REINSURANCE CREDIT RISK MODELLING
— DFA APPROACH —

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This paper presents a unifying stochastic approach to modelling the reinsurance credit risk in a DFA environment. The approach relies on the key ideas of defining the reinsurer default as the outcome of an asset impairment event and modelling default events being dependant on the current state of global reinsurance market. In this modelling approach the state-conditional default rates based on reinsurance credit rating are calibrated by using industry available impairment rates for reinsurers and matching the model implied default dependency structure to the target one of the underlying assets. The proposed model overcomes common issues arising from applying existing investment banking concepts to calibrating credit risk in (re)insurance sector. This paper proposes and discusses a DFA model setup to quantify the costs for reinsurance credit risk.

Keywords: reinsurance credit risk, copula and reinsurance default dependency structure, DFA simulation model.

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1 INTRODUCTION

General[1] insurers rely heavily on reinsurance to smooth insurance results, and to protect against insolvency. Default by a reinsurer will - potentially - lead to losses to the ceding insurer. A Dynamic Financial Analysis (DFA) model, such as those used to assess solvency and for capital planning in general insurance companies, should address this risk.

As with any debtor default, the loss can be decomposed into:

- the probability of default;
- actual and potential exposure at time to default; and
- eventual recoveries.

This suggests that traditional approaches to assessing debtor default could be transferred to assessment of reinsurance default, and (despite the lack of confidence in these models currently in vogue) our approach borrows some concepts from the literature in credit risk modelling in the bond arena.

There are, however, significant differences between an insurer's exposure to reinsurance default and a typical portfolio of real or synthetic bonds, so some changes must be made.

In addition, modelling reinsurance default as a part of a DFA model presents different implementation issues compared to assessing current credit risk of a bond (or a bond portfolio). Specifically, a DFA model is generally a multi-year, monte-carlo projection covering not only claims currently paid at calculation date, but also the exposure arising from the underlying policies in force at that time and, in many cases, business expected to be written over a number of years in the future.

Differences will arise in relation to probability of default, exposure, and recoveries. In respect to loss given default some of the differences are:

- the number of reinsurers is small (when compared to the number of bond issuers) and so a typical insurer - however prudent - is likely to have a concentrated exposure to individual names;
- by definition reinsurer exposure is specific to one industry sector (insurance) so correlations are likely higher than in a more diversified portfolio;
- the ceding insurer is in the same industry - catastrophe events will weaken the balance sheet of the reinsurers at the same time (potentially) as the ceding insurers portfolio is stressed;
- within the context of a DFA model, some variables used as predictors of risk of default used in rating models (such as a bond issuer's share price) are not available within the confines of the model so are unavailable to be used in modelling underlying default risk.

Consequently, a DFA model of reinsurance default risk should deal with concentration, increased average dependency, tail dependency, and potentially tail dependency between reinsurer creditworthiness and the ability of the ceding company to cope with defaults.

The exposure profile of an insurer (particularly the modelled exposure within a DFA model) is also different. The insolvency of a reinsurer will lead to costs on the part of the insurer:

- any amounts owing as a result of claims settled with the reinsurer but not yet paid will be impaired;

1 In Australia the non-life type of insurance is called 'general' insurance.

- amounts potentially owing in respect of claims incurred but not yet advised to the ceding insurer are potentially not recoverable;
- any potential recoveries in respect of policies in force are unlikely to be met in full;
- the ceding company may be required to purchase replacement cover to remain protected against events from the time of reinsurance default to the time the contract would have expired.

Reinsurance credit risk modelling should address each of these items.

Within the constructs of a DFA model there is a further difficulty, in that exposure may arise in respect of contracts not in force at the calculation date, but expected to be written in respect of future years. It is not clear which reinsurers will be given these contracts, so future exposure to existing credits is unknown. Finally, in respect of potential recoveries, they are frequently commuted prior to contractual payment date - at a discount to full face value in the case where the reinsurer is in distress. The terms of these commutations are frequently not in the public domain.

To some extent the current proposed approach to modelling the reinsurance credit risk addresses these issues. The structure of this paper is as follows. In section 2, the general concepts of quantifying reinsurance credit risk are discussed and an overview of some already existing modelling approaches is provided. In section 3, which forms the main contribution of the paper, a unifying stochastic approach to modelling the reinsurer credit risk and quantifying total costs associated with such risk is discussed. This includes numerical simulation examples extracted from the DFA model output. Finally, brief conclusions are given in section 4.

2 REINSURANCE CREDIT RISK MODELLING: OVERVIEW

The reinsurance credit risk is the risk of the reinsurance counterparty failing to pay reinsurance recoveries in full to the ceding insurer in a timely manner, or even not paying them at all. In a wide sense it is the part of company's overall credit risk. Here we start with several definitions of the credit risk in a broad sense.

APRA[2]: *Credit risk is the risk of loss arising from failure to collect funds from creditors, including reinsurers and intermediaries.*

FSA[3]: *Credit risk is incurred whenever a firm is exposed to loss if a counterparty fails to perform its contractual obligations, including failure to perform them in a timely manner.*

Reinsurance credit risk emerges mainly because the ceding insurer (cedant) pays insurance claims to policyholders before reclaiming reinsurer's part and is liable for the full gross amount. The delay of reinsurance recoveries simply increases the cedant's liquidity risk and is often due to *unwillingness* to pay as the result of a contractual dispute (dispute risk), whereas the failure to pay reinsurance recoveries, either wholly or in part, implies default of the reinsurer. The reinsurance default occurs because of the reinsurer's *inability* to pay what the ceding insurer claims is owed under the terms of reinsurance treaty.

Up until recently, the reinsurance credit risk was modelled deterministically using factor-based charges to expected reinsurance recoveries, where the charge factors were based on adjusted corporate bond default rates. This was done mainly in the reserving area were the calculation of 'bad debt'

2 Australian Prudential Regulation Authority.
3 UK Financial Services Authority.

provisions representing part of reinsurance recoveries expected to be lost due to reinsurance default were taken place. A detailed description of this deterministic approach can be found in Bulmer et al. [2]. This approach is obviously not an ideal one to be used for accurate assessment of reinsurance credit risk, since it has many shortfalls. The most important ones among them are:

- first of all, it is a deterministic approach and, therefore, is far from modelling 'real world' implying proper randomisation and allowance for default dependency;
- it uses corporate bond transition and default rates which are not optimal for modelling the reinsurance default.

On the other hand, taking into account the fact that the reinsurance credit risk is not as significant as insurance risk[4] for general insurers in determining capital requirement, this simplistic approach might serve its purpose under some circumstances. This is especially relevant to well capitalised insurers with little amount of reinsurance on board.

However now, with recent worldwide advances in regulatory requirements[5] to the methods of determining internal solvency capital, those general insurers wanting to use their Internal Capital Models will need to demonstrate their ability to model even the reinsurance credit risk quite accurately among other more significant risks. Therefore, new advanced risk modelling methods would need to be used. One of the way of coming up with some new 'revolutionary' ideas of how to model reinsurance credit risk properly is to look outside the actuarial area and see how risk professionals in other financial sectors, like investment banking, are modelling similar risks. Risk managers/analysts in investment banking are dealing with investment credit risk which has got some similarities with reinsurance credit credit. There are many advanced credit risk modeling approaches in the investment banking industry, some ideas of which can be borrowed and applied within the (re)insurance industry. Among those developed credit risk modelling approaches are:

- Merton's models and Moody's KMV;
- Default intensity modelling using stochastic Cox process and martingale methods;
- Rating-based term-structure modelling using Markov chains;
- Credit derivative pricing models (Interest Rate Swaps (IRS), Credit Default Swaps (CDS));
- Li model;
- Modelling dependent defaults.

