

Proxies – Glenn Meyers

## Proxies

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**Abstract.**

This paper illustrates a Bayesian method to estimate the predictive distribution for outstanding loss liabilities that can be applied when there is either insufficient data or little actuarial expertise. The assumptions made are that the unpaid losses can be described by the collective risk model and that the scenarios that make up the prior distribution contain the possible loss ratio and loss development factors. As one should expect, the more data one puts into the method, the tighter the predictive distribution will be.

**Keywords**

Reserving Methods, Reserve Variability, Uncertainty and Ranges, Collective Risk Model, Fourier Methods, Bayesian Estimation

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## 1. Introduction

As the European Union works toward implementing Solvency II, a number of insurers are participating in a series of “Quantitative Impact Studies” to learn about and to test the various proposals contained its provisions. One of the more interesting concepts in these proposals goes under the name of “proxies.” As defined by CEIOPS [1], “the term ‘proxy’ is used to denote simplified methods for the valuation of technical provisions (otherwise known as loss reserves) that are applied when there is only insufficient data to apply a reliable statistical actuarial method, or when there is insufficient actuarial expertise available to the insurer.” In reading through their documents on the subject, the terms “benchmark” and “credibility” appear often.

Insurers in the United States have a strong tradition of using benchmark data. For example, each insurer is required to submit ten-year (Schedule P) loss triangles by line insurance in their annual statements filed with the National Association of Insurance Commissioners (NAIC). Also, insurers are required to submit more detailed data to regulators through a statistical agent. So Americans have a lot of potential benchmarks.

This paper describes a proxy method for loss reserves that could work with the data available in the United States. The examples that follow are for real insurers on some fairly old Commercial Automobile Schedule P data.

For “benchmarks.” these examples use a list of 5,000 scenarios representing the expected loss ratio for each of ten accident years,  $\{ELR\}$ , and ten incremental paid loss development factors  $\{Dev\}$  by settlement lag.

An insurer supplies data consisting of earned premium,  $\{Premium_{AY}\}$ , and incremental paid losses,  $\{X_{AY,Lag}\}$ , by accident year and settlement lag. Usually this will consist of the upper part of a standard incremental paid loss development triangle.

The method uses the data to calculate the likelihood of each scenario, i.e., the probability of the data given each scenario. Making the assumption that each scenario

is equally likely, one can use Bayes' Theorem to calculate the posterior distribution of the scenarios, i.e., the probability of each scenario given the data.

For a given scenario, one can calculate the distribution of losses,  $X_{AY,Lag}$ , for each  $(AY,Lag)$  cell with the collective risk model. In general, the distribution of losses for a single cell is not of interest. Instead, it is the distribution of the sum of losses, usually the sum of losses for future payments that is of interest. This paper demonstrates a method of calculating the distribution of the sum of the losses by cell, taking correlation between the losses in the same accident year into account.

The predictive distribution of losses will be the mixture of the conditional probabilities of the losses given each scenario, weighted by the posterior probability of each scenario.

This paper describes, in some detail, a statistical method that can carry out the procedure described above. It starts by describing a Bayesian methodology to assign posterior probabilities to each scenario. Next it describes how to calculate the predictive distribution of the sum of losses for a selected set of  $(AY,Lag)$  cells. It then finishes by describing how to derive a set of scenarios from the loss triangles of large insurers. The code for a program that implements this procedure in the R programming language, along with a spreadsheet containing the scenarios, are included with this paper.

## 2. The Collective Risk Model

This section describes the particular form of the collective risk model used in this paper. In the past, I have found it easiest to explain the collective risk as a computer simulation and I will continue this practice here.

### Simulation Algorithm 1

For  $AY = 1, \dots, 10$

1. Select  $\chi$  from a distribution with mean 1 and variance  $c$ .
2. For  $Lag = 1, \dots, 10$ 
  - a. Select a random claim count  $N_{AY,Lag}$  from a Poisson Distribution with mean  $\chi \cdot \lambda_{AY,Lag}$ .
  - b. If  $N_{AY,Lag} > 0$ , for  $i = 1, \dots, N_{AY,Lag}$ , select a claim amount,  $Z_{i,Lag}$ , from a claim severity distribution. The claim severity distribution will be different for each settlement lag.
  - c. If  $N_{AY,Lag} > 0$ , set  $X_{AY,Lag} = \sum_{i=1}^{N_{AY,Lag}} Z_{i,Lag}$ . If  $N_{AY,Lag} = 0$ , set  $X_{AY,Lag} = 0$ .

In general, insurers observe losses,  $X_{AY,Lag}$ , in one subset of the 10x10 grid, and they want to predict the distribution of the sum of losses in another subset of the grid. Typically the observed subset of losses is in the upper triangle of losses where  $AY + Lag \leq 11$ . Of most interest is the predictive distribution of the sum of losses,  $X_{AY,Lag}$  for  $AY + Lag > 11$ . It is often of interest to obtain separate predictive distributions by accident year.

In this paper, we will assume that the contagion parameter,  $c$ , defined in Step 1 above is known. We will use  $c = 0.01$ . Meyers [2], gives one way to estimate this parameter from data similar to that used in this paper. In principle, it is possible to add different values of  $c$  to the scenarios used in this paper's methodology.

