Stochastic re-reserving in multi-year internal models – An approach based on simulations

Contact Information

Author:	Dr. rer. nat. Dorothea Diers
Affiliation:	Provinzial NordWest Holding AG (German Insurance Group) in cooperation with Ulm University (Germany)
Position:	Qualified Actuary (Aktuar DAV), Member of the German Actuarial Society (DAV), responsible for Non-Life DFA models (Group-wide) and Lecturer at Ulm University (DFA-research project at Ulm University)
Postal Address:	Group Controlling, Provinzial-Allee 1, Germany-48131 Münster
E-mail:	dorothea.diers@provinzial.de
Phone:	+49 (0)251/219-2994
Fax:	+49 (0)251/219-2437

Abstract

Only high-quality internal models optimally reflecting the risk situation facing the company allow insurers to assess the level of risk capital required. This importantly involves measuring and evaluating reserve risk as a part of insurance risks. In literature there is a wide variety of methods for stochastic reserving such as the Mack method, Bootstrap method, regression approaches, Bayesian methods, etc. All these approaches are based on an *ultimo view,* so that the uncertainty of full run-off of the liabilities is quantified. In contrast Solvency II requires the quantification of the one-year reserve risk. In addition the investment results, which have to be added to insurance results, are also based on a one-year view, which means that actually many internal models show an ultimo view for insurance results and the one-year view for investment results. So at the moment there is a discussion in academic literature and in insurance practice, how this one-year reserve risk can be quantified. This paper presents the idea of *re-reserving* which can to be applied in modelling reserve risk and premium risk. Based on this approach we can quantify one-year risk capital and multi-year risk capital. We compare the results of the re-reserving method with the results of the analytic approach recently shown in *Merz / Wüthrich* (2008).

The second part of this paper deals with the use of multi-year internal models in *value and risk-based management*. A sample model (based on the re-reserving approach) was applied to test the effectiveness of management strategies on corporate performance indicators such as EVA (economic value added) and RoRAC (return on risk-adjusted capital).

Keywords: Reserve risk, stochastic reserving, re-reserving, Solvency II, internal models, value and risk-based management

Contents

1. Introduction

Increasing natural catastrophes, difficult capital market environment and fundamental changes in supervisory requirements (Solvency II for European Union member countries) have placed increasing challenges on management strategy in insurance companies. Currently, we are seeing a paradigm shift in corporate management from classical turnover orientation to modern management techniques such as *value and risk-oriented management*. Management is faced with the challenge of allocating and managing capital resources efficiently. This requires a suitable insurance portfolio structure in combination with an adequate asset allocation towards the insurance cash flows. In this context the appropriate use of diversification effects plays an important role.

Internal risk models can play an important part in management decisions in value-and risk-based management strategy. Insurers with suitable internal models are in a position to calculate the risk capital to be put up by the company at large as well as strategic sub-segments on an individual basis according to the risk structure of the company. This enables companies to address issues affecting risk-bearing capability and profitability in the company at large as well as sub-portfolios down to individual product level. So companies are able to assess the amount of risk that can be taken in individual company units and the returns that can be reached from a specified risk position. Increasing transparency in the risk situation, identification of high-risk factors, and identification of segments that generate or decrease shareholder value are essential ingredients in generating a strategic value- and risk-based company management approach aimed towards a long-term and sustainable increase in shareholder value. So these models should also be used in a company's "Own Risk and Solvency Assessment" (ORSA).[1]

Although non-life insurance contracts have short periods in contrast to life insurance, risk-return strategy should follow calculations that span several years. In the actual literature several questions concerning the use of internal models in a multi-year management context are not answered to date.[2] The aim of this paper is to present a multi-year model approach,

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[1] See *CEIOPS* (2008).
[2] In *Diers* (2008b) a model approach of a multi-year model is presented.

which allows the quantification of one-year risk capital of the first simulated year as required by Solvency II. Moreover we calculate a "multi-year" risk capital, to have a higher probability of settling all losses that occur within the entire period of simulated *n* years without needing external capital sources (Section 2).

Only high-quality internal models optimally reflecting the risk situation facing the company allow insurers to assess the level of risk capital required according to the corporate risk structure. This importantly involves measuring and evaluating reserve risk as a part of insurance risks. In literature there is a wide variety of methods for stochastic reserving such as the *Mack* method, Bootstrap method,[3] regression approaches,[4] Bayesian methods,[5] etc. All these approaches are based on an *ultimo view,* so that the uncertainty of full run-off of the liabilities is quantified. The same holds for premium risk. In contrast Solvency II requires the quantification of the one-year reserve risk. In addition the investment results are also based on a one-year view, which means that actually many internal models show an ultimo view for insurance results and the one-year view for investment results. So at the moment there is a discussion in academic literature and in insurance practice, how this one-year reserve risk can be quantified.[6] This paper presents the idea of *re-reserving* which can be applied in modelling reserve risk and premium risk. Based on this approach we can quantify one-year risk capital and multi-year risk capital as defined in Section 2. We compare the results of the re-reserving method with the results of the analytic approach shown in *Merz / Wüthrich* (2008) (Section 3).

In value and risk-based management, strategies should be selected in such a way as to fulfil the requirements on risk-capital coverage with economic capital while achieving the highest possible return. One goal is to ensure effective risk diversification, which is hardly possible without the help of internal models. So for example management has to decide which strategy might improve the risk and return situation of a company if not enough risk capital is available – changing the asset allocation by lowering share quota or lowering insurance risk by extending reinsurance cover, or any suitable combination of these or other strategies.

A sample model was applied to examine the effectiveness of management strategies on corporate performance indicators such as EVA (economic value added) and RoRAC (return on risk-adjusted capital). Here we used a very detailed and fully developed internal model presented in *Diers* (2007a), which is extended by the multi-year approach described in section 2 and the stochastic re-reserving approach for modelling premium and reserve risk described in Section 3. So we can quantify the effects of different management strategies on a non-life insurer's risk and return profile in a one-year and a multi-year view.

The aim of this study is to define a multi-year model approach which allows to define one-year risk capital and multi-year risk capital and to give an idea how to use such a fully developed internal model in the management context. We want to give a realistic and helpful idea of defining a suitable balance between reinsurance cover and asset allocation (such as share quota level), which can only be achieved by considering the risk and return situation of the whole company instead of managing investment results and insurance results separately.

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[3] See for example *England / Verrall* (2002).
[4] See for example *Christofides* (1990).
[5] See for example *England / Verrall* (2002) and *England / Verrall* (2006).
[6] See Merz / Wüthrich (2008), Diers (2007b) and *Ohlsson / Lauzeningks* (2008).