A very good description of these investment credit risk modelling approaches can be found in Schönbucher [12] and Lando [9]. Many of these approaches were employed in building up well known investment banking software packages for credit risk assessment, among which are JP Morgan's CreditMetrics™, Moody's KMV™, McKensey's CreditPortfolioView™and CreditRisk+™.

At the time of writing this paper there were two pioneering actuarial papers that attempted to employ some of these investment banking modelling approaches to modelling reinsurance credit risk in insurance industry.

4 Mainly underwriting and catastrophe risks.

5 For example, in Australia (APRA's IMB method), EU (proposed Solvency II SCR) and the UK (FSA's ICA).

Those are papers by Shaw [13] and Flower et al. [6] presented at the GIRO Convention of the UK Institute of Actuaries in 2007. In Flower et al. [6] authors use market data of CDS prices, i.e. yield margins on corporate bonds over and above risk-free rates, to derive the term structure of reinsurance default rates by credit rating. Their proposed modelling approach also explicitly allows for correlation between reinsurance defaults. In Shaw[13] the author uses corporate bond default rates as proxies to reinsurance default rates, and borrows the idea from investment banking industry of correlating reinsurance default events via reinsurers' asset return correlation. In a way, it is a good idea of doing this as it creates some default dependency structure and avoids direct correlation of defaults which is problematic because of lack of credible historical default data.

Both papers are very practical and have their own built-in toy models with many numerical examples. However, they have the following key shortfalls which the current paper is trying to overcome:

- they use corporate bond default rates and CDS prices respectively to derive reinsurance default rates;
- they do not explicitly formulate the modelling of defaults (especially in a multi-period setting) with some defined dependency structure including tail dependency characteristics.

Generally speaking, the calibration of the reinsurance credit risk model can come from one of three paradigms:

1. Calibration to observed prices;
2. Calibration to observed parameters; and / or
3. Calibration to 'emergent properties' or 'stylized facts'.

PARADIGM ONE: Calibration to observed prices is used in risk neutral pricing to fit a model of prices of securities (or derivatives of securities), so as to derive prices of similar instruments. For example, call options with a new strike can be calibrated via model to the prices of call options with slightly different strike prices. Calibration to prices is key to that part of financial economics that deals with the price of things. Its strength is that as a *relative* pricing paradigm, it implicitly captures the price of risk embodied in the underlying securities. In the case of reinsurance default, the suggestion has been to calibrate the model to credit default swaps of the underlying issuer, or a 'similar' issuer.

As a way to determine the characteristics of the underlying securities (specifically, default probability and loss given default in the case of a reinsurer) it has several shortcomings.

SHORTCOMING ONE: It relates to model mis-specification, and how that impacts on model derived estimates. If a model is mis-specified, then any attempt to derive the characteristics of the underlying will be biased as a result of the mis-specification. For example, the Black-Scholes formula to price an option on a stock makes an assumption about the volatility of the stock - that it is constant over the term of the option. If this is not true, then the volatility assumption that replicates the price of traded options will differ as the term of the option varies. This results in the volatility 'smile' or volatility 'smirk' that exists in the options market.

SHORTCOMING TWO: It relates to the complexity of the dynamics of the market for a given security. In the case of corporate bonds (similar in many ways to reinsurance default) or credit default swaps, the 'price' is the spread to risk free (or swap curve) of the instrument. This credit spread captures (amongst others):

- probability of default;
- loss given default;
- probability of credit migration to a lower credit rating (which may force sale, if the owner is restricted in further debt issue);
- liquidity premium of the bond compared to the risk free alternative;
- funding uncertainty of the marginal investors (how likely are they to be able to fund the instrument through to maturity, if liquidity is not guaranteed);
- the price of risk, or level of risk aversion in the economy, at any particular point in time.

Of these, the probability of default and the loss given default are relevant for our purpose of modelling the real world creditworthiness of the issuers, the others are not. Consequently, for our purpose we do not calibrate to prices[6].

PARADIGM TWO: The second paradigm refers to maximum likelihood estimators (or robust estimators) based on direct observation of data. We use direct observation to estimate our unconditional default rates. In contrast to Shaw [13] and Flower et al. [6], the current paper derives reinsurance default rates from AM Best asset impairment rates for reinsurers provided in [1].

PARADIGM THREE: The last paradigm - calibration to 'emergent properties' or 'stylized facts' - lies in the realm of Simulated Method of Moments or Indirect Inference (see books by Gouriéroux and Jasiak [8] and Campbell et al. [3] on Financial Econometrics). In essence, recognising that our model is a simplification of the real world, we accept that maximum likelihood estimators of the parameters of the model chosen will still lead to model specification error. We have, from observation or some other source, some views on how the model output should behave (e.g. correlation or tail dependence of reinsurance default events). We calibrate the model such that the desired properties are recovered. Here in this paper we show how to model the default dependency structure in a multi-period setting within DFA environment by

- allowing transition of the (re)insurance market into 'stressed' state with fixed transition rate; and
- calibrating 'normal' and 'stressed' default rates such that the model implied default dependency structure matches the target one defined by a Gaussian copula.

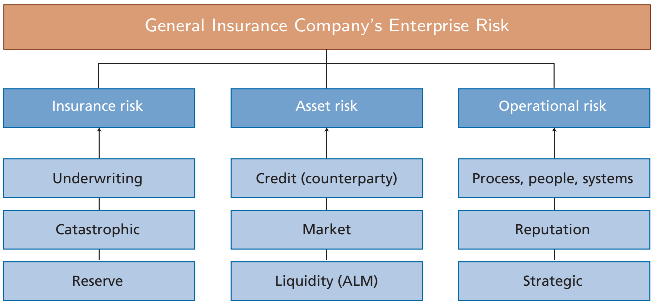
3 MODEL SETUP AND NUMERICAL EXAMPLES

3.1 *Introduction*

In the proposed modelling approach we consider quantification of reinsurance credit risk for a generic general insurance company that uses a DFA

6 Of course, in other circumstances we may be interested in more than just probability of default and loss given default. We may, for example, be interested in testing the economic balance sheet in twelve months time. In this context, we would be less interested in the real world probability of default and loss given default, and more interested in the cost of laying off the risk. Within this framework, calibration to observed prices is more relevant.

model in determining capital requirements that would particularly allow for reinsurance credit risk emerging over a one year time horizon of writing business as well as the run-off of this risk to extinction. In this paper, to provide numerical illustrations of reinsurance credit risk quantification authors used the DFA Internal Capital Model being a customised version of MoSes™Property-Casualty Financial Model (PCFM). The modelling of the reinsurance credit risk in DFA environment is usually done in a separate module within the existing DFA model. For example, in the model used here the so called *Reinsurance Default* module is responsible for modelling reinsurance credit risk. The diagram below represents the risk taxonomy in the DFA Internal Capital Model.