We paper also require that the claim severity distributions be given. This paper uses a Pareto distribution of the form:

$$F_{Lag}(z) = 1 - \left( \frac{\theta}{z + \theta} \right)^\alpha.$$

The parameters by lag are given in the following table<sup>1</sup>. The units are in thousands of dollars. Individual claims were capped at \$1,000,000. Note that the expected claim severity increases with settlement lag. This agrees with the general observation that larger claims take longer to settle.

Settlement Lag	Expected Claim Severity	Coefficient of Variation	$\theta$	$\alpha$
1	5	3.00	4.5244	1.8978
2	20	3.50	6.4195	1.2099
3	75	2.00	39.1784	1.3639
4	100	1.75	62.9849	1.4565
5	125	1.60	84.1527	1.4715
6	150	1.55	77.6372	1.2481
7	175	1.50	68.9063	1.0566
8	185	1.45	78.6878	1.0837
9	195	1.40	91.6444	1.1261
10	200	1.35	117.6357	1.2623

### 3. Calculating the Likelihood of the Data

While simulation is a good way to describe the collective risk model, it is difficult to use the method to calculate the probability density of a random loss,  $X_{AY,Lag}$ . For the examples in this paper, a loss triangle has 55  $(AY,Lag)$  cells. As we shall see below, the probability density must be calculated three times for each cell for each of 5,000

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<sup>1</sup> My employer, ISO, provides claim severity distributions appropriate for a variety of lines of insurance based on data reported by a large number of insurers. ISO declined to allow use of their distributions for this paper. The claim severity distributions I selected for this paper, while reasonable, should not be used for real applications.

scenarios. This gives us 825,000 probability densities to calculate for each predictive distribution. Speed is important.

Let  $\mu_{Lag}$  and  $\sigma_{Lag}$  denote the mean and standard deviation of claim severity,  $Z_{Lag}$ . With computational speed in mind, this paper uses a function,  $ODNB(x|\lambda,\mu,\sigma)$ , which approximates the density of a compound Poisson distribution with an overdispersed negative binomial distribution. This function is described in Appendix B of Meyers[4]. To summarize, the first two moments of the compound Poisson distribution, implied by  $\lambda$ ,  $\mu$  and  $\sigma$ , match the first two moments of the approximating distribution,  $ODNB(x|\lambda,\mu,\sigma)$ .

This paper uses a three-point discrete distribution for  $\chi$  (see Step 1 of Simulation Algorithm 1) with  $\chi_1 = 1 - \sqrt{3c}$ ,  $\chi_2 = 1$ ,  $\chi_3 = 1 + \sqrt{3c}$  and corresponding probabilities,  $w_1 = 1/6$ ,  $w_2 = 2/3$  and  $w_3 = 1/6$ .

Given the  $\{ELR_{AY}\}$  and  $\{Dev_{Lag}\}$  parameters of a scenario, we calculate the likelihood of the observed data in a loss triangle as follows.

1. For each  $AY$

- a. For each  $Lag$ , calculate  $\lambda_{AY,Lag} = Premium_{AY} \cdot ELR_{AY} \cdot Dev_{Lag} / \mu_{Lag}$ .
- b. Calculate  $L_{AY,i}(\{x_{AY,\cdot}\}) = \prod_{Lag} ODNB(x_{AY,Lag} | \chi_i \cdot \lambda_{AY,Lag}, \mu_{Lag}, \sigma_{Lag})$
- c. Calculate  $L_{AY,\cdot}(\{x_{AY,\cdot}\}) = \sum_{i=1}^3 w_i \cdot L_{AY,i}(\{x_{AY,\cdot}\})$ .

2. The likelihood for the entire loss triangle is given by:

$$L(\{x_{AY,Lag}\} | Scenario) = \prod_{AY} L_{AY,\cdot}(\{x_{AY,\cdot}\}).$$

The complications in Step 1 account for the “correlation” between settlement lags in the same accident year implied by Step 2a in Simulation Algorithm 1. Step 2 in the likelihood calculation is simpler because we assume that the losses are independent between accident years for a given scenario.

**4. The Posterior Distribution of the Scenarios**

The likelihood,  $L(\{x_{AY,Lag}\} | Scenario)$ , can be interpreted as the probability of the observed data,  $\{x_{AY,Lag}\}$ , given the scenario. We now use Bayes' Theorem to invert the probabilities to find the posterior probabilities of the scenarios.

$$\Pr(Scenario | \{x_{AY,Lag}\}) \propto L(\{x_{AY,Lag}\} | Scenario) \cdot \Pr(Scenario).$$

Given a loss triangle, one first calculates the likelihood,  $L(\{x_{AY,Lag}\} | Scenario)$ , for each of the 5,000 scenarios. Then, under the assumption that each  $\Pr(Scenario)$  is equally likely, the posterior distribution of each scenario is given by:

$$\Pr(Scenario | \{x_{AY,Lag}\}) = \frac{L(\{x_{AY,Lag}\} | Scenario)}{\sum_{Scenario} L(\{x_{AY,Lag}\} | Scenario)}.$$

We now calculate the posterior distribution for the four different loss triangles shown in Exhibit 1. The data in these triangles were taken from Schedule P, Part 3C - Commercial Auto Liability, of the NAIC Annual Statement. The data were reported as cumulative net paid losses. Occasionally the cumulative paid loss decreased, and I dropped negative incremental paid losses from the triangle.

