Decomposing total risk of a portfolio into the contributions of individual assets

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Abstract. Analysing the concentration risk in the portfolio is one of the important problems for the risk management of financial institutions. Several measures are proposed in order to quantify the concentration risk, here we consider the risk contribution (RC) of asset j, RC_j, which is defined by $RC_j \equiv a_j \partial R_p / \partial a_j$, where a_j is the holding amount of asset and R_p is the total risk of the portfolio. RC_j satisfies the additivity, that is, the sum of RCs of all assets are equal to the total risk of the portfolio R_p, and we can select many famous risk measures as R_p, for example, standard deviation, Value at Risk (VaR) and Expected Shortfall (ES). However, the exact and robust estimation of RC is very difficult. One of the hopeful methods to overcome this problem is the "hybrid method." In this method, assuming that the future prices of individual assets are conditionally independent with respect to the risk factors, we generate some scenarios of the risk factors by Monte Carlo simulation, and calculate the conditional distribution of the future value of the portfolio and RCs of individual assets analytically by using the saddlepoint approximation. The hybrid method gives much more reliable estimates of VaR and RCs to VaR than the ordinary Monte Carlo simulation, however, the accuracy of the estimates of ES and RCs to ES is not so good. In this article, we summarize the hybrid method in more general settings, and propose a more accurate estimation method of ES and RCs to ES. Since our proposed estimation method is based on the universal mathematical relation between VaR and ES, it can be applicable to many risk evaluation models. We also show some numerical examples to confirm some merits of our method.

Keywords: risk management, marginal risk contribution, VaR (Value at Risk), ES (Expected Shortfall), conditional independence, saddlepoint approximation.