In that risk structure the reinsurance credit risk is the part of total counterparty credit risk, and is not as significant as insurance risk for insurance companies. However, it has to be modelled accurately if one chooses to use an Internal Capital Model according to APRA's requirements on Internal Model-Based capital adequacy assessment or the proposed Solvency II SCR.

In high level terms, the *Reinsurance Default* module of the Internal Capital Model uses the following key modelling assumptions to calculate the cost arising from the reinsurance default:

- *Exposure* - a small number of representative 'proxy reinsurers' is created to capture the company's exposure to reinsurers default. The company's exposure to a proxy reinsurer varies by exposure type, e.g. by catastrophe (cat) and non-cat, small and large cat events. In the event of default, the defaulted proxy reinsurer will be replaced by a new proxy reinsurer of the same quality. We model proxy reinsurers to by pass the issue of future exposure. In effect we model the credit policy, not its realisation at any one point in time;

- *Default event* - the default of any proxy is assumed to occur at the beginning of any projection time period, that is assumed here to be a quarter, and is modelled as a binary event using Bernoulli random variable with the default rate dependent on the state ('normal' or 'stressed') of the global reinsurance market;

- *Cost of Default* - the Loss Given Default is calculated by applying a recovery rate to the exposure to proxy reinsurers (i.e. reinsurance recoveries) at the end of previous period and the replacement cost of unexpired reinsurance cover.

In the model the *Reinsurance Default* module interacts with other modules, such as

- *Reinsurance* module - by importing potential reinsurance recoverables and unexpired ceded premiums;

- *Economic Scenario Generator (ESG)* module - by importing rates of economic inflation and discounting rates;

- *Accounts* module - by exporting cost of default by business class level to model accounts. These will be used to calculate underwriting profit in the income statement, and write-off the reinsurance recoverables as part of assets by default cost in the balance sheet.

3.2 *Exposure and loss given default assumptions*

3.2.1 *Exposure matrix*

We develop a small number of a number of representative *proxy reinsurers* to capture the company's exposure to reinsurer default. The proxy reinsurers are chosen so as to capture the credit exposure and concentration levels typical of company's reinsurer exposure. These representative exposures will remain static over the life of the projection. These exposures will be stored in an *Exposure Matrix*: - one for catastrophe exposure below a certain vertical exposure level, say 50% of total limit of reinsurance programme, one for catastrophe exposure greater than that threshold, and (possibly) one for long-tailed (non-catastrophe) exposures. The exposure threshold can be determined as the dividing point where the company's catastrophe reinsurer profile differs materially between the top of the *catastrophe tower*[7] and the bottom of the tower.

The Exposure Matrix also provides the assumptions about the proxy reinsurers' shares of unearned reinsurance premium (or Deferred Reinsurance Expense (DRE) from the company's point of view). This is used to calculate the cost of replacing the reinsurance cover when one or several proxy reinsurers defaults. The illustrative example of Exposure Matrix is provided in Table 1. In this table, for example, bucket number 1 would be modelled as a proxy reinsurer with a rating AA to which company has the following exposures:

- 27.5% of total exposure to potential cat reinsurance recoverables when cat event is below $1.5Bn;
- 29.8% of total exposure to potential cat reinsurance recoverables when cat event is above $1.5Bn;
- 23.4% of total exposure to unearned ceded premium;
- 15.8% of total exposure to potential non-cat reinsurance recoverables.

3.2.2 *Patterns of potential recoveries from proxy reinsurer*

For each 'proxy reinsurer' we will store at any time a vector of potential (expected) reinsurance recoveries at that time. This will initially be based on current exposures, and will be updated in response to simulated catastrophes in the *Cats* module by applying an appropriate payment pattern to the vectors once a simulated catastrophe occurs. For the sake of simplicity we will assume that each catastrophe is shared amongst the 'proxy reinsurers' based on the Exposure Matrix. The hypothetical patterns of quarterly payment rates of reinsurance recoveries separately for cat and long-tail non-cat claims are provided below.

Exhibit 1. *Reinsurance Recovery Pattern for Cat Events*

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9
10%	20%	20%	15%	10%	10%	5%	5%	5%

7 The tower of layers of catastrophe reinsurance programme.

Table 1: *Exposure Matrix*

BUCKET	CREDIT RATING	< $1.5BN	> $1.5BN	UEP	NON CAT
1	AA	27.5%	29.8%	23.4%	15.8%
2	A+	7.5%	5.0%	4.7%	3.7%
3	A	7.5%	9.0%	7.9%	3.7%
4	AA-	7.5%	4.5%	18.7%	22.1%
5	AA-	10.0%	5.5%	7.1%	2.7%
6	A-	0.0%	0.0%	0.0%	0.0%
7	AA-	5.4%	0.0%	3.4%	0.0%
8	AA-	0.0%	20.5%	0.0%	0.0%
9	A+	1.0%	1.2%	0.4%	0.0%
10	A-	10.0%	1.3%	7.8%	0.0%
11	AA+	2.5%	0.0%	2.4%	15.3%
12	AA-	2.0%	6.0%	1.9%	0.0%
13	A	2.0%	1.7%	1.6%	8.8%
14	A	5.0%	2.5%	3.2%	0.0%
15	A-	4.0%	2.5%	2.7%	0.0%
16	A-	3.0%	0.6%	5.0%	0.0%
17	A+	1.0%	5.4%	5.2%	13.5%
18	A-	1.0%	0.0%	0.8%	12.4%
19	CASH (AAA)	3.1%	4.5%	3.9%	0.6%
20	NOT RATED (BBB)	0.0%	0.0%	0.0%	1.4%
TOTAL		100%	100%	100%	100%

Exhibit 2. *Reinsurance Recovery Pattern for Large Long Tail Claims*

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11
5%	6%	6%	6%	6%	5%	5%	5%	5%	4%	4%

Q12	Q13	Q14	Q15	Q16	Q17	Q18	Q19	Q20	Q21	Q22
4%	4%	3%	3%	3%	3%	3%	3%	2%	2%	2%

Q23	Q24	Q25	Q26	Q27	Q28
2%	2%	1%	1%	1%	1%

The illustrative exhibit below shows how the recovery vector in respect of 'Bucket 1' may change as a result of a catastrophe in the first quarter. The catastrophe is assumed to generate a $100 million overall recovery.

Exhibit 3. *Expected Reinsurance Recoveries (in $M)*

	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12
Initial	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	0.0	0.0
Q1		12.8	15.5	15.5	14.1	12.8	12.8	11.4	11.4	11.4	0.0	0.0

The 'Initial' recoveries are assumed to be the recoveries in reinsurance recoveries at model start date. The recoveries after the first quarter are calculated (using time period Q3/ recovery period Q2 as an example) as:

- Recoveries expected before the first quarter catastrophe ($10 M); plus
- Overall catastrophe recovery ($100M); times
- Bucket 1 share of recoveries (27.5%); times
- Expected proportion recovered in recovery period (20%);

This gives 10 + 100×27.5%×20% = $15.5 M shown.