Exhibit 1  
Sample Loss Development Triangles (000)

Insurer 1											
AY	Premium	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5	Lag 6	Lag 7	Lag 8	Lag 9	Lag 10
1	3,874	478	587	398	114	111	78	22	4	1	
2	4,755	702	500	458	353	150	93		12		
3	3,960	799	566	447	57	162		15			
4	3,094	802	605	308	67	22	1	1			
5	2,694	690	538	119	78	118	130				
6	1,958	526	277	289	50	17					
7	1,548	488	318	78	13						
8	1,710	540	367	114							
9	1,742	518	405								
10	1,740	564									

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Exhibit 1 (Continued)  
Sample Loss Development Triangles (000)

Insurer 2											
AY	Premium	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5	Lag 6	Lag 7	Lag 8	Lag 9	Lag 10
1	14,830	2,106	1,699	2,097	92	771	154	115	43	84	1
2	18,356	2,561	3,313	1,214	1,697	712	181	217	3	-	
3	20,178	3,456	3,342	2,141	1,817		195	192	412		
4	20,495	3,797	3,552	3,468	1,766	889	270	269			
5	22,078	4,581	2,893	3,370	1,512	570	143				
6	23,130	3,857	3,158	1,902	1,433	1,198					
7	22,662	3,837	2,593	1,853	1,717						
8	24,768	5,165	4,432	3,131							
9	27,437	5,394	4,882								
10	31,382	6,151									

Insurer 3											
AY	Premium	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5	Lag 6	Lag 7	Lag 8	Lag 9	Lag 10
1	48,844	5,886	6,128	4,739	5,442	1,741	1,183	678	158		303
2	52,622	5,632	7,487	6,630	5,195	2,465	1,149	459	372	184	
3	53,507	6,419	4,697	10,567	5,715	2,280	2,031	191	296		
4	56,949	7,300	8,939	9,495	6,966	3,960	787	1,060			
5	58,611	8,249	11,302	9,038	5,687	3,452	1,483				
6	61,692	8,074	9,454	7,913	3,455	3,485					
7	66,755	8,747	10,542	11,235	4,356						
8	81,119	10,258	15,376	11,697							
9	98,632	15,540	23,594								
10	91,311	14,289									

Insurer 4											
AY	Premium	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5	Lag 6	Lag 7	Lag 8	Lag 9	Lag 10
1	154,769	22,459	28,908	19,921	21,410	10,174	2,989	4,612			
2	166,929	19,945	14,711	29,697	24,723	9,032	6,564				
3	132,704	10,036	42,385	20,782	3,156	16,903		143	173		
4	212,198	21,380	40,015	37,410	12,050	5,553	6,711	1,087			
5	218,364	22,505	52,592	31,367	24,107	18,357	9,619				
6	235,788	26,633	47,039	33,072	24,141	11,471					
7	253,306	19,292	64,884	64,489	32,434						
8	210,428	28,867	57,014	48,262							
9	222,016	37,892	59,734								
10	212,360	33,500									

Figures 1 and 2 represent the posterior distributions for the  $\{ELR_{AY}\}$  and  $\{Dev_{Lag}\}$  parameters graphically as paths over time as measured by accident year and settlement lag, respectively. The lighter colored paths represented a random sample of 500 paths taken from the 5,000 scenarios that make up the prior distribution for each insurer. The darker colored paths represent the scenarios that account for 99.5% of the posterior probability. We see from these plots that the posterior probabilities are concentrated on fewer scenarios as the size of the insurer increases.

