The Insurance Risk in the SST and in Solvency II: Modelling and Parameter Estimation

by Alois Gisler
AXA Winterthur Insurance Company and ETH Zurich

Abstract:
Both, the Swiss Solvency Test (SST) and solvency II in the EU are the framework of a new, risk based solvency regulation. In this paper we concentrate on the insurance risk. We will compare the two models and discuss what is common and what is different in the two models. Emphasis will be lead on the estimation of the parameters and some new parameter estimators will be presented. In this context we will also address the problem of how to combine individual and industry-wide data by means of credibility. Another special discussion point will be diagonal effects in the reserve risk such as the impact of inflation and super-imposed inflation.

1 Introduction

As early as in 2004 Switzerland developed the first version of the Swiss Solvency Test (SST), a new risk based solvency regulation. The first field test with 10 insurers already took place in 2004. Since 2008 all Swiss companies have to do the SST and from 2011 on the target capital or solvency capital requirement according to the SST will be in force.

In the meantime the EU has developed solvency II, and in 2008 the 4th quantitative impact study QIS has been carried through.

Both, the SST and solvency II have the same aim, namely to install a risk based solvency regulation and to require a solvency capital which is based on the risks of the company, be it on the asset side or be it on the insurance side.

It is now interesting to see the two systems and modelling frameworks, where they are equal and similar and where there are major differences.

In this paper we will concentrate on the non-life insurance risk. In Section 2 we will define the insruance risk, and in Section 3 we will investigate how they are modelled in the two systems. Section 4 will be devoted to parameter estimators, where some new parameter estimators will be presented.

2 The Insurance Risk

By the insurance risk in non-life we denote next year's technical result TR defined by

$$TR = P - K - C^{CY} - C^{PY} \tag{1}$$

where

$$\begin{aligned} P &= \text{earned premium}, \\ K &= \text{administrative costs}, \\ C^{CY} &= \text{total claim amount current year}, \\ C^{PY} &= \text{total claim amount previous years}. \end{aligned}$$

The total claim amount previous years is defined as

$$C^{PY} = -CDR,$$

where CDR is the *claims development result*

$$CDR = R - PA^{PY} - R^{31.12.,PY.},$$

where

$$\begin{aligned} R &= \text{claims reserves per 1.1. (best estimate)}, \\ PA^{PY} &= \text{payments for claims of previous years}, \\ R^{31.12.,PY.} &= \text{claims reserves per 31.12. for claims of previous years}. \end{aligned}$$

(1) can also be written as

$$\begin{aligned} TR &= \left(P - K - E\left[C^{CY}\right]\right) - \left(C^{CY} - E\left[C^{CY}\right]\right) - C^{PY} \qquad (2) \\ &\simeq \underbrace{E\left[P - K - E\left[C^{CY}\right]\right]}_{\text{expected technical result}} - \left(C^{CY} - E\left[C^{CY}\right]\right) - C^{PY} \qquad (3) \end{aligned}$$

In (2) it is assumed that the claims reserves are best estimate reserves and that therefore

$$E\left[C^{PY}\right] = 0.$$

Premiums and administrative costs of next year can usually be forecasted with high accuracy and the risk involved in these two components are negligible compared to C^{CY} and C^{PY}, which are the main risk drivers of the technical result. Replacing P and C by their expected values (forecasts at the beginning of the year) leads to (3).

From (3) it is seen that the technical result is composed of three components:

- the expected technical result,
- $C^{CY} - E\left[C^{CY}\right]$, which is the deviation of the total claim amount current year (CY) from its expected value,
- C^{PY}, the total claim amount previous years (PY).

$C^{CY} - E\left[C^{CY}\right]$ is referred to as *premium risk* in solvency II and as the *risk of claim amount current year* in the SST, and C^{PY} is called *reserve risk* in solvency II and in the SST. In the next two Sections we are going to consider in more detail how these risks are modelled in the SST and in solvency II.

In the SST as well as in solvency II, a *standard model* is in place, but companies might use another *internal model* subject to approval from the supervision authority. The topic of this paper is the modelling of the insurance risk in the standard non-life model.

The SST as well as solvency II distinguishes between different lines of business (lob) $i = 1, 2, \ldots, I$. We will use the index i to denote lob i, whereas a dot in the index always indicates summation over the corresponding index. For instance, C_i^{CY} will denote the claim amount CY of lob i and $C_\bullet^{CY}$ the claim amount CY summed up over all lines of business. The segmentation into lob considered in the standard models can be found in Appendix A.1.

Finally, in the SST as well as in solvency II, reserves are discounted with the risk-free yield rate. Thus the reserves have to be interpreted as discounted reserves. The same is the case for related random variables. For instance the claim amount CY has to be interpreted as the discounted claim amount CY.

3 Modelling of the Insurance Risk

3.1 Modelling of the Insurance Risk in the SST

In the SST standard model the C^{CY} is split into $C^{CY,n}$, the claim amount caused by the bulk of so called "normal" claims and $C^{CY,b}$, the claim amount caused by big claims or big claim events. In a short term notation we will call $C^{CY,n}$ the normal claim amount and $C^{CY,b}$ the big claim amount. All these elements ($C^{CY,n}$, $C^{CY,b}$, C^{PY}) are modelled by a stoschastic model named the *analytical insurance risk model.* The *analytical insurance risk model* as well as an analogous analytical model for the market risk are designed to describe adequately the reality except for very seldom extraordinary situations. Therefore, in addition to the analytical model the SST also allows to take into account very seldom and extraordinary situations by means of so called *scenarios SC_k,* which are characterised by face amounts c_k and assigned probabilities p_k for their occurrences. There are *financial scenarios* producing a big market loss (e.g. scenario Nikkei drop), *insurance scenarios* causing a big insurance loss (e.g. explosion in an industrial complex) and *mixed scenarios* producing both an insurance and a market loss (e.g. a pandemia). A clear separation between insurance and market risk is not possible for the mixed scenarios. We allocate to the insurance risk all insurance scenarios and those mixed scenarios, where the insurance loss is the dominant one and bigger than the market loss. The insurance scenario risk is then the sum

$$SC_\bullet^{ins} = \sum_{k=1}^{K} SC_k,$$

where K is the number of scenarios allocated to the insurance risk.

From (3) we obtain

$$\begin{aligned} TR \;=\; & \underbrace{E\left[P - K - E\left[C^{CY}\right]\right]}_{\text{expected technical result}} - \left(C_\bullet^{CY,n} - E\left[C_\bullet^{CY,n}\right]\right) - \left(C_\bullet^{CY,b} - E\left[C_\bullet^{CY,b}\right]\right) \\ & - C_\bullet^{PY} - SC_\bullet^{ins}. \end{aligned}$$

The SST is distribution based. At the end one wants to determine the distribution for Δ = change of the risk bearing capital within a one-year period. The risk measure used in the SST is the 99% mean expected shortfall. Transferred to the insurance risk this means that we want to determine the distribution function of the technical result TR, and we define the target capital or solvency capital required for the insurance risk by

$$\begin{aligned} SCR_{ins} &= ES_{99\%}[-TR] \\ &= ES_{99\%}^{mean}[-TR] - E[TR], \end{aligned}$$

where, for any random variable X, $ES_{99\%}^{mean}[X] = ES_{99\%}[X - E[X]]$ and where $ES_{99\%}[X]$ denotes the 99% expected shortfall. Note that the solvency capital required as defined above is the necessary capital of the insurance risk on a stand-alone basis (before diversification with the market risks).

3.1.1 Modelling of the normal claim amount current year in the SST

Model Assumptions 3.1 (normal claim amount SST) *For each line of business i it is assumed that the current year (=next coming year) is characterised by its risk characteristics $\boldsymbol{\Theta}_i^T = (\Theta_{1i}, \Theta_{2i})$, where Θ_{1i} and Θ_{2i} are independent with $E[\Theta_{1i}] = E[\Theta_{2i}] = 1$, and that conditional on $\boldsymbol{\Theta}_i^T = (\Theta_{1i}, \Theta_{2i})$*

- *the normal claim amount $C_i^{CY,n}$ is compound Poisson distributed*
- *with Poisson parameter $\lambda(w_i, \Theta_{1i}) = w_i \lambda_i \Theta_{1i}$, where w_i is a known weight and where λ_i is the a priori expected claim frequency,*
- *and with claim severities $Y_i^{(\nu)}$ having the same distribution as $\Theta_{2i} Y_i$, where Y_i has a distribution $F_i(y)$ with $E[Y_i] = \mu_i$.*

Remarks:

- The underlying risk characteristics $\boldsymbol{\Theta}_i^T = (\Theta_{1i}, \Theta_{2i})$ reflect the "state of the nature" in the next coming year for lob i. Indeed things like weather conditions, economic situation, change in legislation, etc. might have an impact on the claim number and the claim severity. These "conditions" may vary from year to year and the imposed changes increase the risk and affect big companies as much as small companies, i.e. this risk cannot be diversified. The "true claim frequency" in the coming year will be $\lambda_i(\Theta_{1i}) = \lambda_i \cdot \Theta_{1i}$, hence Θ_{1i} is the random factor by which next year's "true underlying claim frequency" will deviate from the a priori expected claim frequency λ_i. The interpretation for Θ_{2i} is quite analogous, but with respect to the "true underlying expected value of the claim severity".
- Note, that no distributional assumption about the claim severities is made in model assumptions 3.1. But it will later be assumed that

  $$C_\bullet = C_\bullet^{CY,n} + C_\bullet^{PY}$$

  can be approximated by a lognormal distribution with the corresponding first and second moments (see Section 3.1.3). Another idea would be to assume a lognormal distribution on the level of lines of business. This would be a slightly different model, since the sum of lognormal distributions is not lognormal any more. But the difference would presumably be rather small without a significant impact on the result, but it would have the disadvantage to be much more complicated and the distribution of $C_\bullet$ could not be expressed by a simple analytical formula any more.