3.2.3 *Loss given default assumptions*

The default recoveries from outstanding potential reinsurance recoverables can be modelled either deterministically or stochastically via introducing a random recovery rate with *Beta* distribution on interval from zero to one. In this setup we elect deterministic approach to modelling recoveries. The following average loss rates given default were derived from the industry study of reinsurance default recovery rates conducted by GIRO Working Group of the UK Institute of Actuaries (see Bulmer et al. [2])

Exhibit 4. *Average loss rates given default*

AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	NR
0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.8

3.3 *Unconditional default rates*

Recommended total unconditional default rates of reinsurers were derived from the AM Best research study [1] titled "Securitisation of Reinsurance Recoverables" published in August 2007. The study defines the reinsurer default as an asset impairment event. This definition of reinsurer default is different from the financial default of issuer generally used in the credit markets. The credit markets consider an issuer default as having occurred when there is a shortfall in interest and/or principal payments on its obligation. Under financial distress reinsurers often go into runoff and usually enter into a commutation agreement with cedants, and therefore it is often unclear if the default defined in financial sense has taken place or not. This definition of reinsurance default excludes unwillingness to pay recoveries due to disputes in interpretations of contract terms. This portion of reinsurer credit risk can be well controlled/mitigated via active management and therefore is not modelled here.

AM Best derived the probabilities of reinsurers default from an original study spanning 1977 to 2006. The resulting probabilities of default are considered to cover periods during which the global reinsurance market was in stressed and normal states. Original raw cumulative default rates over ten years period were disaggregated into a set of forward quarterly rates, and smoothed[8] appropriately so that the average rate over ten years for the smoothed rates is the same as the original ones. The following exhibit shows smoothed unconditional forward annual default rates for proxy reinsurers by credit rating.

Exhibit 5. *Smoothed unconditional annual forward rates of default (in %)*

YEAR	AAA	AA+	AA	AA-	A+	A	A-	BBB+	NR
1	0.063	0.192	0.318	0.446	0.587	0.702	0.836	1.506	4.124
2	0.101	0.232	0.350	0.476	0.609	0.719	0.879	1.586	4.335
3	0.116	0.248	0.361	0.487	0.617	0.725	0.895	1.617	4.413
4	0.131	0.264	0.373	0.497	0.626	0.731	0.912	1.648	4.494
5	0.146	0.279	0.384	0.508	0.634	0.737	0.928	1.678	4.571
6	0.162	0.296	0.396	0.519	0.643	0.743	0.945	1.710	4.655
7	0.182	0.317	0.412	0.533	0.654	0.751	0.967	1.752	4.762
8	0.200	0.336	0.426	0.546	0.664	0.758	0.987	1.790	4.859

8 Our thanks to Lisa Simpson of PwC Australia for her work in calibrating the unconditional default rates.

3.4 *Dependency structure*

Actuaries, please do not correlate. Go forth and copulate! But only if you can not do anything better.

— Unknown

In the case of two reinsurers, the 2×2 contingency table (see the definition of it on page 13) uniquely describes the default behaviour. In many respects, it doesn't matter how we get to the target contingency table, what counts is the emerging table itself. Following the general approach to modelling credit risk, we have assumed for convenience that the return on an underlying reinsurer's net assets follows a normal distribution. Reinsurance default is defined as an event where the value of the reinsurer's net assets falls below a certain threshold. The default value threshold depends on the state of the reinsurance market at any point of time, with a higher threshold (more likely as we breach from above) in times where the reinsurance market is stressed. There are two ways to implement a dependency structure within this framework:

- we can add a copula to the underlying asset values of two or more reinsurers, which will directly influence the underlying asset values; and/or
- there will be some dependency arising from the fact that default rates increase globally following a transition into a stressed state.

The former approach forms the basis of many default models - Moody's KMV™, CreditMetrics™and the Li model are examples. In each case the underlying copula is a Gaussian copula. In our case we implement the dependency structure by controlling the calibration of the relativities between stressed and unstressed default rates. The higher the differential between the two, the higher is the level of dependency between them (and the higher the level of tail dependence between the two). We do this for a number of reasons, mostly computational. The overall model is parsimonious and runs efficiently without the need to simulate large numbers of correlated random variables. When we come to calibration targets we choose to replicate the contingency table we would have got had we implemented a model with Gaussian copula and using unconditional default rates. We arrive at the same destination, but via a less computationally intensive route. It is important to note that it is the base asset return correlation that meaningfully represents the default dependency structure under the assumption of normality of asset returns, and not actual default (Pearson) correlation implied by the model. The Pearson default correlation is significantly lower than the assumed base asset return correlation, and carries little useful information about the default dependency structure. We recommend that the calibration target for asset return correlation be set at 25%, which incorporates 17.5% mean sample estimate of asset return correlation taken from the KMV empirical study plus a margin to deal with potentially increased correlation from the reinsurance industry (for more details please refer to CEOPS e.V. QIS3 study [4] on calibration of credit risk).

3.5 *Transition probability*

The *Reinsurance Default* module incorporates a scenario where the global reinsurance market transits into a 'stressed' state following a significant global catastrophe event. In that state all reinsurers are more prone to default. In this modelling approach we use Swiss Re data [14] of the 40 most costly insurance losses spanning 1970-2005 to get an estimate of the frequency of severe cat events causing the market stress. Using data we have estimated that the top four of these severe cat events would have likely triggered significant reinsurer downgrades across the global (re)insurance industry. This would mean that the frequency can be assumed to be approximately 1 event every 10 years, i.e. 10% p.a.

The use of a 'stress' scenario that affects all reinsurers is a useful method by which co-dependency between reinsurer defaults can be allowed for. It is assumed that once the global reinsurance market transits into stressed state, it remains stressed for some period of time. The reasonable duration of 'stress' state is considered to be between 1 to 4 years. For example, industry research shows that following Hurricane Katrina, there were several downgrades of reinsurers for a period of approximately 18 months whilst those reinsurers rebuilt their balance sheets. In this paper for illustrative purpose we assume the duration of 'stress' state equals two years.

3.6 *Modelling default using normal and stressed default rates*

As was mentioned at the beginning of this section, the default of any proxy reinsurer is modelled at the beginning of projection period as a binary event using Bernoulli random variable with the default rate dependent on the state ('normal' or 'stressed') of the global reinsurance market. To proceed and use this default modelling approach one needs to calibrate the state-conditional 'normal' and 'stressed' default rates using already available smoothed unconditional default rates provided on page 10.

This can be done in many ways. Here we discuss two possible calibration methods. The first method, simplistic one, assumes that the stressed default rates were set such that they represent approximately two credit market scale downgrades from the unconditional default rates. This simplistic assumptions gives a direct solution to this calibration problem. However, it often results in the poor adherence of the dependency structure implied by the model to the target one defined by a Gaussian copula.

The second method overcomes this drawback by changing 'normal' and 'stressed' rates to hit the target dependency structure defined by a Gaussian copula. The rest of this subsection outlines the key steps of the latter calibration method along with modelling implementations of default events.

3.6.1 *Modelling implications of default*

… I will remember that I didn't make the world, and it doesn't satisfy my equations.

