parameterized model, we have to consider also the region i (i.e. the relative to this circle) and the depth of the earthquakes.

It can be easily verified, see also Zimbidis et al. (2007) that the pricing procedure is quite complex and requires dozens of logical functions and many subroutines. Moreover, it should be mentioned that a catastrophic event might diminish our capital if and only if the maximum magnitude level of the earthquake is above 6.6 in all the circles and all the depths. Looking very carefully in Table 1, the possibility of losing capital is far below of 3%. That means, we have a more that 97% Capital guarantee CAT bond which makes it quite attractive for conservative investors.

4. Discussion of the Greek Earthquake Data

4.1 Basic framework from Extreme Value Theory

By analogy with the univariate case, the traditional approach to define multivariate extremes is to base it on component wise maxima. Thus, the model focuses on the statistical behavior of maxima if $\left\{\left(X_{i,1}, X_{i,2}, \ldots, X_{i,p}\right) : i = 1, 2, ..., n\right\}$ are i.i.d. p-variate random vectors with joint distribution function F and

$$\mathbf{M}_n = \left(M_{n,1}, M_{n,2}, \ldots, M_{n,p}\right) = \left(\max_{1 \le i \le n} X_{i,1}, \max_{1 \le i \le n} X_{i,2}, \ldots, \max_{1 \le i \le n} X_{i,p}\right) \qquad (3)$$

is the vector of maxima of each component of $n = 365$ independent random variables having a common unknown distribution function (d.f.) F and measures the magnitude, the depth etc of earthquakes during the 365 days of each year in the boarder area of Greece for the period $[t, t+1)$. So the sequence of $M_{n,j}$ represents the i^{th} annual maximum for the j^{th} component of the process over 43 years of observation (see Table 1).

This analysis is based on the series of annual maximum magnitude of the earthquakes in the broader area of Greece, over the period 1966-2008 as described in the following Table 1 (Y: year, LA: latitude, LO: longitude, D: depth, M: magnitude).

Table 1

Annual Maximum Earthquakes in the Broader Area of Greece

Y	LA	LO	D	M	Y	LA	LO	D	M	Y	LA	LO	D	M	Y	LA	LO	D	M
1966	39.10	21.60	0	5.9	1977	35.00	23.10	0	5.9	1988	37.90	20.96	4	5.5	1999	38.15	23.62	30	5.4
1967	41.40	20.50	0	6.6	1978	40.80	23.30	33	6.0	1989	37.24	21.12	1	5.4	2000	36.00	22.01	5	5.4
1968	39.50	24.80	0	6.7	1979	42.00	19.00	0	6.8	1990	39.13	20.38	38	5.5	2001	40.05	20.44	27	5.3
1969	38.60	28.50	0	6.6	1980	39.20	23.90	0	6.0	1991	35.13	26.32	1	5.3	2002	35.56	26.73	104	6.1
1970	38.30	22.60	0	5.4	1981	39.20	25.30	0	6.3	1992	35.51	22.38	93	5.8	2003	40.41	26.09	33	5.4
1971	39.60	27.20	0	5.3	1982	39.90	24.50	0	6.4	1993	39.25	20.57	5	5.4	2004	34.46	23.26	24	6.0
1972	35.30	23.60	50	6.1	1983	40.08	24.81	22	6.6	1994	41.15	21.26	5	5.9	2005	37.58	20.86	22	5.6
1973	35.20	23.60	0	5.5	1984	35.36	23.31	46	5.9	1995	40.18	21.71	39	6.1	2006	36.21	23.41	69	6.4
1974	36.30	28.90	0	5.0	1985	39.24	22.89	13	5.3	1996	36.11	27.52	38	6.1	2007	34.90	22.75	34	5.1
1975	40.40	26.10	80	5.7	1986	37.10	22.19	1	5.5	1997	37.26	20.49	5	6.1	2008	37.98	21.51	25	6.5
1976	37.30	20.40	7	5.6	1987	38.37	20.42	1	5.4	1998	35.99	21.98	5	5.5					

A Revised Catalogue of Earthquakes in the Broader Area of Greece for the Period 1966-2000 &
Site Data from the Institute of Geodynamics, National Observatory of Athens (IG-NOA) for the Period 2001-2008

In theory, the distribution of $M_{n,j}$ can be derived exactly for all values of n:

$$\Pr\left[M_{n,1} \le z_1, M_{n,2} \le z_2, \ldots, M_{n,p} \le z_p\right] = F^n\left(a_{n,1}z_1 + b_{n,1}, \ldots, a_{n,p}z_p + b_{n,p}\right) \to G\left(z_1, z_2, \ldots, z_p\right) \tag{4}$$

where $\frac{M_{n,j} - b_{n,j}}{a_{n,j}} \le z_j$, with $a_{n,j} > 0$, $b_{n,j}$ for $j = 1, 2, \ldots, p$ for a p-variate distribution G with no degenerate marginals. If this holds for suitable choices of a_n and b_n, then we say that G is a multivariate extreme value distribution and F is in the domain of attraction of G, written as $F \in D(G)$. By setting all z_j but one to infitine in (4), we obtain that $F_j \in D(G_j)$ for $j = 1, 2, \ldots, p$, i.e.

$$F_j^n\left(a_{n,j}z_j + b_{n,j}\right) \to G_j\left(z_j\right), \text{ for } j = 1, 2, \ldots, p, \tag{5}$$

where F_j, G_j are the j^{th} marginal distribution of F and G, respectively. However, in our case, as a first step, we assume that there is total independence between the component wise maxima, as for instance the local magnitude of an earthquake has nothing to do with the depth of it.