Figure 1

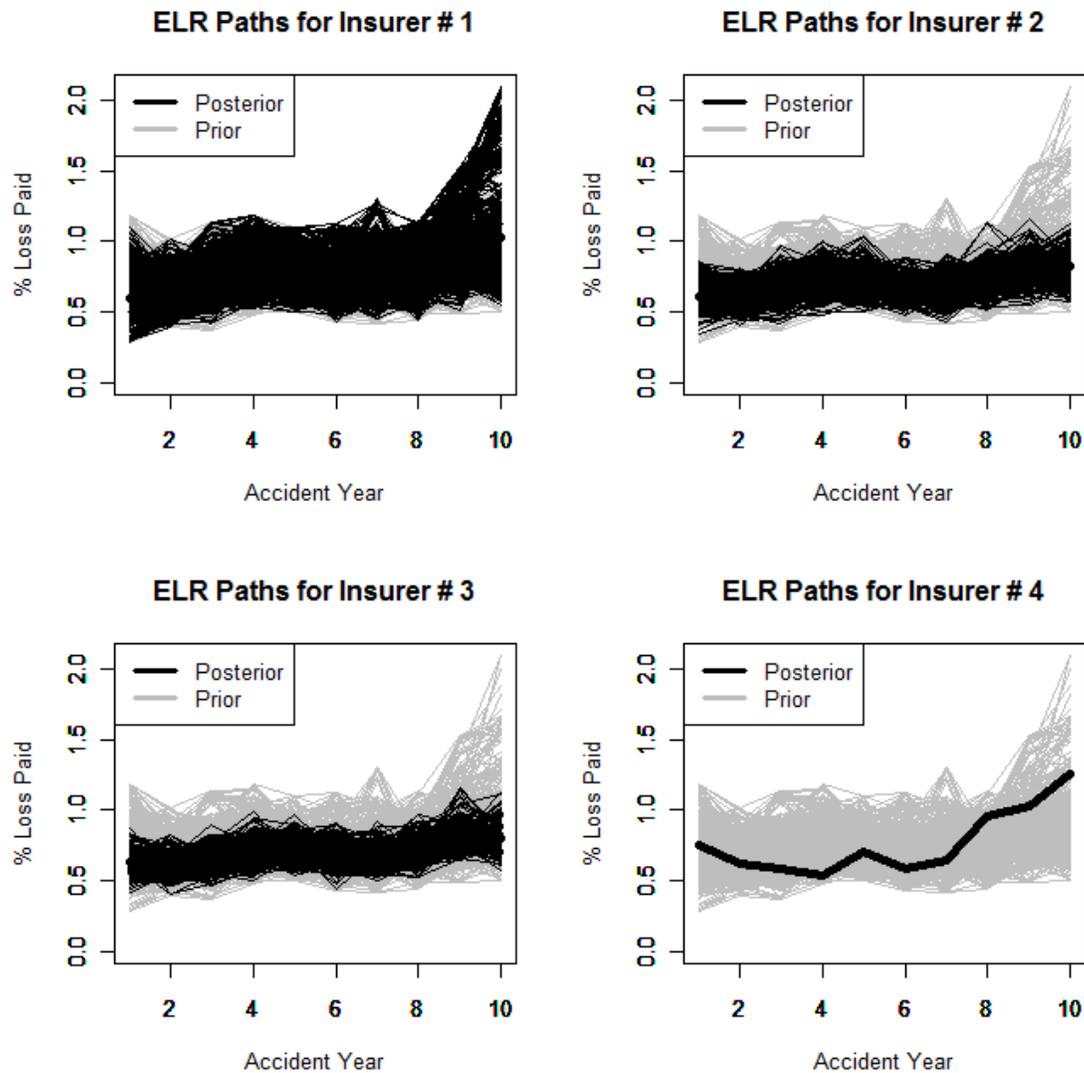
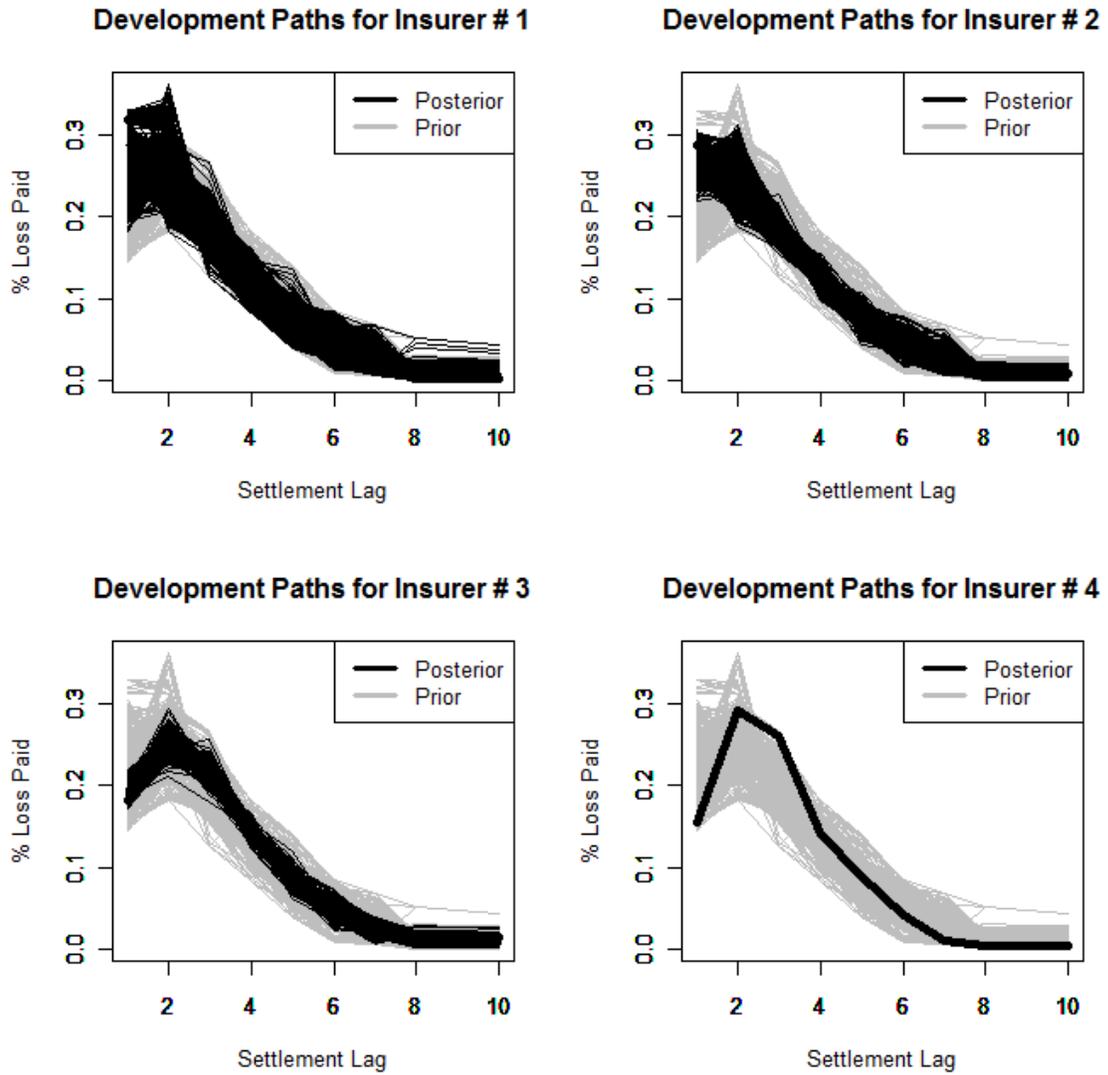