We introduce

$$\widetilde{P}_i = E\left[C_i^{CY,n}\right],$$

which is the pure risk premium for normal claims, and

$$X_i = \frac{C_i^{CY,n}}{\widetilde{P}_i} \tag{4}$$

the corresponding loss ratio.

It follows from appendix A.3, formula (65) , that

$$\sigma_i^2 := \mathrm{Var}\left(X_i\right) = \sigma_{i,param}^2 + \frac{\sigma_{i,fluct}^2}{\nu_i}, \tag{5}$$

where

$$\begin{aligned} \sigma_{i,param}^2 &= \mathrm{Var}\left(\Theta_{1i}\right) + \mathrm{Var}\left(\Theta_{2i}\right) + \mathrm{Var}\left(\Theta_{1i}\right)\cdot \mathrm{Var}\left(\Theta_{2i}\right) & (6)\\ &\simeq \mathrm{Var}\left(\Theta_{1i}\right) + \mathrm{Var}\left(\Theta_{2i}\right), & (7)\\ \sigma_{i,fluct}^2 &= CoVa^2\left(Y_i^{(v)}\right) + 1. & (8) \end{aligned}$$

and where

$$\begin{aligned} &CoVa\left(Y_i^{(v)}\right) = \text{the coefficient of variation of the claim severities,}\\ &\nu_i = w_i\lambda^{(i)} = \text{a prori expected number of claims.} \end{aligned}$$

Since $\widetilde{P}_i = \nu_i\mu_i$ we can also write (5) as

$$\sigma_i^2 := \mathrm{Var}\left(X_i\right) = \sigma_{i,param}^2 + \frac{\widetilde{\sigma}_{i,fluct}^2}{\widetilde{P}_i}, \tag{9}$$

where

$$\widetilde{\sigma}_{i,fluct}^2 = \mu_i\sigma_{i,fluct}^2.$$

Note that $\sigma_i^2 = CoVa^2\left(C_i^{CY,n}\right)$ is composed of two components, the *parameter risk* and the *random fluctuation risk*. The parameter risk is independent of the size of the company whereas the fluctuation risk decreases with the size of the company respectively with the number of a priori expected claims.

The *standard values* provided by the supervision authority for $\sigma_{i,param}^2$ and $\sigma_{i,fluct}^2$ in the SST 2008 (comparable to the *market values* in solvency II) can be found in Appendix A.2. Companies might deviate from them based on estimates from their own data. Usually there is sufficient data in most of the companies to estimate $\sigma_{i,fluct}^2$, but there is not yet a sound methodology to estimate $\sigma_{i,param}^2$ from the own data. Therefore most companies use the default values for $\sigma^2_{i,param}$. Estimators for estimating $\sigma^2_{i,param}$ from the company-own data will be presented in Section 4.2.2.

Model assumptions 3.1 have resulted in the variance structure given by (5). Only this variance structure is then further used in the SST standard model. Hence, alternatively to model assumptions 3.1, we can formulate the model-assumptions directly by the variance condition (9).

Model Assumptions 3.2 (normal claim amount SST, alternative version) *It is assumed that*

$$\sigma_i^2 := \mathrm{Var}\left(X_i\right) = \sigma^2_{i,param} + \frac{\widetilde{\sigma}^2_{i,fluct}}{\widetilde{P}_i}. \tag{10}$$

For calculating the variance of the total normal claim amount (summed up over all lines of business) we have to make assumptions on the correlation of the normal claim amount of different lines of business. Let

$$\begin{aligned} \rho_{ij}^{CY} &:= \mathrm{Corr}\left(C_i^{CY,n}, C_j^{CY,n}\right), \\ X_\bullet &:= \frac{C_\bullet^{CY,n}}{\widetilde{P}_\bullet}. \end{aligned}$$

Then we obtain

$$\mathrm{Var}\left(C_\bullet^{CY,n}\right) = \sum_{i,j=1}^{I} \widetilde{P}_i\, \sigma_i \widetilde{P}_j\, \sigma_j \rho_{ij}^{CY} \tag{11}$$

and

$$\sigma^2 := \mathrm{Var}\left(X_\bullet\right) = \frac{1}{\widetilde{P}_\bullet^2} \sum_{i,j=1}^{I} \widetilde{P}_i\, \sigma_i \widetilde{P}_j\, \sigma_j \rho_{ij}^{CY}. \tag{12}$$

It is convenient to write (11) in matrix notation. Let

$$\begin{aligned} \mathbf{X} &= \left(X_1, X_2, \ldots, X_I\right)^T, \\ \mathbf{W}_{CY} &= \left(\widetilde{P_1}\sigma_1, \widetilde{P_2}\sigma_2, \ldots, \widetilde{P_I}\sigma_I\right)^T, \\ \mathbf{R}_{CY} &= \mathrm{Corr}\left(\mathbf{X}\ ,\ \mathbf{X}^T\right). \end{aligned}$$

Note that $\mathbf{R}_{CY}$ denotes the correlation matrix of $\mathbf{X}$ with the entries

$$\mathbf{R}_{CY}\left(i,j\right) = \rho_{ij}^{CY}.$$

(11) written in matrix notation becomes

$$\mathrm{Var}\left(C_\bullet^{CY,n}\right) = \mathbf{W}_{CY}^T \cdot \mathbf{R}_{CY} \cdot \mathbf{W}_{CY}. \tag{13}$$

The correlation matrix $\mathbf{R}_{CY}$ as provided by the supervision authority for the standard model in the SST 2008 can be found in Appendix A.2.

.

3.1.2 Modelling of the Reserve Risk in the SST

Let

$$\begin{aligned} L_i &= \text{outstanding claims liabilities at 1.1. for lob } i, \\ R_i &= \text{best estimate of } L_i \text{ per 1.1.} = \text{best estimate reserve}, \\ \widetilde{R_i} &= PA_i^{PY} + R_i^{31.12.,PY} = \text{best estimate of } L_i \text{ per 31.12.}, \end{aligned}$$

where PA_i^{PY} are the claim payments for previous years' claims and $R_i^{31.12.,PY}$ are best estimate reserves per 31.12. of previous years' claims (see Section 2).

Note that R_i is known at the beginning of the year, whereas $\widetilde{R_i}$ is still a random variable (PA_i^{PY} and $R_i^{31.12.,PY}$ will only be known at the end of the year).

The claim amount PY becomes

$$C_i^{PY} = \widetilde{R_i} - R_i.$$

As for the current year risk, we introduce something analogous to the loss ratio, but with the ingoing reserves R_i (instead of the premiums) as "weight", that is we consider

$$Y_i = \frac{\widetilde{R_i}}{R_i}.$$

Note that $\mathrm{E}[Y_i] = 0$ and that $C_i^{PY} = (Y_i - 1)R_i$.

Model Assumptions 3.3 (reserve risk SST) *It is assumed that*

$$\tau_i^2 := \mathrm{Var}\,(Y_i) = \tau_{i,param}^2 + \frac{\tau_{i,fluct}^2}{R_i}. \tag{14}$$

Remarks:

- The SST distinguishes between parameter risk and random fluctuation risk, but the model assumptions 3.3 are not written down anywhere in an official document of the SST. As pointed out in [7] the question of how to quantify the reserve is risk is not yet fully answered. Thus the variance structure (14) is my own interpretation and is not yet an integral part of the current SST. I think that from a modelling point of view there are good reasons to assume the above structure, even if this structure is not reflected in the current standard parameters, where the coefficients of variation of the random fluctuation risk do not depend on the size of the reserves. Note that (14) has the same structure as (10).

- As for the current year risk it is again assumed that the variance consists of two components, the parameter risk independent of the weight (=volume of reserves) and the random fluctuation risk which is inversely proportional to this weight. The parameter risk reflects the estimation error which is a risk measure of the deviation of the reserve R_i from the true expected value $E\left[L_i\right]$ of the outstanding liabilities L_i, whereas the random fluctuation risk encompasses the pure random fluctuation of the ultimate around $E\left[L_i\right]$, which is also called *process error* in claims reserving.

The table with the standard values for $\tau^2_{i,param}$ and $\tau^2_{i,fluct}$ provided by the supervision authority for the SST 2008 can be found in Appendix A.2.

In order to calculate the variance for the reserve risk summed up over all lines of business, we have again to make assumptions about the correlation of the reserve risk of different lob. According to the current SST standard assumption it is assumed that there is no correlation between the reserve risk of different lob.

Discussion and remarks on the correlation assumption for PY risks in the SST

- The current standard assumption of no-correlation between the reserve risks of different lines of business is questionable. Calendar year factors may affect the reserves of several lines of business simultaneously and impose a positive correlation. An obvious calendar year factor is certainly inflation, but there might also be others like change in legislation.
- A change of the correlation assumption can be done well within the SST modelling frame-work. Hence the no-correlation assumption does not question the SST-standard-model as such, but is rather a matter of the parameter choice within the SST standard model.