2. Results by calendar year

2.1 Multi-year model approach

We have based the model design on an internal simulation model in non-life insurance, modelling strategic insurance segments and asset classes based on economic principles and simulating the results considering suitable dependencies.[7] The economical result, $EcRes_t$, prediction for a future year t, can be expressed by the change in economic capital $EcCap$ within the respective year:[8]

$$(1) \qquad EcRes_t = EcCap_t - EcCap_{t-1}$$

$$= EcResLiab_t + EcResAs_t - O_t - A_t,$$

where

$EcResLiab_t$ = net insurance result at time t,
$EcResAs_t$ = investment result at time t,
O_t = result from operational risk at time t,
A_t = tax at time t.[9]

The net insurance result is calculated using underwriting result (2.1) and claims development result (2.2):

$$(2) \qquad EcResLiab_t$$

$$= P_t - C_t - U_t \qquad (2.1)$$

$$- F_t + F_{t-1}, \qquad (2.2)$$

whereas:

P_t = premiums earned at time t,
C_t = costs (acquisition costs, administration costs, internal settlement costs) at time t,
U_t = prediction for the ultimate claim losses of the simulated accident year at time t,
F_t = prediction for the ultimate claim losses of former accident years ($<t$) at time t,
F_{t-1} = prediction for the ultimate claim losses of former accident years ($<t$) at time t-1.

All insurance-related figures such as premiums, ultimate losses, and costs are to be seen as net figures, i.e. after reinsurance, and external settlement costs are modelled together with the claims. In the multi-year model not only claims but also premiums are modelled stochastically

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[7] See *Diers* (2007a) for details. Refer to *Diers* (2008b) in the multi-year model design.
[8] Economical capital is defined as the difference between the market value of assets and market value of technical provisions (best estimate plus market value margin). We have selected a somewhat simplified representation such as by ignoring other assets and liabilities, while focussing on the result before earnings distribution to shareholders.
[9] See *Diers* (2007a) for tax modelling.

in order to represent the effects of premium cycles.[10]

Internal models begin by modelling gross insurance results[11] and the reinsurance results are matched to individual agreements. This enables evaluation of the gross insurance results in return and risk aspects, and assessment of individual reinsurance results or alternative reinsurance structures for efficiency.

The ultimate claim loss prediction in the simulated accident year (U_t) and former years (F_t) in time t may be calculated using methods such as *re-reserving* from the ultimate model. Stochastic cash-flows are essential for this purpose (see Section 2.2 for details). So the one-year reserve risk is calculated using the simulated empirical distribution of $F_0 - F_1$, where we condition on all observations up to time t=0.

For modelling the economic investment results we use capital market scenarios that arise from real-world models. In the asset model management rules (e.g. sales priorities, rebalancing rules) have to be taken into account. Like the insurance result that can be calculated according to insurance segments, the economic investment result can also be modelled down to the most detailed asset level (asset class) depending on the model's depth.

The economic investment results, $EcResAs_t$, can be calculated according to the results $EcResAs_t^a$ from the individual asset classes $a \in A$ at the end of the year t:[12]

$$EcResAs_t = \sum_{a \in A} EcResAs_t^a \quad \text{and} \tag{3}$$

$$EcResAs_t^a = MWB_t^a - MWA_{t-1}^a + Re_t^a - Exp_t^a, \tag{4}$$

with market value MWB_t^a from a at the end of year t before reissue or sale, market value MWA_{t-1}^a from a at the beginning of year t after reissue or sale, revenues and repayments Re_t^a from a in year t, and costs related to investments Exp_t^a, which should also be allocated to individual asset classes.[13] To enable investment management within the year, the economic capital result – as required – may also be calculated within the year (e.g. monthly).[14]

Insurance practice provides the amount for investment, and would also be able to yield a risk-free return. For this reason, the risk-free return on discounted insurance reserves as well as other liability positions should be added to insurance results and debited from the investment result. The same applies to risk-free returns on equity that should be matched to equity. The risk capital required for each segment can now be calculated from these figures.

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[10] See for example *Cummins* and *Outreville* (1987) for an analysis on premium cycles. Dependencies between premium cycles and various market indexes should be modelled in an appropriate way.

[11] Modelling on net values would not be an option due to the changing reinsurance structures alone.

[12] All of the cost positions related to the investment need to be taken into account in the investment result. A refers to the set of all asset classes.

[13] Revenues refer to interest, including cumulative interest accrued, dividends and rents, referring to revenues after (interest certificate) failure. Repayments are only listed in bonds. If the balance of revenues and repayments on assets and insurance-related cash flow is positive, the total is entered again as an asset position (which may involve rebalancing). Negative totals result in asset sales.

[14] In this case insurance-related cash flow must also be calculated within the year (e.g. monthly).

Apart from that, the capital providers will demand a risk-oriented minimum return on the capital they have placed – referred to as capital costs. This requirement is represented as a deduction term in various indicators, such as *economic value added* (EVA), resulting in the expected return beyond the capital costs.

Using simulation methods a large number of random observations can be simulated from the model in order to create the empirical distributions of insurance results, investment results, etc. Depending on the parametric situation, a certain number – such as 100,000 – of simulations are carried out.

2.2 Simulation of claim development results by re-reserving

The ultimate development result is defined as:

(5)
$$\begin{aligned} DevResUlt &= F_0 - F_\omega \\ &= R_0 - R_\omega , \end{aligned}$$

where F_ω refers to the sum of cash flows from previous accident years, which are simulated up to the final settlement of claims at $t=\omega$, where we condition on all observations up to time $t=0$. F_0 refers to the associated best estimate. R_0 and R_ω refer to the corresponding reserves. The ultimate reserve risk can be calculated using the simulated empirical distribution of *DevResUlt*. The literature describes a variety of approaches for modelling the ultimate reserve risk.[15]

In contrast, Solvency II requires a one-year risk-perspective, where the change in reserves and therefore also the development result for the next year is to be calculated. In addition the investment results, which have to be added to insurance results, are also based on a one-year view, which means that actually many internal models show an ultimo view for insurance results and the one-year view for investment results. So a consistent view of investment results and investment results is required.

In 2007 *Association Internationale des Sociétés d'Assurance Mutuelle* (AISAM) and *Association of European Cooperative and Mutual Insurers* (ACME) initiated an international study which aims at clarifying how the reserve risk should be calculated over one-year horizon (*AISAM / ACME* 2007). *Wüthrich*, *Merz and Lysenko* (2008) presented an analytical approach towards calculating the claims development result for the next calendar year and its prediction uncertainty based on the chain ladder model. However, the analytical approach is often not sufficient in internal models, because (simulated) cash flows are needed for future claim settlements.

As shown in Section 2.1, multi-year internal models yield the following simulation results for the development results per calendar year *t*:

(6)
$$DevRes_t = F_{t-1} - F_t$$

[15] There is a wide variety of methods for stochastic reserving such as the *Mack* method, Bootstrap method (see for example *England / Verrall* (2002)), regression approaches (see for example *Christofides* (1990)), Bayesian methods (see for example *England / Verrall* (2002) and *England / Verrall* (2006)), etc.

$$= R_{t-1} - I_t - R_t,$$

whereas R_{t-1} refers to the opening reserve estimate at the beginning and R_t refers to the closing reserve estimate at the end of the simulated year *t*. I_t refers to the payments (cash-flow) during the year *t*, where we condition on the "observed" part of the claim triangle.