Figure 2



## 5. The Predictive Distribution of Outcomes

This section shows how to predict the distribution of the sum of losses in the lower triangle of the 10 x 10 grid,  $AY + Lag > 11$ . At a high level, this method works as follows.

1. For each scenario, calculate the distribution of the outcomes,  $X_{AY,Lag}$ , for each  $(AY,Lag)$  cell using the scenario's  $ELR_{AY}$  and  $Dev_{Lag}$  parameters and the collective risk model.

2. Calculate the distribution of the sum  $\sum_{AY=2}^{10} \sum_{Lag=12-AY}^{10} X_{AY,Lag}$  for each scenario.

3. The predictive distribution is the posterior probability weighted mixture of each scenario's distribution calculated in Step 2 above.

In principle, and perhaps even in practice, one could implement the procedure above by randomly selecting a scenario in proportion to its posterior probability and using Simulation Algorithm 1. This paper uses a faster and more accurate method that involves Fast Fourier Transforms (FFT).

The mathematics for calculating an aggregate loss distribution with an FFT is described in detail by Klugman, Panjer and Willmot [2]. More detailed descriptions of using FFT's in a loss reserve setting are in Meyers [4] and Meyers [5].

The first step involves discretizing the claim severity distribution. This can be done to any desired degree of accuracy.

We deal with the correlation between settlement lags in a given accident year by invoking a result due to Mildenhall [6]. His result is that the distribution of the sum of aggregate losses, linked by a common  $\chi$  as is done in Simulation Algorithm 1, has the same distribution as a single aggregate loss distribution, where the severity distribution is a mixture of the settlement lag distribution, weighted by the expected claim count.

What follows are the steps in the calculation of the predictive distribution of outcomes.

1. For each settlement lag, calculate the FFT,  $\phi(\bar{\mathbf{q}}_{Lag})$ , where  $\bar{\mathbf{q}}_{Lag}$  is the discretized claim severity distribution for  $Z_{Lag}$ .
2. Set  $\phi_{Post} = 0 + 0 \cdot i$  (A complex vector with the same length as  $\bar{\mathbf{q}}_{Lag}$ )
3. For each scenario, do the following.
  - a. Set  $\phi_{Scenario} = 1 + 0 \cdot i$  (A complex vector with the same length as  $\bar{\mathbf{q}}_{Lag}$ )
  - b. For each accident year,  $AY$ , do the following.
    - i. Set  $\phi_Z = 0 + 0 \cdot i$  (A complex vector with the same length as  $\bar{\mathbf{q}}_{Lag}$ )
    - ii. For each  $Lag$  in  $AY$ , do the following.
      1. Set  $\lambda_{AY,Lag} = Premium_{AY} \cdot ELR_{AY} \cdot Dev_{Lag} / E[Z_{Lag}]$ .
      2. Set  $\phi_Z = \phi_Z + \lambda_{AY,Lag} \cdot \phi(\bar{\mathbf{q}}_{Lag})$ .
    - iii. Set<sup>2</sup>  $\phi_{Scenario} = \phi_{Scenario} \cdot \left( 1 - c \cdot \left( \sum_{Lag} \lambda_{AY,Lag} \right) \cdot \left( \phi_Z / \sum_{Lag} \lambda_{AY,Lag} - 1 \right)^{-1/c} \right)$ .
  - c. Set  $\phi_{Post} = \phi_{Post} + \phi_{Scenario}$  times the Posterior Probability of the Scenario.
4. The discretized posterior,  $\bar{\mathbf{p}}$ , of the outcomes is the inverse FFT of  $\phi_{Post}$ .

Let  $\{x_i\}$  be the discrete points at which the vector  $\bar{\mathbf{p}} = \{p_i\}$  takes on its values. Then one can calculate a myriad of summary statistics for the predictive distribution of outcomes.