Let

$$\begin{aligned}
Y_\bullet &= \frac{\widetilde{R_\bullet}}{R_\bullet}, \\
\mathbf{Y} &= (Y_1, Y_2, \ldots, Y_I)^T, \\
\mathbf{W}_{PY} &= (R_1\tau_1, R_2\tau_2, \ldots, R_I\tau_I)^T, \\
\mathbf{R}_{PY} &= \operatorname{Corr}\left(\mathbf{Y}\ ,\ \mathbf{Y}^T\right).
\end{aligned}$$

Then we obtain

$$\operatorname{Var}\left(C_\bullet^{PY}\right) = \mathbf{W}_{PY}^T \cdot \mathbf{R}_{PY} \cdot \mathbf{W}_{PY}, \tag{15}$$

and

$$\tau^2 := \operatorname{Var}(Y_\bullet) = \frac{1}{R_\bullet^2}\left(\mathbf{W}_{PY}^T \cdot \mathbf{R}_{PY} \cdot \mathbf{W}_{PY}\right). \tag{16}$$

The no-correlation standard assumption means that $\mathbf{R}_{PY}$ is the identity matrix. Then (16) becomes

$$\tau^2 = \frac{1}{R_\bullet^2}\sum_{i=1}^{I} R_i^2\tau_i^2. \tag{17}$$

3.1.3 Modelling of the sum of normal claim amount CY and claim amount PY in the SST

Let

$$S_i = C_i^{CY,n} + \widetilde{R_i}, \tag{18}$$

$$Z_i = \frac{C_i^{CY,n} + \widetilde{R_i}}{\widetilde{P_i} + R_i} = \frac{\widetilde{P_i}X_i + R_iY_i}{\widetilde{P_i} + R_i}. \tag{19}$$

Whereas $\widetilde{P}_i$ and R_i can be considered as the weights attached to X_i and Y_i respectively, the corresponding weight for Z_i is the sum

$$V_i = \widetilde{P}_i + R_i. \tag{20}$$

From (19) we immediately see that

$$\varphi_i^2 := \mathrm{Var}\left(Z_i\right) = \frac{\left(\widetilde{P}_i\sigma_i\right)^2 + 2\widetilde{P}_i\sigma_i R_i\tau_i \mathrm{Corr}\left(C_i^{CY}, C_i^{PY}\right) + \left(R_i\tau_i\right)^2}{V_i^2}. \tag{21}$$

At the end we are interested in the variance of the total claim amount CY and PY summed up over all lines of business. Denote by

$$\mathbf{R}_{CY,PY} = \mathrm{Corr}\left(\mathbf{C}^{CY}\ ,\ \mathbf{C}^{PY^T}\right)$$

the correlation matrix between the claim amounts CY and the claim amounts PY with entries

$$\mathbf{R}_{CY,PY}\left(i,j\right) = \mathrm{Corr}\left(C_i^{CY}, C_j^{PY}\right).$$

As correlation matrix for the joint random vector of claim amounts CY and claim amounts PY we obtain

$$\begin{aligned} \mathbf{R} &= \mathrm{Corr}\left(\begin{pmatrix} \mathbf{C}^{CY} \\ \mathbf{C}^{PY} \end{pmatrix}, \begin{pmatrix} \mathbf{C}^{CY} \\ \mathbf{C}^{PY} \end{pmatrix}^T\right) \\ &= \begin{pmatrix} \mathbf{R}_{CY} & \mathbf{R}_{CY,PY} \\ \mathbf{R}_{CY,PY} & \mathbf{R}_{PY} \end{pmatrix}. \end{aligned}$$

Let

$$S_\bullet = C_\bullet^{CY} + \widetilde{R}_\bullet, \tag{22}$$

$$Z_\bullet = \frac{C_\bullet^{CY} + \widetilde{R}_\bullet}{\widetilde{P}_\bullet + R_\bullet}. \tag{23}$$

Then we obtain

$$\mathrm{Var}\left(S_\bullet\right) = \begin{pmatrix} \mathbf{W}_{CY} \\ \mathbf{W}_{PY} \end{pmatrix}^T \cdot \mathbf{R} \cdot \begin{pmatrix} \mathbf{W}_{CY} \\ \mathbf{W}_{PY} \end{pmatrix}, \tag{24}$$

$$\mathrm{Var}\left(Z_\bullet\right) = \frac{1}{V_\bullet^2}\left\{\begin{pmatrix} \mathbf{W}_{CY} \\ \mathbf{W}_{PY} \end{pmatrix}^T \cdot \mathbf{R} \cdot \begin{pmatrix} \mathbf{W}_{CY} \\ \mathbf{W}_{PY} \end{pmatrix}\right\}, \tag{25}$$

where

$$V_\bullet = \widetilde{P}_\bullet + R_\bullet\,.$$

Since

$$Z_\bullet = \sum_{i=1}^{I} \frac{V_i}{V_\bullet} Z_i,$$

we can also express the variance of $Z_\bullet$ in terms of the Z-variables, i.e.

$$\begin{aligned}\operatorname{Var}(Z_\bullet) &= \sum_{i,j=1}^{I} \frac{V_i V_j \varphi_i \varphi_j}{V_\bullet^2} \operatorname{Corr}(Z_i, Z_j) \qquad (26)\\ &= \frac{1}{V_\bullet^2} \mathbf{V}^T (\mathbf{W}_{CY}^T \cdot \mathbf{R}_{CY} \cdot \mathbf{W}_{CY} + 2 \cdot (\mathbf{W}_{CY}^T \cdot \mathbf{R}_{CY,PY} \cdot \mathbf{W}_{CY}) + \mathbf{W}_{PY}^T \cdot \mathbf{R}_{PY} \cdot \mathbf{W}_{PY}) \mathbf{V},\end{aligned}$$

where $\mathbf{V} = (V_1, V_2, \ldots, V_I)^T$.

(25) is valid for any correlation matrix $\mathbf{R}$. The standard assumption in the current SST is that there is no correlation between current year risks and previous year risks, which means that $\mathbf{R}_{CY,PY}$ is a zero-matrix (matrix with only zeros). Together with the assumption that the reserve risks of different lob are not correlated we obtain

$$\operatorname{Var}(Z_\bullet) = \frac{\widetilde{P_\bullet}^2 \sigma^2 + R_\bullet \tau^2}{\left(\widetilde{P_\bullet} + R_\bullet\right)^2}. \qquad (27)$$

Discussion and remarks on the correlation assumption between CY and PY risks:

- The current standard assumption in the SST that CY risks and PY risks are not correlated, is questionable. There might well be *calendar year factors* affecting both, the CY and the PY results. The most obvious of these factors is claims inflation, which has an impact on both, CY claims and PY claims.

Model Assumptions 3.4 (normal claim amount CY plus claim amount PY) *It is assumed that $S_\bullet$ has a lognormal distribution with*

$$\begin{aligned}E[S_\bullet] &= \widetilde{P_\bullet} + R_\bullet,\\ \operatorname{Var}(S_\bullet) &= (\mathbf{W}_{CY}, \mathbf{W}_{PY})^T \cdot \mathbf{R} \cdot (\mathbf{W}_{CY}, \mathbf{W}_{PY}).\end{aligned}$$

Remark:

- This model assumption has to be interpreted in the way that a lognormal distribution is a sufficiently accurate approximation to the distribution of $S_\bullet$ for solvency purposes.

3.1.4 Modelling of the Big Claim Amount Current Year in the SST

Big claims are defined as single individual claims or claim events (natural catastrophes) with a claim amount exceeding a certain threshold (e.g. 1 m CHF or 5 m CHF). For both types of big claims it is assumed that the following model assumptions hold true.

Model Assumptions 3.5 (big claim amount SST)

i) The big claim amount for line of business i has a compound Poisson distribution

$$C_i^{CY,b} = \sum_{\nu=1}^{N_i^b} Y_{i\nu}^b,$$

where the number of big claims N_i^b is Poisson-distributed with Poisson parameter λ_i^b and where the claim severities $Y_{i\nu}^b$, $\upsilon = 1, 2, ..., N_i^b$, are independent and independent from N_i^b with distribution function F_i.

ii) $\left\{C_i^{CY,b} : i = 1, 2, \ldots, I\right\}$ *are independent.*

Remarks:

- The claim severity distributions F_i are essentially assumed to be Pareto with Pareto parameters α_i. Upper limits can easily be taken into account by truncation of the Pareto distributions. Also xs-reinsurance is no problem to handle by this model.
- Big claim events like hail, wind-storm, flood etc. usually affect several lines of business. However, this is not relevant in this context. Such an event claim might then be associated to the lob with the highest expected claims load. The essential assumption is that all these big claims are independent and compound Poisson.

From these assumptions it follows that

$$C_\bullet^{CY,b} = \sum_{i=1}^{I} C_i^{CY,b}$$

is again compound Poisson with parameter

$$\lambda = \lambda_\bullet^b = \sum_{i=1}^{I} \lambda_i^b$$

and claim severity distribution

$$F = \sum_{i=1}^{n} \frac{\lambda_i^b}{\lambda_\bullet^b} F_i.$$

The compound Poisson-distribution of $C_\bullet^{CY,b}$ can for instance be calculated by means of the Panjer-algorithm.

3.1.5 Aggregation

Let

$$\widetilde{T} = C_\bullet^{CY,n} + C_\bullet^{CY,b} + C_\bullet^{PY}$$

be the total claim amount before scenarios. According to model-assumptions 3.4, the distribution of $S_\bullet = C_\bullet^{CY} + C_\bullet^{PY}$ is lognormal and according to model-assumptions 3.5, the distribution of $C_\bullet^{CY,b}$ is compound Poisson. Thus the distribution function $\widetilde{F}(x)$ of $\widetilde{T}$ is obtained by convoluting the lognormal distribution of $S_\bullet$ with the compound Poisson distribution of $C_\bullet^{CY,b}$.

3.1.6 Taking the scenarios into account

Insurance scenarios SC_k^{ins} are situations or events producing an estimated loss c_k (face amount) with probability p_k. In the SST standard model, beside prescribed scenarios, the company should also think about company specific scenarios. Hence the number K of the scenarios considered by a company is not fixed. However, in the standard SST it is assumed that the scenarios $SC_k^{ins}, k = 1, 2, \ldots, K$ exclude each other, i.e. that only one of the scenarios can materialize during the next year. This exclusion property was made because of simplicity reasons and is not such a big restriction as it might seem on the first side. If one wants to incorporate the situation, that e.g. SC_i^{ins} and SC_j^{ins} might occur in the same year, one can add a new scenario SC_{new}^{ins} with face amount $c_{new} = c_i + c_j$ and a corresponding probability p_{new}, for instance $p_{new} = p_i p_j$ if the the two scenarios are assumed to be independent.

Let

$$T = C_\bullet^{CY,n} + C_\bullet^{CY,b} + C_\bullet^{PY} + SC_\bullet^{ins},$$

be the total claim amount including scenarios, and denote by $F(x)$ the distribution of T. Define

$$\begin{aligned} p_0 &= 1 - \sum_{k=1}^{K} p_k \text{ and} \\ c_0 &= 0. \end{aligned}$$

Then

$$F(x) = \sum_{k=0}^{K} p_k \widetilde{F}(x - c_k),$$

where $\widetilde{F}(.)$ is the distribution before scenarios.