So for t = 1 we can write:

$$DevRes_1 = R^{D_J} - I^{D_J} - R^{D_{J+1}},$$

where all observations up to time t=0 are given by D_J. We only consider former accident years up to t=0. So in the example given in Fig. 1 in the years 2007ff we simulate the claim development of accident years up to 2006.

The idea of re-reserving is considered in *Ohlsson* and *Lauzeningks* (2008) and *Diers* (2007b). In the following we want to describe this modelling approach based on re-reserving using three steps.

Step 1: Calculating the opening reserve estimate R^{D_J}

First, a claim reserving model is set as a base for the ultimate stochastic reserving process. This involves stochastic reserving methods that yield simulated future cash flows, such as bootstrap approaches or Bayesian methods.[16]

The first step is to assess the opening reserve estimate $R_0 = R^{D_J}$, which can be calculated from the underlying reserving model and should agree with the actuary's best estimate for outstanding claims in $t = 0$.

Step 2: Calculating the payments I^{D_J}

The second step is to simulate payments I^{D_J} for the next calendar year t=1 for each former accident year using the underlying ultimate stochastic reserving process. So we (only) simulate the next diagonal in the development triangle (see Fig. 1). The sum of claim payments on the simulated diagonal equals to I_1. This level of knowledge matches the "actuary in the box" at the end of year t=1. So using simulation methods we get an empirical distribution of I^{D_J}, which – with growing simulation number – converges to the theoretical distribution I^{D_J}.

Step 3: Calculating the closing reserve estimates $R^{D_{J+1}}$

Step 2 is used as a basis for the third step, which is to carry out a best estimate for the closing reserve $R^{D_{J+1}}$ in each simulation path according to the underlying reserving model which is assumed to be set.[17] This process is called *re-reserving*. So we obtain $DevRes_1$.

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[16] See for example *England / Verrall* (2002) and *England / Verrall* (2006).

[17] The description of reserving methods to simulate the next diagonal and the consideration of tails are not the scope of this paper.

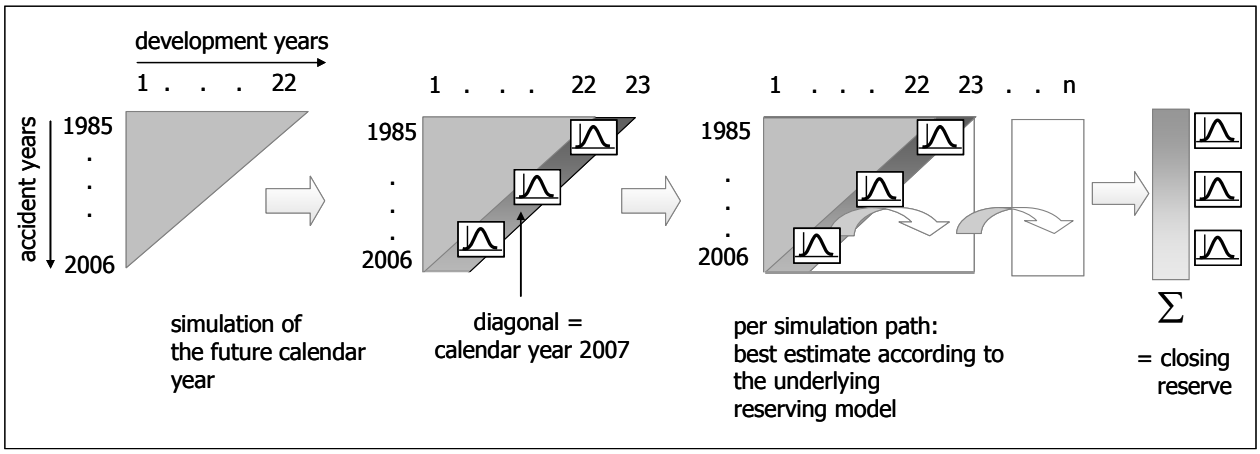


Fig. 1: Re-reserving method (conditional on t=0)

To calculate $DevRes_2$ in each simulation R_1 represents the opening reserve estimate for the second simulated calendar year $t = 2$. Now, payments I_2 are simulated for the second future calendar year $t = 2$ according to the underlying stochastic reserving method and we use the re-reserving-method again, etc. So re-reserving is a method based on the underlying stochastic ultimate reserving model. We have not entered into a discussion on possibilities and limitations of the "actuary in the box" at this point; however, this discussion is necessary.

Fig. 2 shows cumulative cash flows referring to a motor third-party segment generated from 100,000 simulations from one accident year (2005) in the ultimate (left) and one-calendar-year view (right). The example cash flows highlighted in white in the left and right parts of the diagram show results from the same simulation path. The simulated payments for the third development year 2007, which correspond to the first simulated year, agree in both views. The payments in 2005 and 2006 are known, and are therefore deterministic. Since the view on the left-hand side represents the ultimate view, further payments (from 2008 onwards) are simulated up to the final claim settlement using Mack bootstrap.[18] The one-calendar-year view on the right-hand part shows the best estimate calculated at the end of 2007 in each simulation path according to the underlying reserving model ("actuary in a box").

The cash flow highlighted in white, for example, shows a very high payment volume for the fifth development year that has been underestimated in the one-year view in the right-hand part. This leads to a reserve estimate that is "too low" in this simulation path.

[18] See *England* and *Verrall* (2006).