— The Modellers' Hippocratic Oath [5]

According to the model setup the default event of each proxy reinsurer in force is modelled at the beginning of every quarter by Bernoulli random variable rate of which is dependent on the pre-generated global market state ('normal' or 'stressed'). If a particular proxy reinsurer defaults it is replaced by another one of the same credit quality. Our aim is to construct the default event over the period of four quarters for one fixed proxy reinsurer. This can be done through a combination of two competing independent 'risks': 'stressed' default vs. 'normal' default, controlled via a random time of market transition into stressed state independent of default event. The following formula formalises the latter proposition:

$$D = D_n(Z-1) + \left[1 - D_n(Z-1)\right] D_s(5-Z), \qquad (1)$$

where for $m \in 0, 1, 2, 3, 4$

$$D_n(m) \sim Be\left[1 - (1-q_n)^m\right], \qquad (2)$$

$$D_s(m) \sim Be\left[1 - (1-q_s)^m\right], \qquad (3)$$

are respectively conditional 'normal' and 'stressed' defaults over the period of m quarters modelled by Bernoulli random variables; q_n and q_s are quarterly normal and stressed default rates respectively; $Z \sim TruncGeom[p]$ is the

Truncated Geometric random variable over the period of four quarters with the quarterly transition rate p. It has got the following distribution

$$\mathbb{P}[Z = k] = \begin{cases} (1-p)^{k-1}p, & k = \overline{1,\dots,4}; \\ (1-p)^4, & k = 5 \end{cases} \tag{4}$$

where $k = \overline{1,\dots,4}$ is the ordering number of quarter when for the first time the market transits into 'stressed' state, $k = 5$ indicates that the market remains in normal state during the first four quarters. In essence, the default event is modelled as a stochastic mixture of two Bernoulli random variables of stressed/normal defaults mixed by global reinsurance market state represented by a Truncated Geometric random variable over the four quarters with a given quarterly transition rate. For example, if the market transits into stressed state at the beginning of the second quarter (i.e. $k = 2$), then the proxy reinsurer either

1) defaults with normal default rate during the first quarter; or
2) survives the default till the beginning of the second quarter in the market being in unstressed environment, and then either defaults with stressed default rate in the period from quarter two to quarter four or survives the default.

In this instance the default is modelled by $D_n(1) \sim Be[q_n]$ in the first quarter, and by $D_s(3) \sim Be\left[1-(1-q_s)^3\right]$ in the period of the remaining three quarters. The unconditional annual default rate can then be easily calculated using conditional expectation formula:

$$\begin{aligned} \mathbb{E}[D] &= \mathbb{E}_Z\left[\mathbb{E}[D|Z]\right] \\ &= \mathbb{E}_Z\left[1-(1-q_n)^{Z-1}(1-q_s)^{5-Z}\right]. \end{aligned} \tag{5}$$

This formula represents total unconditional annual default rates by credit rating that were derived from the AM Best research study [1] of impairment rates for reinsurers, and then further smoothed. These smoothed unconditional default rates, given on page 10, are constrained to remain unchanged along with the market transition rate. Under these constraints, the quarterly normal/stressed default rates were calibrated such that the model implied default dependency structure matches the target one described by a Gaussian copula with the base asset return correlation of 25%.

3.6.2 *Default dependency structure*

The dependency structure of random Bernoulli defaults D^{c_1} and D^{c_2} of two distinct proxy reinsurers with credit rating c_1 and c_2 is uniquely defined by their 2×2 contingency table[9]

$P_{10}^{c_1c_2}$	$P_{11}^{c_1c_2}$
$P_{00}^{c_1c_2}$	$P_{01}^{c_1c_2}$

where $P_{kl}^{c_1c_2} = \mathbb{P}\left[D^{c_1} = k;\, D^{c_2} = l\right], \quad k,l = \overline{0,1}$ is a joint probability of default/survival for two reinsurers. The joint default probability P_{11} uniquely

9 Here we are dealing with binary random variables, and hence use discrete copulae to define their dependency structure. A good treatment of discrete copulae can be found in Genest and Nešlehová [7].

determines the complement of the contingent table, since $P_{10} + P_{11}$ and $P_{01} + P_{11}$ are marginal default rates that are fixed. Below we provide formulae needed to populate the contingency table for the target default dependency and the one implied by the model.

TARGET DEFAULT DEPENDENCY: The target dependency is defined by a Gaussian copula with a given base asset return correlation . Therefore, the joint default probability for two distinct proxy reinsurers with credit rating c_1 and c_2 is defined by a Gaussian copula in the following way:

$$P_{11}^{c_1 c_2} = \int_{-\infty}^{R_1} \int_{-\infty}^{R_2} \frac{1}{2\pi\sqrt{1-\rho_A^2}} \exp\left\{-\frac{x_1^2 + x_2^2 - 2\rho_A x_1 x_2}{2(1-\rho_A^2)}\right\} dx_1\, dx_2, \tag{6}$$

where $R_i = \Phi^{-1}\left(\mathbb{E}[D^{c_i}]\right),\ i = \overline{1,2}$; ρ_A is the base asset return correlation. The variable x_i represents asset return for i-th reinsurer. The asset return below a given threshold R_i will lead to default. Assuming the asset returns are normally distributed, one can express R_i as a percentile, i.e. $R_i = \Phi^{-1}(P^{c_i})$, where $P^{c_i} = \mathbb{E}[D^{c_i}]$ are the marginal unconditional probability of default.

DEFAULT DEPENDENCY IMPLIED BY THE MODEL: The joint probabilities are provided below

$$\begin{aligned} P_{kl}^{c_1 c_2} &= \mathbb{P}\left[D^{c_1} = k;\ D^{c_2} = l\right] \qquad (7)\\ &= \sum_{z=1}^{5} \mathbb{P}\left[D^{c_1} = k \,|\, Z = z\right] \mathbb{P}\left[D^{c_2} = l \,|\, Z = z\right] \mathbb{P}\left[Z = z\right],\quad k, l = \overline{0,1}, \end{aligned}$$

where $\{D^{c_i} \,|\, Z = z\} \sim Be\left(1 - (1 - q_n^{c_i})^{z-1}\,(1 - q_s^{c_i})^{5-z}\right),\quad i = \overline{1,2}.$

3.6.3 *Default (Pearson) correlation*

In addition to the base asset return correlation between the net assets of two proxy reinsurers, the formula below (sometimes referred to as *phi*) is the Pearson Correlation coefficient between the defaults of two proxy reinsurers. To calculate the model implied default correlation it is required to calculate variance/covariance matrix of reinsurance defaults. The following uses the conditional variance/covariance formula to calculate default correlation for two distinct proxy reinsurers with credit rating c_1 and c_2 .

$$\phi_{12} = \frac{\mathrm{Cov}\left[D^{c_1}, D^{c_2}\right]}{\sqrt{\mathrm{Var}\left[D^{c_1}\right] \cdot \mathrm{Var}\left[D^{c_2}\right]}}, \tag{8}$$

where

$$\begin{aligned} \mathrm{Cov}\left[D^{c_1}, D^{c_2}\right] &= \mathbb{E}_Z\left[\mathrm{Cov}\left[D^{c_1}, D^{c_2} \,|\, Z\right]\right] \\ &\quad + \mathrm{Cov}\left[\mathbb{E}\left[D^{c_1} \,|\, Z\right], \mathbb{E}\left[D^{c_2} \,|\, Z\right]\right] \\ &= \mathrm{Cov}\left[\mathbb{E}\left[D^{c_1} \,|\, Z\right], \mathbb{E}\left[D^{c_2} \,|\, Z\right]\right] \\ &= \mathrm{Cov}\left[(1 - q_n^{c_1})^{z-1}\,(1 - q_s^{c_1})^{5-z}, (1 - q_n^{c_2})^{z-1}\,(1 - q_s^{c_2})^{5-z}\right]; \end{aligned}$$

$$\begin{aligned} \mathrm{Var}\left[D\right] &= \mathbb{E}_Z\left[\mathrm{Var}\left[D \,|\, Z\right]\right] + \mathrm{Var}\left[\mathbb{E}\left[D \,|\, Z\right]\right] \\ &= \mathbb{E}_Z\left[\mathrm{Var}\left[D \,|\, Z\right]\right] + \mathrm{Var}\left[(1 - q_n)^{z-1}\,(1 - q_s)^{5-z}\right]; \end{aligned}$$

$$\begin{aligned} \mathrm{Var}\left[D \,|\, Z\right] &= (1 - q_n)^{Z-1}\left(1 - (1 - q_n)^{Z-1}\right)\left\{1 - 2\left(1 - (1 - q_s)^{5-Z}\right)\right\} \\ &\quad + (1 - q_n)^{Z-1}\left(1 - (1 - q_s)^{5-Z}\right)\left\{1 - (1 - q_n)^{Z-1}\left(1 - (1 - q_s)^{5-Z}\right)\right\}. \end{aligned}$$