3.2 Modelling of the Insurance Risk in Solvency II

For comparison with the SST we consider a non-life insurance company working in only one region. In the terminology of solvency II the solvency capital required for the insurance risk is denoted by SCR_{nl}, where the index nl stands for non-life.

In solvency II, one also distinguishes between *current year risk*, which is called *premium risk*, and *previous year risk,* which is called *reserve risk*. Contrary to the SST, the claim amount CY is not split into a normal claim amount (caused by the bulk of normal claims) and a big claim amount (caused by big claims or big claim events). However, solvency II considers in addition to the CY- and PY-risk the category of *CAT-risks,* which are modelled with a face amount (analogous to the scenarios in the SST) indicating the expected loss of natural catastrophes and man-made catastrophes. Thus the CAT-risks cover to some extent the big claim events taken into account by the big-claim modelling in the SST and also allow to consider situations taken into account as scenarios in the SST.

The risk measure in solvency II is the 99.5% value at risk. Solvency II is, however, not distribution based . At the end one comes up with a figure, the solvency capital required (SCR) and not with a distribution from which the SCR is obtained.

Solvency II also distinguishes between different lines of business $i = 1, 2, \ldots, I$. As in the SST there are provided "standard parameters" called "market parameters", which are used in the standard solvency II calculation.

Solvency II is a framework with formulas how to calculate the SCR. From these formulas we can derive the underlying model assumptions behind solvency II leading to these formulas.

In the next Section we introduce the formulae in solvency II for calculating the SCR and in the following Sections we then discuss the model assumptions behind these formulas.

3.2.1 The Calculation of the SCR in Solvency II

As in the whole paper we concentrate on the non-life insurance risk and hence on the solvency capital required named SCR_{nl} in solvency II.

We will use the analogous notation as in Section 3.1, that is

$$\begin{aligned} X_i &= \frac{C_i^{CY}}{P_i} = \text{loss ratio CY}, \\ Y_i &= \frac{\widetilde{R_i}}{R_i}, \\ \sigma_i^2 &= \operatorname{Var}(X_i), \\ \tau_i^2 &= \operatorname{Var}(Y_i), \end{aligned}$$

where

P_i = premium

R_i = reserve at the beginning of the year = best estimate of the outstanding liabilities L_i.per 1.1.

$\widetilde{R_i}$ = a posteriori best estimate per 31.12.of L_i.

Remark:

- The premium P_i is further specified in solvency II (earned, written, ...).

The following formula is used for the standard deviation of the premium risk.

$$\sigma_i = \sqrt{\alpha_i \cdot \sigma_{i,ind}^2 + (1 - \alpha_i) \cdot \sigma_{i,M}^2}, \tag{28}$$

where α_i is a credibility weight depending on the lob and on the number of historical years of data available, $\sigma_{i,M}$ is a given standard parameter (market wide estimate for the premium risk) and $\sigma_{i,ind}^2$ is a company-specific estimate of σ_i^2 based on the company's own historical data. Hence σ_i^2 is a credibility weighted mean between the company specific estimate $\sigma_{i,ind}^2$ and the market estimate $\sigma_{i,M}^2$. For estimating $\sigma_{i,ind}^2$, denote by P_{ij} and X_{ij} the premiums and the observed loss ratios of the individual company in lob i in year j and by n_i the number of historical data available and taken into account for lob i. Then the following formula is used in solvency II.

$$\sigma_{i,ind}^2 = \frac{1}{n_i - 1} \sum_{j=1}^{n_i} \frac{P_{ij}}{P_{i\bullet}} (X_{ij} - \overline{X_i})^2, \tag{29}$$

where

$$\overline{X_i} = \sum_{j=1}^{n_i} \frac{P_{ij}}{P_{i\bullet}} X_{ij}.$$

The standard deviations τ_i for the reserve risk are given standard parameters which are the same for all companies and which do not depend on the size of the company.

Next the premium plus reserve risk per line of business is considered. As in Section 3.1.3 we introduce

$$\begin{aligned} Z_i &= \frac{1}{V_i}\left(P_i X_i + R_i Y_i\right), \\ \text{where } V_i &= P_i + R_i. \end{aligned}$$

Solvency II assumes, that $\mathrm{Corr}\left(X_i, Y_i\right) = \rho_{CY,PY} = 50\%$ for all i.Then

$$\varphi_i := \sqrt{\mathrm{Var}\left(Z_i\right)} = \sqrt{\frac{\left(P_i\sigma_i\right)^2 + 2\rho_{CY,PY} P_i\sigma_i R_i\tau_i + \left(R_i\tau_i\right)^2}{V_i^2}}. \tag{30}$$

Next it is assumed in solvency II that

$$\mathrm{Corr}\left(Z_i, Z_j\right) = \rho_{ij} \tag{31}$$

where the correlations ρ_{ij} are given standard values equal for all companies. For

$$Z_\bullet = \sum_{i=1}^{I} \frac{V_i}{V_\bullet} Z_i$$

we then obtain

$$\varphi^2 = \mathrm{Var}\left(Z_\bullet\right) = \sum_{i,j=1}^{I} \frac{V_i V_j \varphi_i \varphi_j}{V_\bullet^2} \rho_{ij}, \tag{32}$$

which is the same as (25).

The formula used in solvency II for calculating the SCR of the combined premium and reserve risk is

$$\begin{aligned} SCR_{pr+res} &= V_\bullet \left(\frac{\exp\left(\Phi^{-1}\left(0.995\right) \cdot \sqrt{\log(\varphi^2+1)}\right)}{\sqrt{\varphi^2+1}} - 1 \right) && (33) \\ &= V_\bullet \, VaR_{0.995}^{mean}\left(\Psi\right) && (34) \end{aligned}$$

where

$$\begin{aligned} \Psi &= \text{logormal distributed random variable with } E\left[\Psi\right] = 1 \text{ and } \mathrm{Var}\left(\Psi\right) = \varphi^2, \\ VaR_{0.995}^{mean}\left(\Psi\right) &= 99.5\% \text{ value at risk of } \ \Psi - E\left[\Psi\right], \\ V_\bullet &= P_\bullet + R_\bullet, \\ \Phi\left(x\right) &= \text{standard normal distribution.} \end{aligned}$$

The CAT-risks are similar to the scenarios in the SST. A CAT-risk CAT_k is characterised by a face amount c_k to be interpreted as the expected loss for the company if this catastrophe happened. For natural catastrophes the amounts c_k for a specific company are proportional to the underlying premium ($\widetilde{c}_t P_t$ if only lob t is hit by the event resp. a corresponding sum over several lob in the case that several lob are hit). The solvency capital required is for the total of the cat-risks is then calculated by

$$SCR_{CAT} = \sqrt{\sum_{k=1}^{K} c_k^2}. \tag{35}$$

Finally, the SCR for the non-life insurance risk including the CAT risks is

$$SCR_{nl} = \sqrt{SCR_{pr+res}^2 + SCR_{CAT}^2}.$$

3.2.2 Modelling of the Claim Amount CY in Solvency II

In (28) neither $\sigma_{i,M}^2$ nor the credibility weight α_i depends on the size of the company. Therefore we conclude that the following implicit model assumption regarding the CY claim amount is behind solvency II.

Model Assumptions 3.6 (CY claim amount solvency II) *It is assumed that*

$$\mathrm{Var}\,(X_i) = \sigma_i^2.$$

Model assumptions 3.6 should be compared with the model assumptions 3.2 in the SST. From this comparison we see that in solvency II the variance of X_i does not depend on the size of the company, or in the terminology of the SST, there is only a *parameter risk* and *no random fluctuation risk.*

Discussion and Remarks:

- The assumption that the variance of the loss ratio is independent of the size of the company is questionable. From a conceptual modelling point of view the assumptions in the SST that there is a parameter risk not depending on the size of the company and a random fluctuation risk being inversely proportional to P_i seems to be more realistic.

- Solvency II resp. formula (28) is from a pure mathematical point of view not fully stringent in the sense that (29) is a best unbiased estimator linear in $(X_{ij} - \overline{X_i})^2$ under the model assumption that $\mathrm{Var}\,(X_i) = \sigma_i^2 / P_i$. Under the model assumptions 3.6 the non weighted mean

$$\sigma_{i,ind}^2 = \frac{1}{n_i - 1} \sum_{j=1}^{n_i} (X_{ij} - \overline{X_i})^2,$$

where

$$\overline{X_i} = \frac{1}{n_i} \sum_{j=1}^{n_i} X_{ij}$$

would be a better estimator than (29) .

3.2.3 Modelling of the Reserve Risk in Solvency II

Model Assumptions 3.7 (PY claim amount solvency II) *It is assumed that*

$$\mathrm{Var}\left(Y_i\right) = \tau_i^2.$$

Model assumptions 3.7 should again be compared with the model assumptions 3.3 in the SST. From this comparison we see that in solvency II the variance of Y_i does not depend on the size of the company, or in the terminology of the SST, there is only a *parameter risk* and *no random fluctuation risk.*

Discussion and Remarks:

- We can add here the same remark as for the CY risk. The assumption that the variance of the reserve risk is independent of the size of the company is rather questionable.