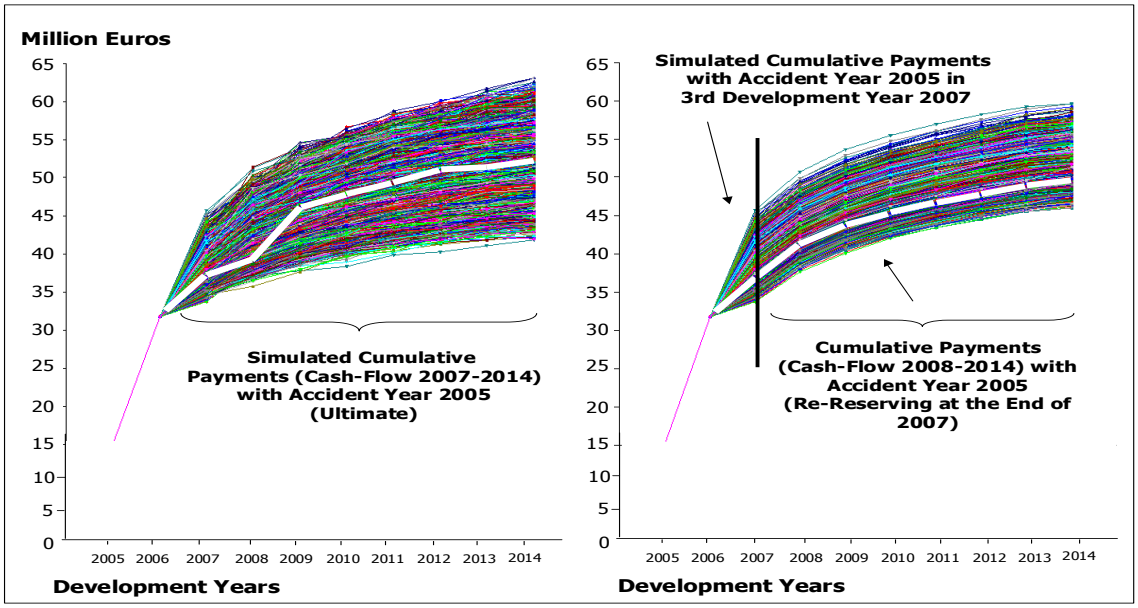
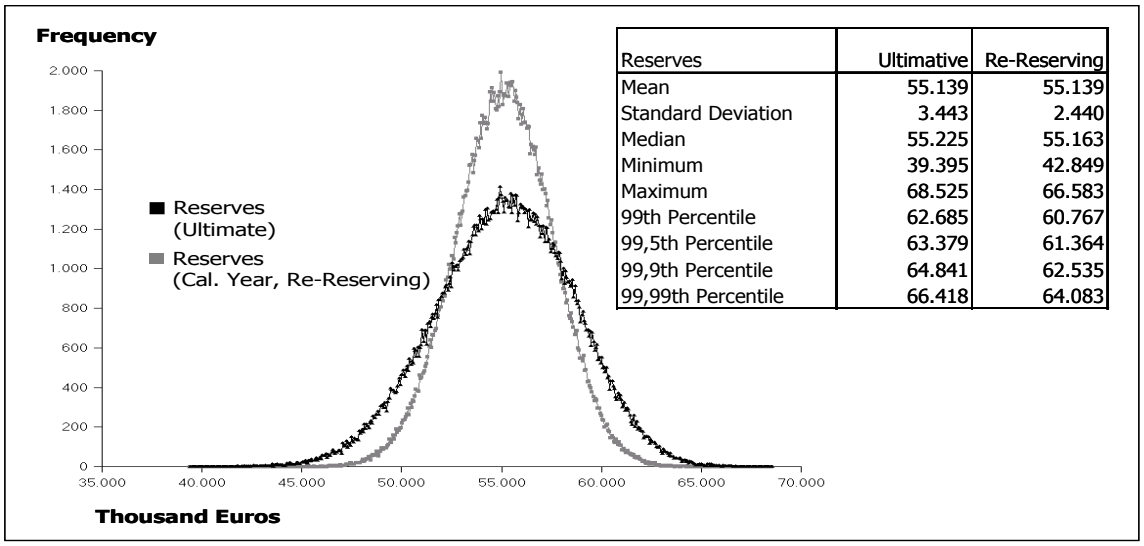


Fig. 2: Simulated cumulative cash flows in the 2005 accident year referring to motor third-party segment in ultimate (left) and one-calendar-year views (right: cash flow patterns by re-reserving)

Fig. 3 represents the frequency distribution in the ultimate and one-calendar-year views. Here we refer to the motor third-party payment triangle given by *Kortebein et al*. (2008). To make matters easier, we have not selected a tail. We have used the Mack bootstrap with Mack's bias correction for stochastic reserving, and a normal distribution for adding process risk. The re-reserving method can now be applied to generate a one-calendar-year view using this ultimate reserving method as a basis. The standard deviation in the calendar-year view is reduced to around 71%.



Reserves	Ultimative	Re-Reserving
Mean	55.139	55.139
Standard Deviation	3.443	2.440
Median	55.225	55.163
Minimum	39.395	42.849
Maximum	68.525	66.583
99th Percentile	62.685	60.767
99,5th Percentile	63.379	61.364
99,9th Percentile	64.841	62.535
99,99th Percentile	66.418	64.083

Fig. 3: Frequency distributions for reserves for a motor third-party segment in one-calendar-year and ultimate views

So we use the probability distribution of *DevResUlt* to calculate risk capital in the ultimate

view and $DevRes_1$ for the one-year reserve risk. With regard to value at risk[19] at a confidence level of 99.5% according to Solvency II, which corresponds to a 200-year event, the risk capital level amounts €8.2 million in the ultimate view and €6.2 million in the calendar year view. This is a reduction in risk capital of 24.4%. If a company calculates risk capital at a higher internal risk level, for example using tail value at risk[20] at 99.8%, the average of the two hundred worst simulation results will be taken for risk capital calculation at a hundred thousand simulations. This will result in a risk capital level of €7.6 million in the calendar-year view, which is also significantly lower than the corresponding value of €9.9 million in the ultimate view (reduction by 24%).

For comparison we applied the re-reserving method to a claim payment triangle (where the first accident year has been completely settled) presented by *Wüthrich /* Merz (2008) and compared our results obtained by re-reserving to those presented in this paper based on the analytical approach. If we simulate claims to ultimate based on Mack bootstrap (using Mack's bias correction and normal distribution for process risk) and use a re-reserving approach based on deterministic Chain-Ladder to investigate variability of the run-off reserve after one year we obtain nearly the same prediction error as presented by *Wüthrich / Merz* (2008) in their example shown in Section 4.

Analogue calculations are essential in assessing underwriting results, which can be transferred to development results in multi-year models such as by simulating the first payment I_t of the new accident year and estimating the related closing reserve CR_t per simulation path as described above from the underlying reserving model.[21] So formula (2.1) can be written as:

(7) $$P_t - C_t - U_t = P_t - C_t - I_t - CR_t.$$

The selected risk measure ρ can now be applied to the random variable $EcRes_1$ in order to determine the one-year risk capital, such as the tail value at risk *TVaR* at the (1–α) percentile:

(8) $$\rho(EcRes_1) = TVAR_\alpha(EcRes_1).$$

2.3 Multi-year risk capital

Usually, management requires that substantial risks (natural catastrophes, development results in long-tail-business) be viewed from an underwriting context of several years in order to address the following issues:

- How many years of catastrophe risks or other major events, such as negative capital market development, can the company economically withstand at a certain confidence

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[19] Value at risk at a high confidence level of 1-α is defined at the (1–α) quantile for the F_L loss distribution, L: $VaR_\alpha(L) := Q_{1-\alpha}(L) = \inf[x \in IR : F_L(x) \geq 1-\alpha]$, with real numbers IR.

[20] $TVaR_\alpha(L) = E[L \mid L \geq VaR_\alpha(L)] = VaR_\alpha(L) + E[L - VaR_\alpha(L) \mid L \geq VaR_\alpha(L)]$, where E refers to the expected value, which is conditional in this case. Tail value at risk is a coherent risk measure for random variables with continuous distribution. We will not be discussing the advantages and disadvantages of risk measures such as *VaR* and *TVaR* here, but this discussion is necessary – see for example *Koryciorz* (2004), *Pfeifer* (2004b) and *Rootzén / Klüppelberg* (1999).