For example:

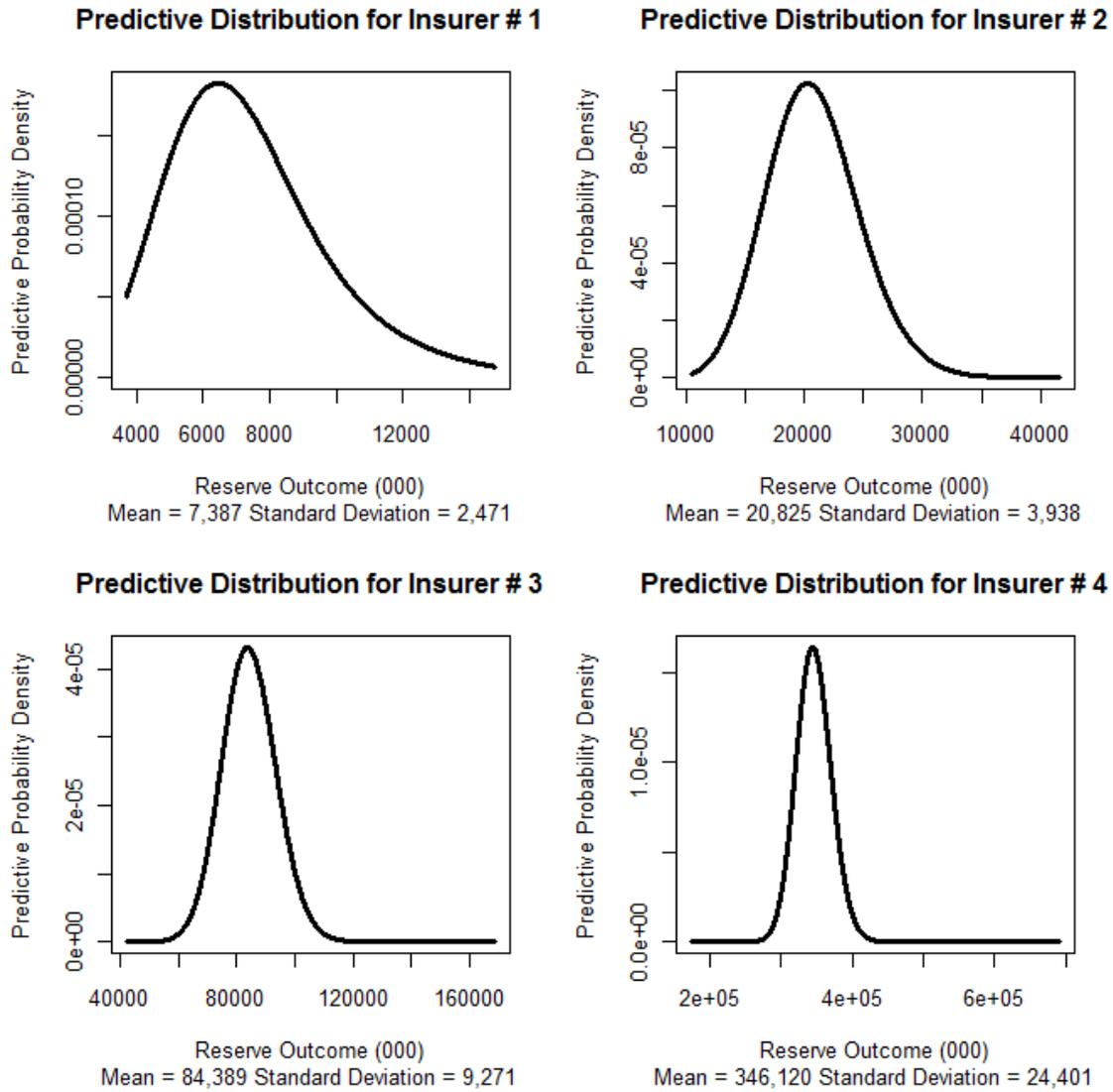
$$\text{Posterior Mean} = \sum_i p_i \cdot x_i \text{ and Posterior Standard Deviation} = \sqrt{\sum_i p_i \cdot x_i^2 - \left( \sum_i p_i \cdot x_i \right)^2}.$$

Figure 3 shows the plots of the predictive distributions and shows some summary statistics.

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<sup>2</sup> The FFT of an aggregate loss distribution with a negative binomial claim count distribution is given by the probability generating function of the negative binomial distribution applied to the FFT of the claim severity distribution. See Klugman *et. al.* [2] for details.

Figure 3



The scales of the horizontal axis in these plots is the same relative to the predictive mean, enabling one to see the decreasing volatility that comes (according to this model) with the increasing size of insurer.

## 6. Constructing the Scenarios

The scenarios used in this paper were constructed by examining the Schedule P paid triangles for 50 large insurers. For each insurer I did the following:

1. Obtained estimates of the  $\{ELR_{AY}\}$  and  $\{Dev_{Lag}\}$  parameters by maximum likelihood for each insurer using a generic numerical maximizing program.
2. Formed a “prior distribution” centered on the maximum likelihood estimate. Then, using the Gibbs sampler (See Meyers [5]) I generated 100 additional  $\{ELR_{AY}\}$  and  $\{Dev_{Lag}\}$  parameter sets. This generated a richer set of scenarios that could conceivably represent the collection of the 50 insurers.
3. Randomly shuffled the  $\{ELR_{AY}\}$  parameter set by Gibbs sample. I did the same, independently, for the  $\{Dev_{Lag}\}$  parameter set. I then bound them together to form a set of 5,000 scenarios.
4. The scenarios generated above contained some obvious outliers. So I then capped each of the  $ELR_{AY}$  and  $Dev_{Lag}$  parameters at the 1<sup>st</sup> and 99<sup>th</sup> percentiles for their respective accident year and settlement lags.

This procedure was fairly time consuming. It took over a week to do this for all fifty insurers. But once it is done, one can calculate the predictive distributions for any insurer in a matter of minutes.