Consequently, we obtain

$$G\left(z_1, z_2, \ldots, z_p\right) \to G_1\left(z_1\right) G_2\left(z_2\right) \cdot \ldots \cdot \le G_p\left(z_p\right) \tag{6}$$

The entire range of possible limit distributions for the rescaled sample maxima is provided by the well known following Theorem of Fisher-Tippett, Gnedenko; see Fisher and Tippett (1928); Gnedenko (1943), Embrechts, Klüppelberg and Mikosch (2003), and Coles (2004).

Theorem 1 (Fisher-Tippett, Gnedenko)

If there exists sequences of constants $\left\{a_{n,j} : a_{n,j} > 0 \ \forall \ n \in \mathbb{N}\right\}$ and $\left\{b_{n,j}\right\}_{n \in \mathbb{N}}$ for $j = 1, 2, \ldots, p$ such that:

$$\Pr\left[M_{n,j}^* \le z_j\right] = \Pr\left[\frac{M_{n,j} - b_{n,j}}{a_{n,j}} \le z_j\right] \to G_j\left(z_j\right) \quad \text{as } n \to \infty, \ z_j \in \mathbb{R}$$

where G_j is a non-degenerate distribution function, then G_j belongs to one of the following families.

$$\text{I. (Gumbel)} \quad G_j\left(z_j\right) = \exp\left\{-\exp\left[-\left(\frac{z_j - b_j}{a_j}\right)\right]\right\}, \quad -\infty < z_j < \infty$$

$$\text{II. (Fréchet)} \quad G_j\left(z_j\right) = \begin{cases} 0, & z_j \leq b_j \\ \exp\left\{-\left(\dfrac{z_j - b_j}{a_j}\right)^{-\gamma_j}\right\}, & z_j > b_j \end{cases}$$

$$\text{III . (Weibull)} \quad G_j\left(z_j\right) = \begin{cases} \exp\left\{-\left[-\left(\dfrac{z_j - b_j}{a_j}\right)\right]^{\gamma_j}\right\}, & z_j < b_j \\ 1, & z_j \geq b_j \end{cases}$$

for parameters $a_j > 0,\ b_j$ and in the case of families II and III, $\gamma_j > 0$.

The unification of the previous three families of extreme value distribution into a single family simplified a lot the statistical implementation. Through the inference of a new parameter ξ_j, the data determine by themselves the most appropriate type of tail behavior, so we avoid completely making a priori judgment for the individual extreme value family. Moreover, uncertainty in the inferred value of ξ succeeds in measuring the lack of certainty as to which of the original three types is most appropriate for a given dataset. For convenience, we restate Theorem 1 in the following modified form.

Theorem 2 (Fisher-Tippett, Gnedenko)

If there exist sequences of constants $\left\{a_{n,j} : a_{n,j} > 0\ \forall\ n \in \mathbb{N}\right\}$ and $\left\{b_{n,j}\right\}_{n\in\mathbb{N}}$ such that:

$$\Pr\left[M^*_{n,j} \leq z_j\right] = \Pr\left[\frac{M_{n,j} - b_{n,j}}{a_{n,j}} \leq z_j\right] \to G_j\left(z_j\right) \text{ as } n \to \infty,\ z_j \in \mathbb{R}$$

for a non-degenerate distribution function G_j, then G_j is a member of the **Generalized Extreme Value (GEV)** family of distributions or **von Mises type Extreme Value** distribution or the **von Mises–Jenkinson type** distribution.

$$G_j\left(z_j\right) = \exp\left\{-\left[1+\xi_j\left(\frac{z_j-\mu_j}{\sigma_j}\right)\right]^{-\frac{1}{\xi_j}}\right\} \tag{7}$$

defined on the set $\left\{z_j : 1+\xi_j\left(z_j-\mu_j\right)/\sigma_j > 0\right\}$, where the parameters satisfy $-\infty < \mu_j < \infty$, $\sigma_j > 0$ and $-\infty < \xi_j < \infty$.

The model has three parameters: ξ_j is a shape parameter, μ_j is a location parameter and σ is scale parameter. The type II and type III classes of extreme value distribution correspond respectively to the cases $\xi_j > 0$ and $\xi_j < 0$ in the parameterization. The subset of the GEV family with $\xi_j = 0$ is interpreted as the limit of (7) as $\xi_j \to 0$, leading to the Gumbel family.

5. Conclusion

In this paper, we review the basic concepts as regards the design and pricing process of catastrophe bonds. We restrict our attention in the earthquake risk within the boarder area of Greece. We discuss the historical data for the earthquakes as regards the different parameters and measures involved as long as the financial and other consequences of them.

Finally and after providing an elementary introduction for both extreme value theory and theory of incomplete markets, we establish a simple formula for pricing a catastrophe bond. Further research is carried forward aiming to a concrete framework for pricing indemnity-linked catastrophe bonds.

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