[21] See *Ohlsson* and *Lauzeningks* (2008) for modelling one-year underwriting risk.

level without needing external capital sources?

- How much risk capital does a company currently provide to maintain a certain confidence level to ensure its status as a going concern for another five years, i.e. taking five future underwriting years into account, without needing external capital sources?

To address issues of this nature, we can use our multi-year model to calculate a "multi-year" risk-capital taking into account n, $n \in IN$, future accident years. This referred to the random *MaxLoss* variable defined as follows:

(9) $$MaxLoss(n) = \underset{1 \le t \le n}{MAXIMUM}\{KumLoss_t\}, \quad \text{with}$$

$$KumLoss_1 = -EcRes_1 \cdot (1 + r_f)^{-1} \qquad \text{and}$$

$$KumLoss_t = KumLoss_{t-1} - EcRes_t \cdot (1 + r_f)^{-t}, \quad 1 \le t \le n.$$

In order to keep a uniform approach on all of the years simulated, all of the events are discounted at the risk-free interest rate of r_f as of the beginning of the simulation period t=0. *MaxLoss* is to be provided for each simulation path at t=0 to enable the insurance company to settle all losses that occur within the entire period simulated of n years without needing external capital sources.

The selected risk measure, ρ, can now be applied to the *MaxLoss*: Ω → IR in order to determine the multi-year risk-capital requirement e.g. for *tail value at risk* (TVaR) at percentile (1–αn):

(10) $$\rho(MaxLoss(n)) = TVAR_{\alpha n}(MaxLoss(n)).$$

The confidence level αn may decrease with increasing values of n. By definition, the multi-year risk capital is always at least as high as the one-year risk capital (discounted) for values of αn = α1 = α. If the insurance company can cover its multi-year risk capital with its own economic capital at t=0, $EcCap_0$, the following will apply:

(11) $$EcCap_0 \ \ge \ \rho(MaxLoss(n)).$$

The company can therefore cover all losses that may incur over the simulation period without external capital supply at a probability of more than 1–αn.[22]

So the multi-year risk-capital concept may take on *the role of a strict constraint* in internal models in addressing strategic issues, which can be used for "Own Risk and Solvency Assessment" (ORSA).

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[22] See *Diers* (2008b) for the multi-year risk capital concept. Note that the optimal level of economic capital should be assessed according to optimal business management strategy in a shareholder-value calculation; see *Gründl / Schmeiser* (2002). A discussion of advantages and disadvantages of different risk measures such as *VaR* or *TVaR* is necessary, but exceeds the purpose of this paper, see for example *Koryciorz* (2004), *Pfeifer* (2004b) and *Rootzén / Klüppelberg* (1999).

3. Case study – Strategic management decisions

Risk-adjusted performance strategy aims towards maximising the risk-return trade-off with constraints such as complying with risk-capital requirements as well as accounting regulations and supervisory regulations.[23] A host of restrictions such as cross-selling, cross-cancellation and price-sales function effects makes solving this optimisation process difficult; we therefore refer to improvement in risk-return trade-off rather than optimisation in the whole company. So we quantify the effects of a variety of potential management strategies on the risk-return-situation for identification and selection of those strategies achieving the most positive effect.

The aim of the following simulation study is to examine the effect of various management strategies on performance indicators using a detailed and fully developed internal risk model from an example company as reference, demonstrating that separately managing assets and liabilities at corporate level is not an effective approach. The simulation example will clearly demonstrate that improving risk-adjusted performance indicators at corporate level is highly dependent on diversification effects. The study will give a quantitative example for finding adequate management strategies using modern management techniques.

For risk-adjusted performance indicators we will apply the one-year return on risk-adjusted capital

(12) $$RoRAC = \frac{Mean}{RiskCapital} = \frac{E(Ec\operatorname{Re}s_1)}{\rho(-Ec\operatorname{Re}s_1)}$$

and risk-adjusted economic value added

(13) $$EVA = EVA\_ra = E(Ec\operatorname{Re}s_1) - r_{cap} \cdot \rho(-Ec\operatorname{Re}s_1)$$

thus applying capital costs to the risk capital.[24]

We will be using the TVaR (tail value at risk) allocation principle for identifying major risk factors. The TVaR allocation principle originates from risk theory, and is based on linearity of expectation values. According to the TVaR principle, the risk capital required for business segments i ($1 \le i \le n$) taking the TVaR as the risk measure is as follows:[25]

(14) $$\rho_{TVaR}(L_i \mid L) = \mathrm{E}[\mathrm{L}_i \mid \mathrm{L} \ge VaR_\alpha(L)].$$

This equation allocates the exact risk amount to the segments as contribution to risk-capital requirement at corporate level. Since we will only be using the allocation principle for identifying major risk factors and not for implementing company segment strategy, we will

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[23] Maximising risk-adjusted performance indicators and maximising shareholder value are only compatible under certain circumstances; cf. *Gründl/Schmeiser* (2002).

[24] We only refer to RoRAC where expectation value and risk capital are greater than zero; otherwise, no meaningful statement can be made.

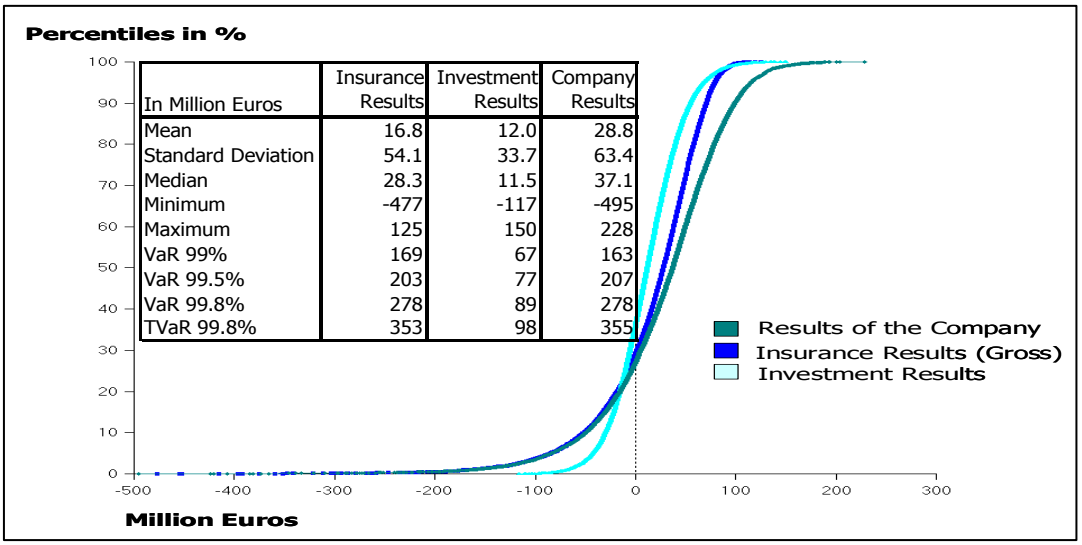
[25] We have defined this allocation principle in connection with *TVaR* as risk measure, which is a coherent allocation principle. There is a more generalised definition, without coherence in the allocation method. See also *Koryciorz* (2004).

not be covering the advantages and disadvantages of allocation principles as applied to value and risk-based management.[26]

Our example internal simulation model is based on a five-year period using 10,000 simulations.[27] Refer to *Diers* (2007a) for the model description of the internal risk model this study is based on. We will be modelling development results using the re-reserving method as described in Section 2. The sales priorities of assets to be sold in order to raise the liquidity required have been set in the asset model through management rules for each scenario. Scenarios can appear where the liquidity requirement is not fulfilled, depending on the level of liquidity required and potential extreme developments on the capital market. One of management's major responsibilities is to ensure that these scenarios are minimised by sufficient reinsurance protection and appropriate asset allocation.