This procedure is meant to be purely illustrative. The general idea is to come up with a rich set of scenarios that represents the range of reasonable outcomes. A jurisdiction lacking the massive data that is reported to American regulators can rely on its best judgment to select the scenarios.

## 7. Conclusion

The purpose of this paper is to illustrate a method an insurer with insufficient data or little actuarial expertise, could use to derive a predictive distribution of its outstanding loss liabilities. It is a mechanical procedure that involves plugging the available data into a computer program that generates a predictive distribution of outcomes.

The appendix contains a program written in the R Programming Language and sample data sets will be provided so that one can try running the program.

As one should intuitively expect, the predictive distributions can be relatively wide for small insurers with little data but they narrow as the insurer becomes large and has ample data.

The assumptions made in using this procedure are that the distribution of losses is described by the collective risk model with an assumed claim severity distribution and that some of the scenarios represent the possible  $\{ELR_{AY}\}$  and  $\{Dev_{Lag}\}$  parameters.

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## Appendix – Code for Calculating the Predictive Distribution

This appendix contains the code that produced Figures 1-3. It is written in the R programming language. It reads three files that are electronically available with this paper.

1. Capped and Shuffled Prior.csv – this file contains the 5,000 prior scenarios.
2. Severity Pareto.csv – this file contains the parameters of the Pareto distributions in Section 2 of this paper.
3. Rectangle Proxies.csv – This file contains the data for the four insurers' loss triangles in Exhibit 1.

### R code

```
# Input
comp=c(1,2,3,4)
pred.ay=c(2,rep(3,2),rep(4,3),rep(5,4),rep(6,5),rep(7,6),rep(8,7),rep(9,8),
          rep(10,9)) #accident years in prediction
pred.lag=c(10,9:10,8:10,7:10,6:10,5:10,4:10,3:10,2:10) #development lags
#
##### Begin Subprograms#####
#
# estimate the step size for discrete loss distribution
#
estimate.h<-function(premium,fftn,limit.divisors){
  h<-min(10*premium/2^fftn,max(limit.divisors)-1)
  h<-min(subset(limit.divisors,limit.divisors>h))
  return(h)
}
#
# Pareto limited average severity
#
pareto.las<-function(x,theta,alpha){
  las<- ifelse(alpha==1,-theta*log(theta/(x+theta)),
              theta/(alpha-1)*(1-(theta/(x+theta))^(alpha-1)))
  return(las)
}
#
# discretized Pareto severity distribution
#
discrete.pareto<-function(limit,theta,alpha,h,fftn){
  dpar<-rep(0,2^fftn)
  m<-ceiling(limit/h)
  x<-h*0:m
  las<-rep(0,m+1)
  for (i in 2:(m+1)) {
    las[i]<-pareto.las(x[i],theta,alpha)
  } #end i loop
  dpar[1]<-1-las[2]/h
  dpar[2:m]<-(2*las[2:m]-las[1:(m-1)]-las[3:(m+1)]) /h
  dpar[m+1]<-1-sum(dpar[1:m])
  return(dpar)
} # end discrete.pareto function
```

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```

#
# total reserve predictive distribution
#
totres.predictive.dist=function(e loss, ay, lag, posterior, esev, phiz) {
  npost=length(posterior)
  phix=complex(2^fftn,0,0)
  for (i in 1:npost){
    phixp=complex(2^fftn,1,0)
    for (j in unique(ay)){
      dpp=rep(0,2^fftn)
      lamp=0
      ayp=(ay==j)
      for (k in 1:length(lag[ayp])) {
        kmin=lag[ayp][k]
        lam=e loss[i,ayp][k]/esev[kmin]
        lamp=lamp+lam
        dpp<-dpp+dp[,kmin]*lam
      } # end k loop
      dpp<-dpp/lamp
      phixp<-phixp*(1-myc*lamp*(fft(dpp)-1))^( -1/myc)
    } # end j loop
    phix=phix+posterior[i]*phixp
  } # end i loop
  pred=round(Re(fft(phix,inverse=TRUE)),12)/2^fftn
  return(pred)
} #end totres predictive density

#
# likelihood for use in gibbs using overdispersed nb approximation
#
ccod.llike1=function(dev,elr) {
  e loss=rdata$premium*dev[rdata$dev]*elr[rdata$ay]
  lam=outer(e loss/esev[rdata$dev],chi)
  siz=lam/scv2
  n1=round(rdata$loss/esev[rdata$dev])
  llike1=dnbinom(n1,mu=lam,size=siz,log=T)
  llike=sum(log(exp(rowsum(llike1,rdata$ay))%*%chiwt))
  return(llike)
} # end ccod.llike1