3.2.4 Modelling of the Sum of CY Risk and PY risk in Solvency II

We use the same notation as in Section 3.1.3 and introduce

$$\begin{aligned} S_i &= C_i^{CY} + \widetilde{R_i}, && (36)\\ Z_i &= \frac{C_i^{CY,n} + \widetilde{R_i}}{P_i + R_i} = \frac{P_i X_i + R_i Y_i}{P_i + R_i}, && (37)\\ V_i &= P_i + R_i. && (38) \end{aligned}$$

As described in Section 3.2.1, it is assumed that

$$\begin{aligned} \mathrm{Corr}\left(X_i, Y_i\right) &= \rho_{CY,PY} = 50\% \text{ for all } i, && (39)\\ \mathrm{Corr}\left(Z_i, Z_j\right) &= \rho_{ij}, && (40) \end{aligned}$$

where the ρ_{ij} are given standard parameters (market values). From (39) and (40) follows that

$$\varphi^2 = \mathrm{Var}\left(Z_\bullet\right) = \sum_{i,j=1}^{I} \frac{V_i V_j \varphi_i \varphi_j}{V_\bullet^2} \rho_{ij}. \tag{41}$$

Discussion and remarks on the correlation assumption in solvency II:

- (40) is assumed to be true for any company. Hence it should be fulfilled for a company which has just started and which has no reserve risk as well as for a company in the run-off and having no CY risk and only a reserve risk. Hence we can conclude that

$$\operatorname{Corr}(X_i, X_j) = \operatorname{Corr}(Y_i, Y_j) = \operatorname{Corr}(Z_i, Z_j) = \rho_{ij}, \tag{42}$$

  which means that the correlation matrices of $\mathbf{X}, \mathbf{Y}$ and $\mathbf{Z}$ are all the same. To assume the same correlation matrix for the CY-risks and the PY-risks is, however, questionable. The reason that there are correlations between lines of business results from the fact that there might be factors affecting different lines of business simultaneously. However, the calendar year factors for CY- and PY-risks are not necessarily the same and if they are the same they might impact CY- and PY-risks differently. Hence the correlation matrix of $\mathbf{X}, \mathbf{Y}$ and $\mathbf{Z}$ are hardly the same in reality. For instance, calendar year factors such as weather conditions which might have an impact on the claim frequency affect only the CY risks but not the reserve risks.

- Using the notation of Section 3.1.3 we can conclude that in solvency II $\mathbf{R}_{CY} = \mathbf{R}_{PY}$ with entries $\mathbf{R}_{CY}(i,j) = \rho_{ij}$ and that the diagonal elements of $\mathbf{R}_{CY,PY}$ are equal to $\rho_{CY,PY} = 50\%$. However the elements of $\mathbf{R}_{CY,PY}$ outside the diagonal depend on the volumes P_i and on R_i, which is a bit strange. It is very difficult to interprete and to see the structure of $\mathbf{R}_{CY,PY}$ resulting from the solvency II correlation assumptions.

Finally the following model assumption is behind formula (33).

Model Assumptions 3.8 (claim amount CY + PY solvency II) *It is assumed that* $S_\bullet - E[S_\bullet]$ *has the same distribution as* $V_\bullet(\Psi - 1)$, *where* Ψ *has a lognormal distribution with*

$$\begin{aligned} E[\Psi] &= 1, \\ \operatorname{Var}(\Psi) &= \frac{1}{V_\bullet^2}\varphi^2 \end{aligned}$$

with $\varphi^2 = \operatorname{Var}(Z_\bullet)$ *as given in formula* (41).

Remarks and discussion:

- Model assumptions 3.8 are a direct consequence of formula (33) and the fact that the risk measure in solvency II is the 99.5% value at risk

- Model assumptions 3.8 should be compared with model assumptions 3.4 in the SST. At first glance it seems as if they were identical. As in the SST it holds that

$$S_\bullet - E[S_\bullet] = V_\bullet\,(Z_\bullet - E[Z_\bullet]).$$

However, $E[Z_\bullet] = 1$ in the SST, whereas in solvency II $E[Z_\bullet]$ is normally smaller than one, because the premiums used for the loss ratios are not the pure risk premiums and hence $E[X_\bullet]$ is the expected loss ratio for CY claims, which is usually smaller than one. Thus in solvency II the random variable $Z_\bullet - E[Z_\bullet]$ is approximated by the random variable $\Psi - 1$, or in other words, $S_\bullet$ is modelled with a lognormal distribution with mean $E[S_\bullet]$, but with a variance which is slightly different from $\mathrm{Var}(S_\bullet)$.

If $S_\bullet$ was modelled with a lognormal distribution with $E[S_\bullet]$ and $\mathrm{Var}(S_\bullet)$ as in the SST, one would obtain instead of (33)

$$\begin{aligned} SCR_{pr+res} &= \mu_Z V_\bullet \left(\frac{\exp\left(\Phi^{-1}(0.995) \cdot \sqrt{\log(\frac{\varphi^2}{\mu_Z} + 1)}\right)}{\sqrt{\frac{\varphi^2}{\mu_Z} + 1}} - 1 \right) \\ &= V_\bullet \, VaR_{0.995}^{mean}(Z_\bullet), \end{aligned}$$

where

$$\mu_Z = E[Z_\bullet].$$

The following table compares $VaR_\alpha^{mean}(\Psi)$ with $VaR_\alpha^{mean}(Z_\bullet)$ for different values of φ with $\alpha = 99.5\%$ and $\mu_Z = 85\%$.

φ	$VaR_\alpha^{mean}(\Psi)$ (a)	$VaR_\alpha^{mean}(Z_\bullet)$ (b)	ratio (c)=(b)/(a)
1%	0.090	0.091	1.008
2%	0.189	0.192	1.016
3%	0.296	0.303	1.024
4%	0.413	0.426	1.032
5%	0.540	0.562	1.041
6%	0.678	0.711	1.050
7%	0.828	0.877	1.059
8%	0.991	1.059	1.068
9%	1.168	1.259	1.078
10%	1.361	1.480	1.088
11%	1.570	1.724	1.098
12%	1.797	1.991	1.108
13%	2.043	2.286	1.119
14%	2.310	2.609	1.130
15%	2.599	2.965	1.140
16%	2.913	3.355	1.152
17%	3.253	3.782	1.163
18%	3.621	4.251	1.174
19%	4.019	4.765	1.186
20%	4.450	5.328	1.197

3.2.5 Modelling and Aggregation of CAT-Risks

The assumptions for the cat-risks leading to formula (35) are as follows:

Model Assumptions 3.9 (CAT risks solvency II) *The CAT-risks CAT_k, $k = 1, 2, \dots I$, are independent and normally distributed with $VaR_{0.995}(CAT_k) = c_k$.*

Discussion and Remark:
The assumption that the cat-risks are normally distributed follows from the quadratic sum in (35) . Of course, this assumption is not adequate for cat-risks, or in other words, the aggregation formula (35) is a practical, but simplified way to calculate the necessary capital for cat-risks.

3.3 Summary and Comparison of the modelling in the SST and in Solvency II

In this Section we shortly summarise the modelling of the insurance risk in the SST and in solvency II and work out, what is common in the two modelling frameworks and where there are major differences.

First we should note that both, the SST and solvency II are parameter-based models. Whereas solvency II is essentially a factor model, the SST is distribution based and the parameters enter in the calculation of the resulting distribution. From a mathematical point of view, the distribution based procedure is not a big complication and does not need more than techniques which are nowadays well known by people having passed the actuarial exams to become a full actuary.

A first difference is the risk measure. In the SST the 99% expected shortfall is used, whereas in solvency II it is the 99.5% value at risk. As shown in [6], the difference is rather small in the case of a lognormal distribution within the relevant parameter range.

A major difference is the modelling of the variance of the CY (premium) and the PY (reserve) risk. In the SST it is assumed, that the variance is the sum of a parameter risk and a random fluctuation risk, where the first is independent of the size of the company and the latter becomes smaller the bigger the company is. In solvency II this variance is assumed to be independent of the size of the company.

A further difference between the SST and solvency II is that for the CY risks the SST models the claims load caused by the bulk of normal claims and the one caused by the big claims separately, whereas solvency II does not make such a distinction.

Major differences also exist in the assumptions about the correlation matrices. The current SST assumptions are such that there are no correlations for the reserve risks between lines of business and that there is also no correlation between CY- and PY- risks, whereas solvency II assumes a correlation of 50% between the premium and reserve risk of the same line of business and that the correlations between lob are the same for the premium risk and the reserve risk. Neither the assumptions in the SST nor the ones in solvency II are fully satisfactory from a conceptual point of view.

Finally, the scenarios in the SST and the modelling of the CAT-risks in solvency II are somehow similar. However, they are aggregated in a fully different manner. In solvency II the corresponding SCR are just added by quadratic summation. In the SST they are fully integrated in the calculation of the resulting distribution. Here the fundamental difference between the distribution based approach in the SST and the factor-model approach and aggregation of the resulting SCR with a correlation matrix becomes most obvious.

4 Parameter Estimation

The standard parameter values in the STT and in solvency II differ substantially (see appendices A and B). This fact by itself reveals that the estimation of the parameters is a problem on its own. In the following we present estimators of the parameters based on the observations of the individual data of the company. For some of the parameters we also show how the individual estimators can be combined with the standard or market values by credibility techniques.

4.1 Estimation of Parameter in Solvency II

The following straightforward estimator is part of the official solvency II guideline and was already encountered in Section 3.2.1, formula (29).

$$\widehat{\sigma}_i^2 = \frac{1}{n_i - 1} \sum_{j=1}^{n_i} \frac{P_{ij}}{P_{i\bullet}} (X_{ij} - \overline{X_i})^2, \tag{43}$$

where

$$\begin{aligned} X_{ij} &= \frac{C_j^{CY}}{P_{ij}} = \text{ observed loss ratio of lob } i \text{ in year } j, \\ \overline{X_i} &= \sum_{j=1}^{n_i} \frac{P_{ij}}{P_{i\bullet}} X_{ij}, \\ n_i &= \text{ number of historical years taken into account.} \end{aligned}$$

An analogous estimator was suggested in [6] for the reserve risk, namely

$$\widehat{\tau}_i^2 = \frac{1}{n_i - 1} \sum_{j=1}^{n_i} \frac{R_{ij}}{R_{i\bullet}} (Y_{ij} - \overline{Y_i})^2, \tag{44}$$

where

$$\begin{aligned} Y_{ij} &= \frac{\widetilde{R}_{ij}^{PY}}{R_{ij}}, \\ \overline{Y_i} &= \sum_{j=1}^{n_i} \frac{R_{ij}}{R_{i\bullet}} Y_{ij}. \end{aligned}$$

Remarks:

- (43) and (44) are a best unbiased estimate of σ_i^2 and τ_i^2 linear in $(X_{ij} - \overline{X_i})^2$ and $(Y_{ij} - \overline{Y_i})$, respectively, under the assumption that $\mathrm{Var}(X_{ij}) = \sigma_i^2 / P_{ij}$ and $\mathrm{E}[X_{ij}] = \mu_X$ and $\mathrm{Var}(Y_{ij}) = \tau_i^2 / R_{ij}$.
- In practice (44) usually underestimates the reserve risk since despite the reserves R_{ij} being best estimate reserves they often contain some smoothing elements.