Our example company mainly underwrites insurance policies with private and low industrial businesses. We have assumed a consistent underwriting policy and asset allocation within the five-year period. In addition we assumed that the claims that will occur in the following five years simulated will be independent concerning the different years. Apart from that, we shall assume that the investment results and insurance results are independent of one another. We have taken tail value at risk (TVaR) of 99.8% as risk measure for one year's risk-capital requirement. Management demands capital costs of r_{cap} = 12%.[28]

The five-year risk-capital requirement will be quantified at TVaR of 99.5%. If the company can cover this five-year risk-capital requirement with its own economic capital $EcCap_0$, it will be able to withstand the next five years without outside capital sources at a TVaR confidence level of 99.5%.



In Million Euros	Insurance Results	Investment Results	Company Results
Mean	16.8	12.0	28.8
Standard Deviation	54.1	33.7	63.4
Median	28.3	11.5	37.1
Minimum	-477	-117	-495
Maximum	125	150	228
VaR 99%	169	67	163
VaR 99.5%	203	77	207
VaR 99.8%	278	89	278
TVaR 99.8%	353	98	355

Fig. 4: Simulated empirical distributions of results $EcRes_1$, $EcResLiab_1$ and $EcResAs_1$[29]

[26] See *Gründl/Schmeiser* (2002). See also the study in *Diers* (2008c), which demonstrates that TVaR, under certain circumstances, can be used successfully in risk-adjusted performance management.

[27] Usually, at least 100,000 simulations are applied in practice. We will be neglecting operational risks in the following.

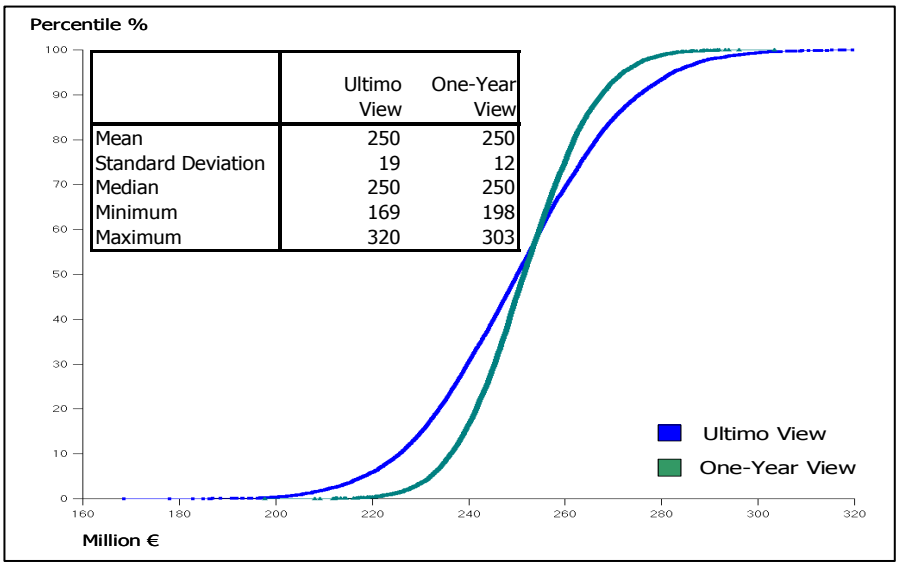
[28] The non-risk interest rate referring to economic capital and liabilities is deducted from the investment results; the non-risk interest rate provides a benchmark for capital investors. Non-risk interest on liabilities is added to the insurance result; non-risk interest on economic capital is entered in a separate position, which will not be considered here; see *Diers* (2008b).

[29] VaR: Value at risk

Figure 4 shows the percentile graphs for the gross insurance results, investment results and results of the company, where no reinsurance is considered. Management wants to find an adequate balance between reinsurance protection and asset allocation.

Ultimo reserve risk versus one-year reserve risk

In our case study we modelled reserve risk using the re-reserving approach as shown in Section 2. The market value of liabilities in our company is €300 million, with a best estimate of €250 million plus market value margin of €50 million. 65% from the best estimate is long tail business, 35% short tail business. Fig. 5a shows the simulated empirical distributions of the reserves in the ultimo and in the one-year view.



	Ultimo View	One-Year View
Mean	250	250
Standard Deviation	19	12
Median	250	250
Minimum	169	198
Maximum	320	303

Fig. 5a: Simulated empirical distributions of reserves in ultimo view and in one-year view

The risk capital for reserve risk which is calculated using the ultimo and the one-year development results can be seen in Fig. 5b. Here the one-year risk capital (for TVaR 99.8%) is 64% of the ultimo risk capital.

	Ultimo View	One-Year View
VaR (99%)	47	31
VaR (99,5%)	51	33
VaR (99,9%)	63	41
TVaR (99,8%)	64	41

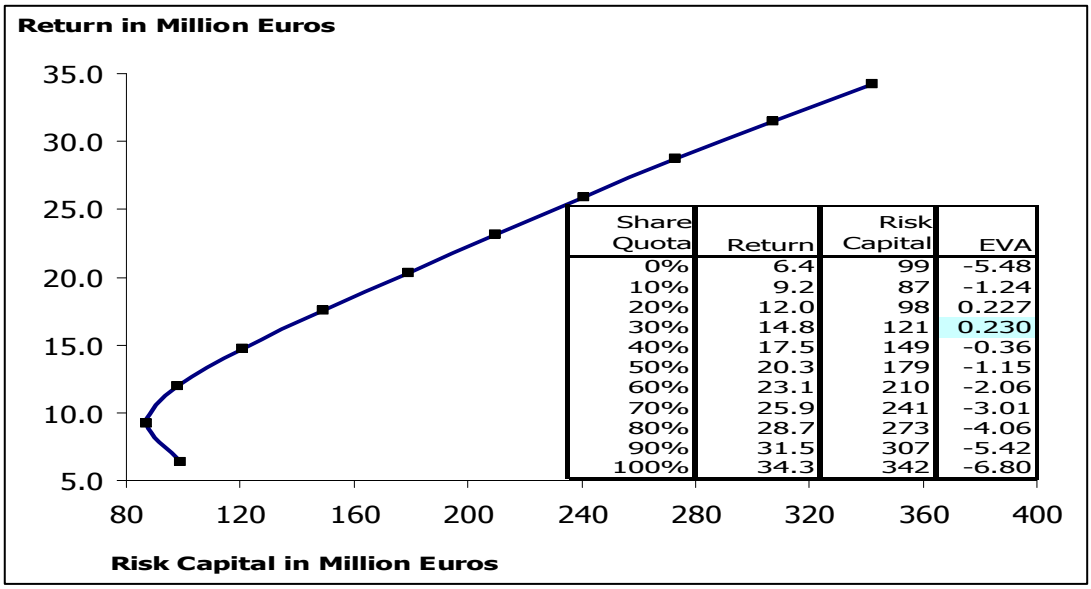
Fig. 5b: Risk capital for reserve risk in ultimo view and one-year view