Alternatively, the Pearson correlation formula for the defaults of two proxy reinsurers can be derived from the first principle taking into account the fact

that
Bernoulli random variables. Expression (8) can be rewritten in the following way:

$$\phi_{12} = \frac{P_{11}^{c_1 c_2} - P^{c_1} P^{c_2}}{\sqrt{P^{c_1}\left(1 - P^{c_1}\right) P^{c_2}\left(1 - P^{c_2}\right)}}, \tag{9}$$

where $P_{11}^{c_1 c_2}$ is joint default probability for two distinct proxy reinsurers with rating c_1 and c_2 defined in (7); $P^{c_i} = \mathbb{E}[D^{c_i}], \ i = \overline{1,2}$ are the marginal unconditional probabilities of default.

3.6.4 *Calibration of stressed/normal default rates*

… I will never sacrifice reality for elegance without explaining why I have done so.

— The Modellers' Hippocratic Oath [5]

Under the constraints that the unconditional annual default rates and the market transition rate remain unchanged, we set the quarterly normal/stressed default rates such that the model implied default dependency structure matches the target one described by a Gaussian copula with the base asset return correlation of 25%. Equivalently, one needs to match the joint default probabilities from the two dependency structures formulated in (6) and (7), since the joint default probability uniquely determines the complement of the contingent table, i.e. the complements of the constant marginal default rates, and thus the joint survival probability. We used the *diagonal calibration*, in which for a given credit rating the normal/stressed default rates were set such that the dependency structure of two distinct proxy reinsurers with the same credit rating matches the target one. Such diagonal approach does not consume as much computational resources as the full calibration would, and is relatively easy in use, especially when being implemented using EXCEL VBA. However, the main implication of using it is that the implied base asset return correlation for proxy reinsurers with different credit rating deviates from the level of 25% actually used in the diagonal calibration. The full calibration of normal/stressed default rates overcomes that drawback. It can be well performed using, for example, the programming suite of MATHEMATICA™.

Below are the exhibits representing the results of the diagonal calibration.

CALIBRATED CONDITIONAL DEFAULT RATES:

Exhibit 6. *Calibrated conditional 'normal' annual forward rates of default (in %)*

YEAR	AAA	AA+	AA	AA-	A+	A	A-	BBB+	NR
1	$6.9{\times}10^{-5}$	0.038	0.089	0.148	0.218	0.277	0.350	0.744	2.513
2	$8.3{\times}10^{-3}$	0.053	0.103	0.162	0.229	0.287	0.374	0.794	2.665
3	0.012	0.059	0.108	0.167	0.233	0.290	0.383	0.813	2.722
4	0.017	0.066	0.113	0.172	0.237	0.293	0.393	0.832	2.780
5	0.022	0.072	0.118	0.177	0.242	0.296	0.402	0.851	2.836
6	0.027	0.079	0.124	0.183	0.246	0.299	0.411	0.872	2.897
7	0.034	0.088	0.131	0.190	0.252	0.304	0.424	0.898	2.975
8	0.041	0.097	0.138	0.197	0.258	0.308	0.435	0.922	3.046

Exhibit 7. *Calibrated conditional 'stressed' annual forward rates of default (in %)*

YEAR	AAA	AA+	AA	AA-	A+	A	A-	BBB+	NR
1	0.987	2.466	3.688	4.839	6.016	6.926	7.951	12.589	27.081
2	1.472	2.870	3.984	5.092	6.189	7.058	8.270	13.107	28.098
3	1.644	3.023	4.087	5.180	6.256	7.103	8.390	13.302	28.473
4	1.816	3.179	4.192	5.271	6.324	7.150	8.512	13.501	28.856
5	1.976	3.325	4.292	5.357	6.389	7.194	8.628	13.689	29.219
6	2.146	3.482	4.400	5.450	6.460	7.242	8.754	13.894	29.611
7	2.360	3.681	4.537	5.569	6.550	7.303	8.914	14.154	30.110
8	2.551	3.859	4.661	5.676	6.632	7.359	9.059	14.390	30.561

EXAMPLES OF CALIBRATED CONTINGENT TABLES: As an illustration consider two distinct proxy reinsurers of the same credit rating, say BBB+. In the exhibits below 1 indicates default, and 0 indicates survival.

1) Contingent table implied by the model (in %)

1	1.42021284314019	0.0854850327281303
0	97.0740892809915	1.42021284314019
	0	1

2) The target dependency structure defined by a Gaussian copula (in %)

1	1.42021284269022	0.085485033178101
0	97.0740892814415	1.42021284269022
	0	1

3) Contingent table for two independent defaults (in %)

1	1.48302661493437	0.0226712609339436
0	97.0112755091973	1.48302661493437
	0	1

BASE ASSET RETURN CORRELATION VS DEFAULT CORRELATION: The calibrated conditional default rates shown above were obtained from using the diagonal calibration, in which the implied default dependency structure is matched to the target one defined by a Gaussian copula with the base asset return correlation of 25% only for pairs of distinct proxy reinsurers having the same rating. Using those calibrated conditional normal/stressed default rates will result in slightly biased value of base asset return correlation for pairs of proxy reinsurers with different credit rating. The example of this is illustrated in Exhibit 8 below.

Exhibit 8. *Model implied base asset return correlation (in %)*

	AAA	AA+	AA	AA-	A+	A	A-	BBB+	NR
AAA	25.0								
AA+	25.0	25.0							
AA	25.1	25.0	25.0						
AA-	25.3	25.1	25.0	25.0					
A+	25.4	25.2	25.1	25.0	25.0				
A	25.5	25.2	25.1	25.0	25.0	25.0			
A-	25.7	25.3	25.1	25.1	25.0	25.0	25.0		
BBB+	26.2	25.7	25.4	25.3	25.2	25.1	25.1	25.0	
NR	28.0	26.9	26.4	26.1	25.9	25.7	25.6	25.3	25.0

Please note that the differences in the base asset return correlation are within the simulation error bounds when using a model run with 10,000 simulation trials. Exhibit 9 shows that the implied Pearson correlation of defaults is much lower than the base asset return correlation.