The above estimators are simple straightforward estimators. In the next Section we will consider estimators of the parameters used in the SST. However, these parameter estimators could also be used in an internal model in solvency II. At the end, in solvency II as well as in the SST, the aim is to estimate the coefficients of variation of the premium and the reserve risk.

4.2 Estimation of the Parameters in the SST

The SST framework does not yet provide formulas for estimating the parameters from individual data. In the following we are going to suggest such estimators.

The SST distinguishes for the premium risk between normal claims CY and big claims CY. However, if one does not want to make this distinction, the estimators developed in Sections 4.2.1 and 4.2.2 are also valid when applied to the total of CY-claims.

4.2.1 Estimation of the random fluctuation risk normal claims CY

In Section 3.1.1 we have seen that

$$\sigma^2_{i,fluct} = CoVa^2\left(Y_i^{(v)}\right) + 1.$$

Hence we have to estimate the coefficient of variation of the claim sizes. For this purpose we calculate first the observed coefficient of variations for different accident years j, that is

$$\widehat{CoVa}_{ij} = \frac{\frac{1}{N_{ij}-1}\sum_{v=1}^{N_{ij}}\left(Y_{ij}^{(\nu)} - \overline{Y}_i\right)^2}{\overline{Y}_i^2}, \qquad (45)$$

where N_{ij} is the number of normal claims of lob i in accident year j, $Y_{ij}^{(\nu)}, v = 1, 2, \ldots, N_{ij}$, the individual claim amounts and $\overline{Y}_i$ the mean of the $Y_{ij}^{(\nu)}$. However, one should have in mind that the individual claim amounts $Y_{ij}^{(\nu)}$ in recent accident years j consist to a great part of case estimates. It is a known fact that, especially in long tail lines of business, (45) underestimates in recent accident years the true ultimate coefficient of variation. Therefore, the values resulting from (45) should first be extrapolated to their ultimate value. The following table shows the development triangle of the coefficient of variation in motor liability of a big Swiss insurance company.

Development triangle CoVa claim amounts in motor liability

AY/DY	0	1	2	3	4	5	6	7	8	9
1998	6.1	7.1	8.1	8.5	9.1	9.7	10.1	10.2	10.2	10.9
1999	6.1	7.6	8.1	9.0	10.0	10.3	10.6	10.6	11.2	11.2
2000	5.8	6.9	8.4	8.9	9.4	9.5	9.5	10.3	10.3	10.3
2001	5.8	7.4	8.9	9.5	9.6	9.6	10.4	10.7	10.7	10.7
2002	5.7	7.8	9.0	9.0	9.2	9.9	10.3	10.6	10.6	10.6
2003	6.6	8.5	8.8	9.1	9.8	10.2	10.6	10.9	10.9	10.9
2004	6.6	8.5	9.2	10.1	10.7	11.1	11.5	11.9	11.9	11.9
2005	5.2	7.2	8.7	9.2	9.7	10.1	10.5	10.8	10.8	10.8
2006	5.4	7.5	8.5	9.0	9.5	9.9	10.3	10.6	10.6	10.6
2007	5.7	7.4	8.4	8.8	9.4	9.7	10.1	10.4	10.4	10.4
									mean	10.8

The values above the diagonals are the observed empirical coefficients of variation resulting from (45) of accident years i calculated at different development years. The values below the diagonals are simple chain ladder forecasts. We can see from this table that for instance in the development years 0 or 1 the coefficients of variation are substantially underestimated compared to the ultimate values. We can also see that the observed coefficients of variation are rather stable and that the mean of the extrapolated coefficients of variation (column 9) over the different accident years is an accurate estimator for the coefficient of variation of the claim sizes. Indeed, this parameter can usually be estimated with great accuracy from the own data.

4.2.2 Estimation of the parameter risk of the normal claim amount CY

For the parameter risk, most companies in Switzerland use the default standard values provided by the supervision authority. One of the reasons is that there has not yet been developed an estimator for the parameter risk for the own data which is based on a sound actuarial basis. In the following we try to fill this gap and we will present such estimators.

We assume that we have observed historical data on the accident years $j = 1, 2, \dots, J$. We consider a specific lob i. To simplify notation we drop in the following the index i and we write for instance X_j for the observation of the specific lob considered in the year j. It is assumed,

i) that each year is characterised by its characteristics $\mathbf{\Theta}_j = (\Theta_{j1}, \Theta_{j2})^T$ and that for each year the model-assumptions 3.1 are fulfilled with underlying claim frequency parameter λ_j and claim severity parameter μ_j,

ii) that the coefficient of variation for the claim sizes $CoVa\left(Y_j^{(\nu)}\right)$ is the same for all years,

iii) that random variables belonging to different years are independent and $\mathbf{\Theta}_{1,}\mathbf{\Theta}_1, \dots, \mathbf{\Theta}_J$ are independent and identically distributed.

From the above assumptions and from (5), (7), (62), (63), we see that we have the following situation:
The random variables X_j are independent with

$$\begin{aligned} \mathrm{Var}\left(X_j\right) &= \sigma^2_{param} + \frac{\sigma^2_{fluct}}{\nu_j} && (46)\\ &\simeq \sigma^2_{param} + \frac{\widetilde{\sigma}^2_{fluct}}{\widetilde{P}_j}, && (47) \end{aligned}$$

where

$$\begin{aligned} \nu_j &= \text{number of a priori expected claims},\\ \widetilde{P}_j &= E\left[C_j^{CY,n}\right],\\ \sigma^2_{fluct} &= CoVa^2\left(Y^{(v)}\right) + 1, \end{aligned}$$

and with

$$\begin{aligned} E\left[X \mid \mathbf{\Theta}\right] &= \Theta_1\Theta_2,\\ \mathrm{Var}\left(X \mid \mathbf{\Theta}\right) &= \frac{1}{w\lambda} \cdot \Theta_1\Theta_2^2\left(CoVa^2\left(Y\right) + 1\right). \end{aligned}$$

Hence the observations X_j, $j = 1, 2, \ldots, J$, fulfil the conditions of the Bühlmann-Straub model (see for instance [1], Chapter 4). However, we do not have the "standard situation" of the Bühlmann Straub model, where we have a collective of risks $i = 1, 2, \ldots, I$, and where for each of these risks observations $X_{ij}, j = 1, 2, \ldots, n$, over several years are available. Here the risks are the different years $j = 1, 2, \ldots, J$ and for each of these "risks", one has only one observation X_j. In the standard situation of the Bühlmann-Straub model, the observations over several years for each risk are used to estimate the within risk variance component. But here the within risk variance is σ^2_{fluct}, which can be estimated otherwise as described in Section 4.2.1.

Therefore we can use the standard estimators in the Bühlmann-Straub model (see e.g. [1], Chapter 4.8) and obtain

$$\widehat{\sigma}^2_{param} = c \cdot \left\{ \frac{J}{J-1} \sum_{j=1}^{J} \frac{w_j}{w_\bullet} \left(X_j - \overline{X} \right)^2 - \frac{J \, \widehat{\sigma}^2_{fluct}}{n_\bullet} \right\}, \tag{48}$$

where

$$\begin{aligned} c &= \frac{I-1}{I} \left\{ \sum_{i=1}^{I} \frac{w_{i\bullet}}{w_{\bullet\bullet}} \left(1 - \frac{w_{i\bullet}}{w_{\bullet\bullet}} \right) \right\}^{-1}, \\ \widehat{\sigma}^2_{fluct} &= \widehat{CoVa}^2 \left(Y^{(v)} \right) \text{ estimator of the coefficient of} \\ & \text{variation of the claim sizes,} \\ n_\bullet &= \text{observed number of claims.} \end{aligned}$$

Whereas the total claim amounts $C_j^{CY,n}$ are known figures and recorded in the files of a company, this is not the case for the corresponding pure risk premiums $\widetilde{P}_j = E\left[C_j^{CY,n} \right]$. Hence, before being able to apply (48) one has to determine $\widetilde{P}_j$, which are used for calculating the X_j.

Denote by

$$LR_j = \frac{C_j^{CY,n}}{P_j}$$

the observed claims ratio for normal claims in year j, where P_j is the earned premium. Under the assumption that

$$E\left[LR_j \right] = \mu_{LR} \tag{49}$$

is the same for all years, it is suggested to use

$$\widetilde{P}_j = \overline{LR} \cdot P_j$$

where

$$\overline{LR} = \sum_{j=1}^{J} \frac{P_j}{P_{i\bullet}} LR_j.$$

Often (49) is not fulfilled because of things like business cycles and premium policies. Then the premiums have first have to be adjusted by such effects, such that the loss ratios calculated with the adjusted premiums should then fulfil (49).

Finally, we have to decide which weights w_j should be used in (48).We suggest to take the $\widetilde{P}_j$ as weights, which is justified by the variance property (47). A numerical example will be given in the next Section 4.2.3.

The estimator (48) is based on the properties and the variance structure of X_j. It does, however not make use of the fact that according to (7)

$$\sigma^2_{param} \simeq \mathrm{Var}(\Theta_1) + \mathrm{Var}(\Theta_2). \tag{50}$$

An alternative approach to (48) is therefore to estimate $\mathrm{Var}(\Theta_1)$ and $\mathrm{Var}(\Theta_2)$ directly, which then involves the claim frequencies and the claim averages. Hence the following estimators are recommended for lines of business where there is a "natural" volume measure, which is necessary that the claim frequencies are comparable over time. This is the case for personal lines like motor liability, household insurance etc., but not for corporate property, corporate liability or industrial fire insurance.