Examining asset allocation (asset-only)

Now we want test different management strategies for efficiency, where we start analysing different asset allocations. Currently, the company has a share quota of 20%. The asset-only examination's aim is to find out which share quota leads to the highest EVA within the

investment results. Figure 6 shows the efficiency curve for investment portfolios using various share quotas. Figure 7 shows the EVA (investment results) applicable.

The asset-only examination yields an optimum share quota of around 30%, which leads to the highest EVA (investment) of €0.23 million.



Share Quota	Return	Risk Capital	EVA
0%	6.4	99	-5.48
10%	9.2	87	-1.24
20%	12.0	98	0.227
30%	14.8	121	0.230
40%	17.5	149	-0.36
50%	20.3	179	-1.15
60%	23.1	210	-2.06
70%	25.9	241	-3.01
80%	28.7	273	-4.06
90%	31.5	307	-5.42
100%	34.3	342	-6.80

Fig. 6: Efficiency curve for investment portfolios: Return (expected investment results) versus risk capital (investment results) at various share quotas

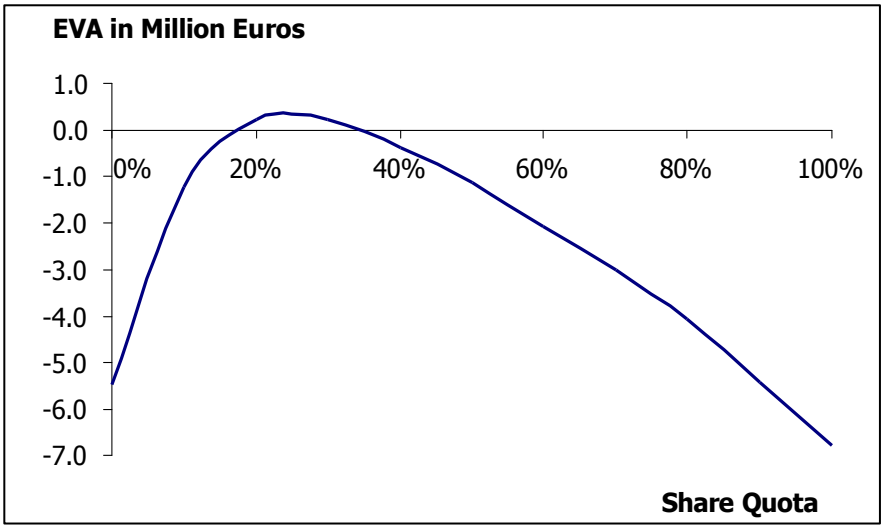


Fig. 7: EVA (investment results) at various share quotas

Examination of asset allocation with regard to company results

Using the current strategy (20% share quota), the company has a negative EVA of –€13.9 million (see Fig. 8). Our aim is to examine which share quota leads to the highest EVA with reference to the combined portfolio from insurance contracts and investments. Unlike the asset-only examination, this examination yields an "optimum" share quota of around 90% leading to the highest EVA (company) at €1.63 million (Fig. 8). It should be noticed that

supervisory regulations on maximum risk quota need to be observed in selecting a share quota value.

Share Quota	Mean (Return)	One Year Risk Capital (Company)	EVA (Company)	TVaR Allocation Risk Capital (Insurance)	TVaR Allocation Risk Capital (Investment)	Diversification Benefit	Shortfall P(KumLoss(1) > $ECap_0$)	Five Years Risk Capital (Company)
0%	23.2	360	-20.0	343	16	20.3%	0.607%	453
10%	26.0	357	-16.8	347	10	18.8%	0.554%	440
20%	28.8	355	-13.9	347	9	21.1%	0.532%	437
30%	31.5	355	-11.0	347	8	25.1%	0.554%	439
40%	34.3	354	-8.2	339	15	29.4%	0.619%	449
50%	37.1	355	-5.5	338	17	33.2%	0.748%	471
60%	39.9	357	-2.9	316	41	36.6%	1.023%	502
70%	42.7	364	-1.0	287	76	38.7%	1.416%	538
80%	45.5	375	0.5	253	122	40.1%	1.987%	581
90%	48.2	388	1.63	216	172	41.1%	2.551%	628
100%	51.0	412	1.58	161	251	40.7%	3.192%	680

Fig. 8: Key numbers for return and risk of the combined portfolio (investment and insurance contracts) and allocated capital (insurance and investment)

In this case a combined portfolio with the highest level of diversification benefit[30] (41.1%) – resulting from the independence of investment results and insurance contracts – leads to the highest EVA. Neither insurance results nor investment results dominate risk-capital requirement in this portfolio in such a strong way as in the other scenarios, as allocation according to the TVaR principle shows. So a risk capital of €216 million is allocated to insurance portfolio and €172 million to investment portfolio.

This analysis shows that asset-only optimisation is not a constructive approach in a value and risk-oriented strategic approach. Instead, the evaluation should be based on the combined portfolio of investments and insurance contracts, where management has the important task of recognising and using diversification potential.

Strategic management process

The next step is to examine the risk-bearing capacity and solvency of the company as a whole. At t=0, the example company has €200 million in economic capital, $EcCap_0$. Management requires coverage of one-year risk-capital requirement at TVaR 99.8% and five-year risk-capital requirement at TVaR 99.5% in an ORSA. So management wants to ensure that the company will be able to withstand five years at the given security level without external capital sources.

According to the internal confidence level, however, the one-year risk-capital requirement, €355 million, would not be covered, leading to a shortfall probability of 0.532%. The five-year risk-capital requirement of €437 million would substantially exceed the company's economic capital (see Fig. 8; actual share quota: 20%).

[30] Diversification benefit between insurance results and investment results is calculated as percentage of the sum of standalone risk capitals.

So company management thinks about reinsurance protection. We calculated reinsurance premiums using technical pricing methods.[31] Reinsurance strategy *A* leads to reduction in risk capital for insurance risks from €353 million (see Fig. 4 for actual strategy) to €227 million,[32] while lowering the expected insurance result from €16.8 million to €13.1 million. This increases RoRAC (insurance results) from 4.8% to 5.8% due to the selection of adequate reinsurance protection, where our company benefits from the diversification effects of the reinsurer.