Exhibit 9. *Model implied default correlation (in %)*

	AAA	AA+	AA	AA-	A+	A	A-	BBB+	NR
AAA	0.68								
AA+	0.96	1.36							
AA	1.11	1.56	1.81						
AA-	1.22	1.72	1.99	2.19					
A+	1.32	1.86	2.15	2.36	2.55				
A	1.39	1.95	2.25	2.48	2.68	2.81			
A-	1.46	2.05	2.36	2.60	2.81	2.95	3.10		
BBB+	1.70	2.40	2.77	3.05	3.29	3.45	3.62	4.24	
NR	2.19	3.08	3.56	3.92	4.23	4.44	4.66	5.45	7.01

3.6.5 *Modelling reinsurance default in run-off*

... Nor will I give the people who use my model false comfort about its accuracy. Instead, I will make explicit its assumptions and oversights.

— The Modellers' Hippocratic Oath [5]

As was mentioned at the beginning of this section, the proposed DFA modelling approach considers quantification of reinsurance credit risk emerging over a one year time horizon of writing business as well as the run-off of this risk to extinction. The risk of reinsurance default in run-off (i.e. after quarter four) is captured through increasing the quarter four default rate. The required overlay of the default rate is calculated as an expected rate of default occurred in the runoff, weighted by relative exposure to reinsurance recoveries (both cat and non-cat) at the beginning of quarter five, and is added to the quarter four normal/stressed default rates. In essence, the actual timing of the default in run-off (i.e. beyond one year period) is not modelled in the DFA model. The run-off default risk is rather condensed over the run-off time and brought back to quarter four through overlaying its quarterly conditional normal/stressed default rates.

On the other hand, calibrating the quarter four default rate overlays will require explicit modelling of the timing of the default in runoff. Mathematically speaking, the default of a proxy reinsurer in run-off is modelled using 'life contingency' approach, in which the proxy reinsurer's survival path depends on random times of market transition into stressed state. For the purpose of illustration, let us assume particular (see Table 2) combined cat and non-cat run-off patterns over the period from quarter 5 to full extinction at the end of quarter 32.

The run-off pattern of a particular proxy reinsurer consists of run-off outstanding reinsurance recoveries at the beginning of each quarter expressed as the percentage, $w(k)$, $k = \overline{5,32}$, of the initial exposure to reinsurance recoveries at the beginning of quarter 5. Then the expected default rate over the period of run-off to full extinction can be modelled in the following way:

$$\theta = \mathbb{E}_\delta\left[\sum_{k=5}^{32}\left(\prod_{j=1}^{k-5}[1-q_s(j+4)]^{\delta(j+4)}[1-q_n(j+4)]^{1-\delta(j+4)}\right)\times[q_s(k)]^{\delta(k)}[q_n(k)]^{1-\delta(k)}\,w(k)\right], \qquad (10)$$

where $\prod_{j=1}^{0} \triangleq 1$; $q_s(k)$ and $q_n(k)$ are calibrated conditional 'stressed' and 'normal' quarterly forward default rates in quarter k; $\delta(k)$ is the '0/1' process

Table 2: *Combined cat and non-cat run-off patterns (in %)*

QUARTER	AAA	AA+	AA	AA-	A+	A	A-	BBB+	NR
5	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
6	69.92	85.11	77.93	79.37	76.81	73.55	75.59	76.10	87.90
7	47.17	72.30	60.42	62.80	58.57	53.17	56.56	57.39	76.92
8	31.01	61.42	47.04	49.92	44.81	38.28	42.37	43.38	67.01
9	20.00	52.18	36.96	40.01	34.60	27.69	32.02	33.09	58.09
10	12.69	44.29	29.35	32.35	27.03	20.25	24.50	25.55	50.09
11	8.00	37.53	23.57	26.37	21.40	15.06	19.03	20.01	42.95
12	5.19	31.73	19.18	21.69	17.23	11.53	15.10	15.98	36.60
13	3.53	26.72	15.76	17.95	14.05	9.07	12.19	12.96	30.98
14	2.66	22.40	13.07	14.94	11.62	7.38	10.04	10.69	26.03
15	2.22	18.67	10.89	12.45	9.68	6.15	8.36	8.91	21.69
16	1.83	15.41	8.99	10.28	7.99	5.08	6.90	7.36	17.91
17	1.50	12.60	7.35	8.40	6.53	4.15	5.64	6.01	14.63
18	1.21	10.18	5.94	6.79	5.28	3.35	4.56	4.86	11.82
19	0.964	8.115	4.734	5.412	4.208	2.673	3.635	3.874	9.429
20	0.757	6.373	3.717	4.250	3.305	2.099	2.855	3.042	7.404
21	0.584	4.919	2.869	3.280	2.551	1.620	2.204	2.348	5.715
22	0.443	3.724	2.172	2.484	1.931	1.227	1.668	1.778	4.327
23	0.328	2.758	1.609	1.839	1.430	0.909	1.236	1.317	3.205
24	0.237	1.991	1.162	1.328	1.033	0.656	0.892	0.951	2.314
25	0.166	1.394	0.813	0.930	0.723	0.459	0.624	0.665	1.620
26	0.111	0.936	0.546	0.624	0.485	0.308	0.419	0.447	1.087
27	0.071	0.597	0.349	0.398	0.310	0.197	0.268	0.285	0.694
28	0.043	0.358	0.209	0.239	0.186	0.118	0.161	0.171	0.416
29	0.024	0.199	0.116	0.133	0.103	0.066	0.089	0.095	0.231
30	0.012	0.100	0.058	0.066	0.052	0.033	0.045	0.048	0.116
31	0.005	0.040	0.023	0.027	0.021	0.013	0.018	0.019	0.046
32	0.001	0.010	0.006	0.007	0.005	0.003	0.004	0.005	0.012

generated via a Geometric r.v. of the timing of market transition into stressed state. At the beginning of each quarter $k \geq 1$ we simulate a transition event:

1) if the market transits into stressed state, then the '0/1' process takes values 1 in this quarter and the following seven quarters, i.e. $\delta(j) = 1, \; \forall j \in \overline{k, k+1, \ldots, k+7}$, and go to quarter $k+8$;

2) otherwise $\delta(k) = 0$ and go to quarter $k+1$.

Since the duration of market's stay in the 'stressed' state is assumed to be two years, the maximum number of market transitions into stressed state over the period of 32 quarters is four. Therefore, the '0/1' process is completely defined by the following Geometric random variables:

$$
\begin{aligned}
\tau_1 &= \zeta_1, \\
\tau_2 &= \tau_1 + 7 + \zeta_2, \\
\tau_3 &= \tau_2 + 7 + \zeta_3, \\
\tau_4 &= \tau_3 + 7 + \zeta_4,
\end{aligned}
$$

where $\zeta_i \sim Geom[p]$ are i.i.d. Geometric random variables quarterly market transition rate p. Each Geometric r.v. can be simulated using the representation Int $\left[\frac{\ln(u)}{\ln(1-p)}\right] + 1$, where u is uniformly distributed on $(0, 1)$.

Before applying these default rate overlays, one needs to adjust them for the difference in expected outstanding reinsurance recoveries at the beginning of quarters four and five, i.e. multiply the expected rate overlays by ratio of quarter four to quarter five outstanding reinsurance recoveries of corresponding credit rating. In this model the expected outstanding reinsurance recoveries are extracted from the *Reinsurance* module.