Let

$$\begin{aligned} F_j &= \frac{N_j}{\nu_j}, \\ \text{where } N_j &= \text{number of claims in year } j, \\ \nu_j &= \text{a priori expected number of claims in year } j. \end{aligned}$$

Note that F_j is a "standardised" frequency with $E[F_j] = 1$. If there are no trends in the claim frequency, that is if

$$E[F_j] = \lambda \text{ for all } j, \tag{51}$$

then one can put

$$\begin{aligned} \nu_j &= \widehat{\lambda} \cdot JR_j, \\ \text{where } JR_j &= \text{number of risks in year } j, \\ \widehat{\lambda} &= \frac{N_\bullet}{JR_\bullet}. \end{aligned}$$

If there are trends, then use $\nu_j = \widehat{\lambda_j} \cdot JR_j$, where $\widehat{\lambda_j}$ is the trend adjusted estimate of the a priori claim frequency in year j.

Under model-assumptions 3.1, where N_j is assumed to be conditionally Poisson, it holds that

$$\mathrm{Var}(F_j) = \mathrm{Var}(\Theta_1) + \frac{\lambda}{\nu_j}$$

and one can show that

$$\widehat{\sigma^2_{\Theta_1}} = \left(c \cdot \frac{\nu_\bullet}{J}\right)^{-1} \left(\frac{V_F}{\overline{F}} - 1\right) \tag{52}$$

where

$$
\begin{aligned}
V_F &= \frac{1}{J-1} \sum_{j=1}^{J} \nu_j \left(F_j - \overline{F}\right)^2, \\
\overline{F} &= \sum_{j=1}^{J} \frac{\nu_j}{\nu_\bullet} F_j, \\
c &= \sum_{j=1}^{J} \frac{\nu_j}{\nu_\bullet} \left(1 - \frac{\nu_j}{\nu_\bullet}\right).
\end{aligned}
$$

is an unbiased estimator of $\mathrm{Var}(\Theta_1)$.

For estimating $\mathrm{Var}(\Theta_2)$, one has to look at the observed claims averages

$$
\overline{Y_j} = \frac{1}{N_j} \sum_{\upsilon=1}^{N_j} Y_j^{(\nu)}
$$

in different years. Because of inflation or possible other trends, one has first to adjust the claim sizes in a given year j by these factors to bring them on the same level, which can be done for instance by linear regression. After these adjustments, the claim sizes and claim averages fulfil the Bühlman-Straub credibility model for claim sizes as presented in Chapter 4.11 of [1] and one can use this theory to estimate $\mathrm{Var}(\Theta_2)$.

4.2.3 Numerical Example

The following table shows the figures in motor liability of a big insurance company in Switzerland. For confidentiality reasons the figures have been multiplied by a constant factor.

year	1	2	3	4	5	6	7	8	9	10
earned premiums	**444'256**	**442'133**	**439'773**	**454'701**	**473'404**	**504'481**	**523'446**	**544'809**	**557'566**	**557'291**
adjusted for business cycle	470'610	492'902	499'742	528'722	530'722	543'037	541'870	544'265	553'141	557'291
number of risks	**606'599**	**602'911**	**592'008**	**597'903**	**605'140**	**599'955**	**596'455**	**598'003**	**592'497**	**595'263**
number of claims	**64037**	**69782**	**66011**	**66894**	**65583**	**65582**	**63328**	**63012**	**60955**	**61397**
claim frequency	10.6%	11.6%	11.2%	11.2%	10.8%	10.9%	10.6%	10.5%	10.3%	10.3%
total claim amount	**443'614**	**492'726**	**485'613**	**489'085**	**480'175**	**511'006**	**478'947**	**452'545**	**476'601**	**473'176**
claims average	6'927	7'061	7'357	7'311	7'322	7'792	7'563	7'182	7'819	7'707
adjusted for trends	7'996	8'013	8'210	8'027	7'909	8'284	7'916	7'402	7'937	7'707
loss ratio	**443'614**	**492'726**	**485'613**	**489'085**	**480'175**	**511'006**	**478'947**	**452'545**	**476'601**	**473'176**
observed	100.1%	99.7%	99.1%	102.5%	106.7%	113.7%	118.0%	122.8%	125.7%	125.6%
with adjusted premiums	106.1%	111.1%	112.7%	119.2%	119.6%	122.4%	122.1%	122.7%	124.7%	125.6%

This data set was one of the many data sets which were evaluated in [5]. In the table below the results for the different estimators are listed, where it was assumed that the number of the a priori expected claims for the CY is 62'000.

Estimator 1 shows the result of the straightforward estimator (43), with and without the business cycle adjustments in the premiums. We see that the difference is quite big between the two results. Indeed, there were quite remarkable business cycles during the last 10 years, such that the risk of the CY claims is overestimated when just using the formula from solvency II and not taking them into account.

Estimator 2 is the resulting estimator when estimating the coefficient of variation with the procedure described in Section 4.2.1 and where the parameter risk is estimated with formula (48). Again the parameter risk is substantially overestimated when not taking into account the business cycles.

Estimator 3 differs from estimator 2 only in the estimation of the parameter risk. Here the variance of Θ_1 and of Θ_2 are estimated by using (52) and the estimator described at the end of Section 4.2. The estimate for $\mathrm{Var}(\Theta_2)$ turned out to be zero. This was the case in nearly all lob investigated in [5], which indicates that the main driver for the parameter risk are random fluctuations from year to year in the claim frequency. Thus it might be a good idea just to concentrate on the claim frequency and to estimate the parameter risk from it. This estimator does not depend on the premiums and hence the results with and without premium adjustments are the same.

The same holds true for estimator 4 where we have just carried through the SST calculations with the SST standard parameters.

In this example , the results in the column "with adjusted premiums" are fairly near to each other, whereas for estimators 1 and 2 there are big differences between the first and the second column.

	with earned premiums	with adjusted premiums
estimator 1	**10.5%**	**5.8%**
$CoVa\left(Y^{(\nu)}\right)$	10.8	10.8
σ_{fluct}	4.4%	4.4%
σ_{param}	10.3%	4.2%
estimator 2	**11.2%**	**6.1%**
σ_{fluct}	4.4%	4.4%
$\sqrt{\mathrm{Var}(\Theta_1)}$	3.8%	3.8%
$\sqrt{\mathrm{Var}(\Theta_2)}$	0%	0%
σ_{param}	3.8%	3.8%
estimator 3	**5.3%**	**5.3%**
$CoVa\left(Y^{(\nu)}\right)$	10	10
$\sqrt{\left(CoVa\left(Y^{(\nu)}\right)^2+1\right)/\nu}$	4.0%	4.0%
σ_{param}	3.5%	3.5%
estimator 4 (with standard parameters SST)	**5.8%**	**5.8%**

4.2.4 Estimation of the Pareto Parameters for Big Claims

We consider again a specific lob i and drop the index i to simplify notation. Assume that $Y_1, Y_2, ...$ are the big claims above a certain threshold c and we assume that they are

Pareto-distributed with Pareto-parameter ϑ.

Assume you have observed n such claims. It is well known that

$$\widehat{\vartheta} = \left(\frac{1}{n-1} \sum_{\nu=1}^{n} \ln\left(\frac{Y_\nu}{c}\right) \right)^{-1} \tag{53}$$

is an unbiased estimator of ϑ with

$$\begin{aligned} E\left[\widehat{\vartheta}\right] &= \vartheta, \\ CoVa\left(\widehat{\vartheta}\right) &= \frac{1}{\sqrt{n-2}}. \end{aligned} \tag{54}$$

The number of observed big claims of an individual company in the considered lob might be rather small. From (54) it is seen that the uncertainty of this estimate then becomes fairly big. Hence, it is desirable to take also into account in some way the standard value of the SST, which can be considered as an estimate gained from industry-wide data.

Indeed, the estimator $\widehat{\vartheta}$ fulfils the Bühlmann-Straub credibility model and we can use credibility techniques to combine the individual experience with the a priori estimate given by the standard value of the SST. The credibility estimator is given by (see [10] or [1], Chapter 4.14)

$$\widehat{\vartheta}^{cred} = \alpha\,\widehat{\vartheta} + (1-\alpha)\,\vartheta_0 \tag{55}$$

where $\widehat{\vartheta}$ is as in (53) and where

$$\begin{aligned} \alpha &= \frac{n-2}{n-1+\kappa}, \\ \vartheta_0 &= \text{standard value from the SST}, \\ \kappa &= CoVa\,(\Theta)^{-2}. \end{aligned}$$

Here $CoVa\,(\Theta)$ denotes the coefficient of variation of the Pareto parameter within the different companies. It could be estimated for instance by the supervision authority from the data of the different companies, or one can assess it in a pure Bayesian way, let's say to assume that it is 25%, which would result in a value of 16 for κ. For instance if your individual estimator is based on 10 observed big claims, then you would give a credibility weight of 32% to your own Pareto-estimate and if your estimator is based on 30 observed big claims, then the credibility weight given to your own estimate would be 62%.

4.3 Estimation of the Reserve Risk

The reserve risk is the same in the SST and in solvency II. Estimator (44) is straightforward, but as already mentioned in Section 4.1, this estimator usually underestimates the reserve risk.

We believe that the reserve risk should be determined based on the techniques used for estimating these reserves. The most known formula for estimating the variance or mean squared error of the reserves is the famous formula of Mack for the chain ladder reserving method (see [8]). However, Mack's formula measures the ultimate reserve risk whereas for solvency purposes we need the one-year reserve risk.

The one-year reserve risk is best understood when we look at the recursive formula for the chain ladder reserving method. This has been done in [2]. The formula derived there coincides with the result found in [9].

In the following, we briefly summarize these results.