#
# begin main program #####
#
h.step=rep(0,length(comp))
plot.range=matrix(0,length(comp),2)
myc=.01
chi=c(1-sqrt(3*myc),1,1+sqrt(3*myc))
chiwt=c(1/6,2/3,1/6)
# get prior distribution parameters
a=read.csv("Shuffled and Capped Prior.csv")
npriors=dim(a)[1]
elr.path=a[,2:11]
dev.path=a[,12:21]
#
# get loss data
#
r=read.csv("Rectangle Proxies.csv")
#
# get severity distributions and their ffts
#
fftn=14
limit.divisors=c(5,10,20,25,40,50,100,125,200,250,500,1000)
limit=1000
sevdist=read.csv("Severity Pareto.csv")
attach(sevdist)
pred=matrix(0,2^fftn,length(comp))
posterior=matrix(0,npriors,length(comp))
o=matrix(NA,npriors,length(comp))
mean.outcome=rep(0,length(comp))
sd.outcome=rep(0,length(comp))

```

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```

#
# calculate predictive distributions and percentile of
# the sum of test diagonal losses for each insurer
#
windows(record=T)
for (i in 1:length(comp)){
  #get reported data
  rdata=subset(r,r$insurer_num==comp[i])
  scv2=(elrm[rdata$dev]-esev[rdata$dev]^2)/esev[rdata$dev]^2
  #get posterior probabilities for dev and elr parameters
  lpost=rep(0,npriors)
  for (n in 1:npriors){
    elr=as.vector(elr.path[n,],mode="numeric")
    dev=as.vector(dev.path[n,],mode="numeric")
    lpost[n]=ccod.llikel(dev,elr)
  }
  posterior1=exp(lpost)
  posterior1=posterior1/sum(posterior1)
  o[,i]=order(-posterior1)
  posterior[,i]=posterior1[o[,i]]
  npost=max(sum(cumsum(posterior1[o[,i]])<.999),1)
  #save time to calculate predictive distribution
  posterior2=posterior1[o[,i]][1:npost]/sum(posterior1[o[,i]][1:npost])
  pred.eloss=matrix(0,npost,length(pred.ay))
  for (j in 1:length(pred.ay)){
    for (n in 1:npost){
      elr=as.vector(elr.path[o[,i],][n,],mode="numeric")
      dev=as.vector(dev.path[o[,i],][n,],mode="numeric")
      pred.eloss[n,j]=rdata$premium[pred.ay[j]]*elr[pred.ay[j]]*dev[pred.lag[j]]
    }
  }
  #get the predictive distribution for the sum of losses in the test diagonal
  h.step[i]=estimate.h(max(rowSums(pred.eloss)),fftn,limit.divisors)
  phiz<-matrix(0,2^fftn,10)
  dp<-matrix(0,2^fftn,10)
  for (k in unique(pred.lag)){
    dp[,k]<-discrete.pareto(limit,theta[k],alpha[k],h.step[i],fftn)
    phiz[,k]<-fft(dp[,k])
  } #end k loop
  pred[,i]=totres.predictive.dist(pred.eloss,pred.ay,pred.lag,posterior2,esev,phiz)
#
# plot of elr paths
#
plot(1:10,elr.path[1,],ylim=range(0,elr.path),
     main=paste("ELR Paths for Insurer #",comp[i]),
     xlab="Accident Year",ylab="% Loss Paid",type="n")
for (j in 1:500){
  par(new=T)
  plot(1:10,elr.path[j,],ylim=range(0,elr.path),main="",
       xlab="",ylab="",col="grey",type="l")
  plotpost=cumsum(posterior[,i])<.995
}
for (j in 1:sum(plotpost)){
  max.post=max(posterior[,i])
  min.post=min(posterior[,i])
  post.width=1+4*(posterior[j,i]-min.post)/(max.post-min.post)
  par(new=T)
  plot(1:10,elr.path[o[,i],][j,],ylim=range(0,elr.path),main="",
       xlab="",ylab="",col="black",type="l",lwd=post.width)
}
legend("topleft",legend=c("Posterior","Prior"),
      col=c("black","grey"),lwd=c(3,3))

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#
# plot of development paths
#
plot(1:10,dev.path[1,],ylim=range(0,dev.path),
     main=paste("Development Paths for Insurer #",comp[i]),
     xlab="Settlement Lag",ylab="% Loss Paid",type="n")
for (j in 1:500){
  par(new=T)
  plot(1:10,dev.path[j,],ylim=range(0,dev.path),main="",
       xlab="",ylab="",col="grey",type="l")
}
plotpost=cumsum(posterior[,i])<.995
for (j in 1:sum(plotpost)){
  max.post=max(posterior[,i])
  min.post=min(posterior[,i])
  post.width=1+4*(posterior[j,i]-min.post)/(max.post-min.post)
  par(new=T)
  plot(1:10,dev.path[o[,i],][j,],ylim=range(0,dev.path),main="",
       xlab="",ylab="",col="black",type="l",lwd=post.width)
}
legend("topright",legend=c("Posterior","Prior"),
      col=c("black","grey"),lwd=c(3,3))
#
# plot density of outcomes
#
x=h.step[i]*(0:(2^fftn-1))
mean.outcome[i]=sum(x*pred[,i])
sd.outcome[i]=sqrt(sum(x*x*pred[,i])-mean.outcome[i]^2)
pred.range=(x>.5*mean.outcome[i])&(x<2*mean.outcome[i])
predb=pred[pred.range,i]
plot(x[pred.range],predb/h.step[i],type="l",col="black",lwd=3,
     ylim=c(0,max(predb/h.step[i])),
     xlim=range(x[pred.range]),
     main=paste("Predictive Distribution for Insurer #",comp[i]),
     xlab="Reserve Outcome (000)",ylab="Predictive Probability Density",
     sub=paste("Mean =",format(round(mean.outcome[i]),big.mark=","),
               "Standard Deviation =",format(round(sd.outcome[i]),big.mark=",")))
}

```