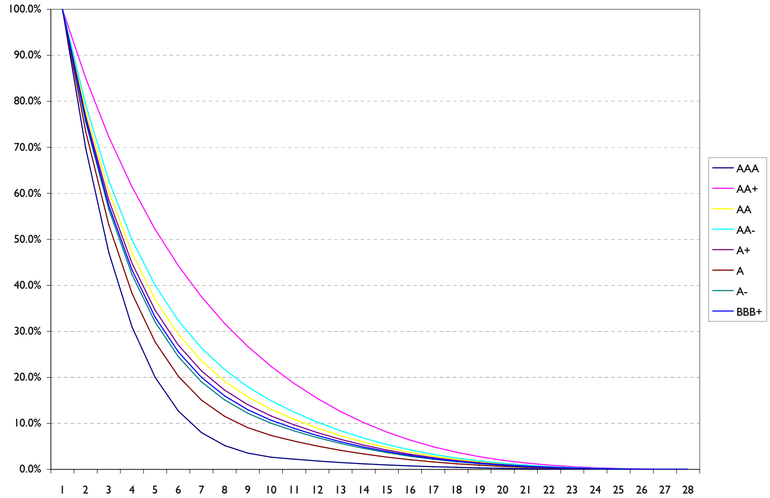
Exhibit 10 below represents unadjusted expected overlays θ for quarter four conditional normal/stressed default rates. These overlays were derived using EXCEL VBA simulation routine.

Exhibit 10. *Quarter four default rate overlay (in %)*

AAA	AA+	AA	AA-	A+	A	A-	BBB+	NR
0.203	0.920	0.834	1.168	1.272	1.275	1.667	2.858	13.179

Please note that the run-off patterns of outstanding reinsurance recoveries plays a special role here. The more flat a particular run-off pattern is, the more exposed to the reinsurance credit risk in run-off the ceding insurer becomes. As the result the default rate overlay might be relatively higher for better credit rating. For instance, in the table above the additional default rate overlay for AA+ is slightly higher than the one for AA. This anomaly is due to the difference in the run-off patterns of combined outstanding cat and non-cat reinsurance recoveries from these two credit rating grades. The runoff pattern for AA+ is flat (see Exhibit 11 below) compared to the ones for other credit grades.

Exhibit 11. *Run-off patterns of outstanding reinsurance recoveries by credit rating*



Under these circumstances the portion of the ceding insurer's outstanding reinsurance recoveries at risk will be greater for AA+ grade in most of the time, and this will outweigh the impact of marginally lower default rates on the calculation of additional quarter four default rate overlay.

3.7 *Distribution of the total reinsurance default cost*

The following exhibits provide key statistics of the distribution of total reinsurance default cost expressed as a percentage of APRA's Minimum Capital Requirement (MCR) that is defined as 1-in-200 years loss.

Exhibit 12. *The distribution of total reinsurance default cost (% of MCR)*

50th percentile	75th percentile	90th percentile	95th percentile	99th percentile	99.5th percentile	99.9th percentile
0.000	0.059	0.996	1.615	3.525	3.525	3.525

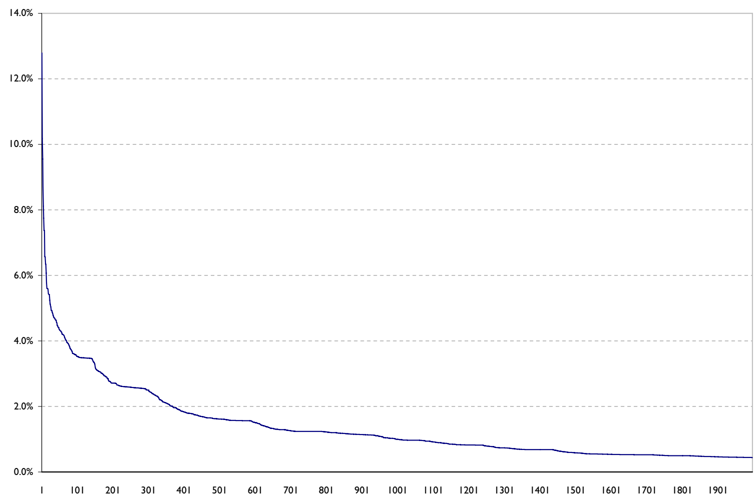
Exhibit 13. *The distribution of total reinsurance default cost given default (% of MCR)*

50th percentile	75th percentile	90th percentile	95th percentile	99th percentile	99.5th percentile	99.9th percentile
0.487	1.132	2.003	2.862	4.674	4.674	4.674

Exhibit 14. *Other key statistics of the distribution*

Average cost of default (% of MCR)	Chance of incurring default cost over 1yr (%)	Average cost of default given default (% of MCR)
0.29	36.70	0.80

Exhibit 15. *Total reinsurance default cost (% of MCR) – worst 20% cases*



It should be noted that the 1-in-200 total reinsurance default cost derived from the single model run of 'standard' number of simulations, say 10,000 simulation trials, is not necessarily fully, if at all, contributing to the overall 1-in-200 capital requirement. This is because such losses are assumed to be independent of the key risks, such as catastrophe and underwriting, which mainly determine the capital requirement. The company's economic capital can be considered as an accounting (scalar) function of a bivariate random vector with two independent marginals: reinsurer credit risk and the aggregation of all other risks. To get a full distribution of a random vector one would need to use, for instance, sequential simulation with very large number of trials. This type of randomization will lead to more accurate estimate of the reinsurance credit risk contribution to the company's 1-in-200 capital requirement. In particular, we believe that as the number of simulation trials increases rapidly that estimate will converge to a positive, most likely immaterial dollar amount.

3.8 *Further developments and improvements*

In this paper, following the general approach to modelling credit risk, we have assumed for convenience that the return on an underlying reinsurer's net assets follows a normal distribution. This assumptions might be weak, as the actual distribution of asset returns may have heavy tails. Therefore, it might be appropriate to use heavier tailed distributions like t-distribution or *Extreme Value (EV)* distribution. This will result in much reacher dependency structure for reinsurance default defined by t and EV copulae[10]. Particularly t copula is the most popular alternative to Gaussian copula. Although its density is quite similar to Gaussian copula density, it is much more concentrated in the upper and lower corners of the contingent table. Having fixed market transition rate and the base asset return correlation, one could tune the degree of upper tail dependence through the t-copula parameter.

4 CONCLUSIONS

... I understand that my work may have enormous effects on society and the economy, many of them beyond my comprehension.

— The Modellers' Hippocratic Oath [5]

In this paper we developed an integrated stochastic approach to modelling the reinsurance credit risk in a multi-period setting within DFA environment. This approach has some advantages over other existing approaches to modelling reinsurance credit risk that are based on ideas borrowed from the investment banking industry. First of all, it uses reinsurers' asset impairment rates rather than corporate bond default rates which are inappropriate in reinsurance credit risk modelling. Secondly, it offers explicit modelling of reinsurance defaults with embedded dependency between them. It imposes a Gaussian dependency structure for reinsurance defaults via introducing reinsurers' asset return correlation. It also assumes that the reinsurance market transits into 'stressed' state with some fixed transition rate, which implies state-dependent default rates for reinsurers. The calibration of the state-dependent 'normal' and 'stressed' default rates constitutes the main contribution of this paper. The results of calibration are discussed and illustrated by a number of numerical examples. This paper also provides numerical illustrations of assessment of the reinsurance credit risk and its contribution to the overall capital requirement for a generic general insurer.

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10 The formal definitions and overview of properties of t and EV copulae can be found in McNeil et al. [10] or Panjer [11].

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