This structural change in insurance results means that the efficient asset allocation strategy for the combined portfolio needs to be revised. Examining the key numbers for return and risk for the combined portfolio shows a share quota of 60% to lead to the greatest EVA (company). Each scenario in Fig. 9 leads to a one-year risk-capital requirement, which exceeds the economic capital of € 200 million. The same is true for the five-year risk capital.

		One Year		TVaR Allocation			Shortfall	Five Years
Share Quota	Mean (Return)	Risk Capital (Company)	EVA (Company)	Risk Capital (Insurance)	Risk Capital (Investment)	Diversification Benefit	P(KumLoss(1) > $ECap_0$)	Risk Capital (Company)
0%	19.5	239	-9.2	212	27	26.5%	0.163%	289
10%	22.3	236	-6.0	215	21	24.9%	0.149%	272
20%	25.1	235	-3.1	213	22	27.7%	0.176%	269
30%	27.9	235	-0.4	211	24	32.3%	0.186%	280
40%	30.6	237	2.2	199	38	37.0%	0.187%	305
50%	33.4	245	4.1	169	76	39.6%	0.554%	342
60%	36.2	258	5.21	139	120	40.8%	0.448%	388
70%	39.0	282	5.18	89	193	39.7%	0.807%	441
80%	41.8	309	4.7	79	230	38.2%	1.527%	499
90%	44.6	338	4.0	64	274	36.7%	2.243%	559
100%	47.4	369	3.1	54	315	35.1%	2.861%	568

Fig. 9: Key numbers for return and risk of the combined portfolio (investment and insurance contracts) and allocated capital (insurance and investment) after reinsurance strategy *A*

So we test the effects of a second reinsurance strategy *B*, with higher liabilities for the reinsurer in event excess of loss contracts for catastrophe events. Reinsurance strategy *B* leads to reduction in risk capital for insurance risks from €227 million to €101 million, while lowering the expected insurance result from €13.1 million to €7.1 million in comparison to reinsurance strategy *A*. This leads to a further increase in RoRAC (insurance results) from 5.8% to 7.0%. Here our company benefits from further diversification effects of the reinsurer.

Strategy *B* leads to a further structural change in insurance results and as a consequence in results of the company. Now a share quota from 30%, which is the scenario with the highest diversification benefit (39.1%), leads to the greatest EVA (company) as Fig. 10 shows. This combined strategy (30% share quota and raising reinsurance cover) fulfils both the requirements on one-year risk capital (€135 million) and five-year risk capital (€200 million),

[31] See *Diers* (2007a) for details.

[32] Strategy *A* leads to a standalone risk capital for insurance results of €227 million. The allocated risk capital for insurance results depends on the asset allocation (share quota), see Fig. 9.

which are covered by the economic capital of the company (€200 million).[33] This strategy will result in an EVA from €5.64 million and a very low shortfall probability.

Share Quota	Mean (Return)	One Year Risk Capital (Company)	EVA (Company)	TVaR Allocation Risk Capital (Insurance)	TVaR Allocation Risk Capital (Investment)	Diversification Benefit	Shortfall P(KumLoss(1) > $ECap_0$)	Five Years Risk Capital (Company)
0%	13.5	125	-1.4	81	43	37.6%	0.00%	179
10%	16.3	121	1.8	77	44	35.6%	0.00%	162
20%	19.1	122	4.4	64	58	38.4%	0.00%	173
30%	21.9	135	5.64	43	92	39.1%	0.00%	200
40%	24.6	159	5.57	30	129	36.4%	0.02%	254
50%	27.4	186	5.1	18	169	33.3%	0.05%	309
60%	30.2	216	4.3	21	195	30.5%	0.21%	368
70%	33.0	246	3.5	20	226	28.0%	0.71%	430
80%	35.8	278	2.5	20	258	25.7%	1.33%	494
90%	38.6	309	1.4	21	289	24.2%	2.01%	558
100%	41.4	342	0.3	16	326	22.8%	2.57%	624

Fig. 10: Key numbers for return and risk of the combined portfolio (investment and insurance contracts) and allocated capital (insurance and investment) after reinsurance strategy *B*

This analysis clearly demonstrates that optimising performance-adjusted indicators cannot be reached by optimising assets or liabilities alone, but requires simultaneous alignment of assets and liabilities while using the diversification potential generated. In our example company this has led to an "optimum" strategy for increasing EVA while performing the necessary decrease in risk-capital requirement using contradictory strategies. One the one side we used strategies to reduce risk capital such as raising reinsurance cover, on the other hand we tried to benefit from high diversification between investment portfolio and insurance portfolio using an increased share quota of 30% (instead of 20% in the actual strategy).

4. Conclusion

The aim of modern management techniques such as value and risk-based management is to evaluate and manage risks and returns in a multi-year period. In the actual literature several questions concerning the use of internal models in a multi-year management context are not answered to date. This study has presented a multi-year model approach and defined a multi-year risk-capital concept that can serve as an internal capital requirement (ORSA) spanning several years.

In the context of multi-year internal models we need techniques to quantify the one-year development result for each period. Moreover Solvency II requires the quantification of one-year reserve risk instead of ultimate reserve risk. In this paper we used the new stochastic re-reserving method based on simulations. We gave a quantitative comparison of the re-reserving method to the analytical methods discussed in the actual literature.[34]

[33] While deciding for an adequate share quota for the company, the regulatory requirements for the maximal risk quota have to be taken into account as a strict constraint.
[34] See Merz / Wüthrich (2008) and Wüthrich / Merz / Lysenko (2008).

It is one of company management's major responsibilities to minimise shortfall probability by adequate underwriting policy, sufficient reinsurance cover and suitable asset allocation strategy. One goal is to ensure effective risk diversification, which is hardly possible without the help of internal models. This study has examined specific management strategies in a five-year time period using a fully developed multi-year risk model. We demonstrated that separate "optimisation" in asset allocation at corporate level is no effective solution, because of the inefficient use of diversification benefits between assets and liabilities. Instead, the two strategic instruments – reinsurance and asset allocation – should be applied as part of a whole concept in order to benefit form diversification effects.

In our example company the "optimum" strategy for increasing EVA while performing the necessary decrease in risk-capital requirement was achieved by using contradictory strategies. So we used risk capital-reducing strategies (raising reinsurance cover) in combination with strategies with increasing risks and increasing returns concerning the investment results (increasing share quota in asset allocation from 20% to 30%) in order to benefit from higher diversification between investment portfolio and insurance portfolio.

The strategies shown here are to serve as examples, where a variety of other alternatives are available with simultaneous effects on the risk-return situation of each company that should also be examined. With this article we want to encourage the use of internal models in strategic risk-adjusted performance management as a basis for decision-making.

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