Assume that at time I, there is given a development triangle or trapezoid

$$\mathcal{D}_I = \{C_{i,j} : 0 \leq i \leq I,\ 0 \leq j \leq J,\ i+j \leq I\},$$

where $C_{i,j}$ denote cumulative claims payments or cumulative incurred claims of accident year i in development year j and where $C_{i,J}$ is the ultimate claim. Let

$$\begin{aligned}
\widehat{C}_{i,J} &= C_{i,I-i} \prod_{j=I-i}^{J-1} \widehat{f}_j, \\
\widehat{R}_i &= \widehat{C}_{i,J} - C^{paid}_{i,I-i}, \\
\widehat{\sigma}_j^2 &= \frac{1}{I-j-1} \sum_{i=0}^{I-j-1} C_{i,j} \left(F_{i,j} - \widehat{f}_j\right)^2.
\end{aligned} \tag{56}$$

$$\begin{aligned}
F_{ij} &= \frac{C_{i,j+1}}{C_{i,j}}, \\
S_j^{[k]} &= \sum_{i=0}^{k} C_{i,j}, \\
\widehat{f}_j &= \frac{S_{j+1}^{[I-j-1]}}{S_j^{[I-j-1]}},
\end{aligned} \tag{57}$$

Note that the $\widehat{f}_j$ are the well known chain ladder factors, $\widehat{C}_{i,J}$ is the chain ladder forecast for the ultimate claim and $\widehat{R}_i$ is the chain ladder reserve for accident year i.

Then in [4] and [9] the following result has been derived for the one-year reserve risk.

Theorem 4.1 (one-year mse) *The one-year mean square error can be estimated as follows:*

i) single accident year i

$$mse\left(\widehat{R}_i\right) = C_{i,I-i}\Gamma_{I-i} + C_{i,I-i}^2 \Phi_{I-i} \tag{58}$$

where

$$\begin{aligned}
\Gamma_{I-i} &= \widehat{\sigma}_{I-i}^2 \left(1 + \frac{C_{i,I-i}}{S_{I-i}^{[i]}}\right) \prod_{j=I-i+1}^{J-1} \widehat{f}_j, \\
\Phi_{I-i} &= \frac{\widehat{\sigma}_{I-i}^2}{S_{I-i}^{[i]}} \prod_{j=I-i+1}^{J-1} \widehat{f}_j^{\,2},
\end{aligned}$$

ii) all accident years

$$mse\left(\widehat{R}_{\bullet}\right)=\sum_{i=I-J+1}^{I} mse\left(\widehat{R}_i\right)+2\sum_{I-J+1\le i<k\le I}^{I} C_{i,I-i}\widehat{C_{k,I-i}}\left(\Gamma_{I-i}+\Phi_{I-i}\right), \qquad (59)$$

where $\widehat{C_{k,I-i}}$ is the chain ladder forecast of $C_{k,I-i}$.

Remarks:

- Since the reserves $\widehat{R}_i$ are proportional to $C_{i,I-i}$, model assumptions 3.1 (or (14), respectively) are consistent with model assumptions 3.3 (or (58) respectively). In (58) the term $C_{i,I-i}\Gamma_{I-i}$ corresponds to the random fluctuation risk and the term $C_{i,I-i}^2\Phi_{I-i}$ to the parameter risk.
- We believe that the parameter risk for the chain ladder reserving method can be treated in a rigorous mathematical way only in a Bayesian set-up (see[2], [3], [4]). With the Bayesian set-up and taking a non-informative prior the resulting estimates are slightly different from Mack's formula for the ultimate risk or the above formula for the one-year risk. Mack's formula and the above formula are then obtained by a first order Taylor approximation from the corresponding results with the Bayesian approach.

When looking at the reserve risk from a solvency point of view, we are interested in the 100- or 200-year adverse event. What are such events that first come into our mind? There are things like a high inflation or a change in legislation like a decrease of the technical interest rate to calculate the lump sums in motor liability insurance, hence situations with a high adverse calendar year effect. But such events are usually not observed in the triangles and not captured by chain-ladder or similar reserving methods. Hence they are also not reflected by the above estimator of the reserve risk. Thus the question arises whether we do the right thing when looking at the reserve risk from a solvency point of view. The answer is that the above method is adequate for the reserve risk except in extraordinary situations with a huge adverse calendar year effect. Hence, it seems absolutely necessary to complement the reserve risk calculation with a scenario of a hyper-inflation or another adverse situation which might happen. Indeed, nobody knows whether the actual finance crisis will not lead to high inflation in some years.

The following example is taken from [5]. First you find below a development triangle in private-liability of a major Swiss insurance company. For confidentiality reasons the figures are multiplied by a constant factor. The figures above the diagonal are the observed cumulative payments, whereas the figures below the diagonal show the chain ladder forecasts.

development triangle cumulative payments

acc. year \ dev. year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Ultimate
1979	3'670	5'817	6'462	6'671	6'775	7'002	7'034	7'096	7'204	7'326	7'395	7'438	7'468	7'541	7'580	7'677	7'794	7'919	8'047	8'051	8'152	8'152
1980	4'827	7'600	8'274	8'412	9'059	9'197	9'230	9'277	9'331	9'435	9'864	9'621	9'654	9'704	9'743	9'745	9'745	9'748	9'749	9'749	9'749	9'749
1981	7'130	10'852	11'422	12'024	12'320	12'346	12'379	12'426	12'460	12'500	12'508	12'547	12'557	12'624	12'639	12'644	12'647	12'666	12'711	12'713	13'063	13'063
1982	9'244	13'340	13'758	13'853	13'894	13'942	13'980	14'001	14'012	14'070	14'085	14'098	14'111	14'118	14'120	14'133	14'141	14'145	14'166	14'167	14'167	14'167
1983	10'019	14'223	15'403	15'579	15'732	15'921	16'187	16'420	16'531	16'559	16'568	16'585	16'597	16'614	16'617	16'619	16'681	16'681	16'687	16'711	16'748	16'748
1984	9'966	14'599	15'181	15'431	15'506	15'538	15'906	16'014	16'537	16'833	16'951	17'038	17'040	17'195	17'298	17'307	17'308	17'308	17'309	17'310	17'311	17'311
1985	10'441	15'043	15'577	15'784	15'926	16'054	16'087	16'107	16'311	16'366	16'396	16'414	16'419	16'426	16'480	16'480	16'480	16'480	16'482	16'482	16'482	16'482
1986	10'578	15'657	16'352	16'714	17'048	17'289	17'632	17'646	17'662	17'678	17'693	17'700	17'706	17'706	17'712	17'713	17'718	17'719	17'719	17'719	17'719	17'719
1987	11'214	16'482	17'197	17'518	18'345	18'480	18'505	18'520	18'578	18'633	18'671	18'689	18'746	18'772	18'774	18'804	18'804	18'804	18'806	18'806	18'809	18'809
1988	11'442	17'621	18'465	18'693	18'882	18'965	20'464	20'522	20'558	20'607	20'611	21'250	21'257	22'505	22'509	22'516	22'518	22'522	22'523	22'527	22'611	22'611
1989	11'720	17'779	18'655	18'940	19'098	19'368	19'970	20'162	20'195	20'415	20'510	20'594	20'657	20'752	20'823	20'899	20'983	21'124	21'622	21'627	21'708	21'708
1990	13'293	20'689	21'696	22'439	22'798	23'054	23'394	23'554	23'888	24'061	24'096	24'301	24'356	24'389	24'391	24'784	24'784	24'794	24'894	24'900	24'993	24'993
1991	15'063	22'917	23'543	24'032	24'156	24'232	24'360	24'410	24'884	24'896	24'968	25'031	25'172	25'459	25'460	25'470	25'734	25'774	25'877	25'883	25'980	25'980
1992	16'986	23'958	25'090	25'392	25'546	26'099	26'129	26'149	26'166	26'231	26'328	26'743	27'023	27'048	27'075	27'168	27'234	27'276	27'386	27'392	27'494	27'494
1993	16'681	24'867	25'871	26'463	26'941	27'120	27'164	27'183	27'250	27'490	27'497	27'561	27'565	27'582	27'706	27'787	27'855	27'898	28'010	28'017	28'121	28'121
1994	17'595	25'152	26'140	27'090	27'568	27'637	27'805	28'003	28'223	28'240	28'245	28'250	28'257	28'297	28'347	28'430	28'499	28'543	28'658	28'665	28'772	28'772
1995	16'547	25'396	26'506	27'043	27'866	27'882	27'903	27'933	28'492	28'545	28'564	28'686	28'695	28'897	28'948	29'033	29'104	29'149	29'266	29'273	29'382	29'382
1996	15'449	22'702	23'909	24'690	26'083	26'525	26'567	26'640	26'695	26'801	26'814	27'018	27'078	27'269	27'317	27'397	27'463	27'506	27'616	27'623	27'726	27'726
1997	18'043	26'918	28'256	28'569	28'964	29'268	29'344	29'393	30'159	30'936	30'966	31'122	31'190	31'410	31'466	31'558	31'635	31'684	31'811	31'819	31'937	31'937
1998	17'655	26'241	27'369	28'063	28'346	28'780	29'024	29'180	29'250	29'575	29'660	29'809	29'875	30'085	30'138	30'227	30'300	30'347	30'469	30'476	30'590	30'590
1999	16'789	25'547	27'099	27'801	27'919	28'052	30'020	30'035	30'456	30'663	30'750	30'905	30'973	31'192	31'247	31'338	31'415	31'463	31'590	31'597	31'715	31'715
2000	15'538	23'830	25'202	26'462	27'056	27'480	27'564	27'612	27'873	28'062	28'143	28'284	28'347	28'546	28'597	28'681	28'751	28'795	28'911	28'917	29'025	29'025
2001	15'113	23'405	26'822	27'711	28'080	29'203	29'647	29'751	30'032	30'236	30'323	30'475	30'542	30'758	30'812	30'903	30'978	31'025	31'150	31'157	31'274	31'274
2002	14'543	22'674	23'603	24'159	24'242	24'425	24'767	24'854	25'089	25'260	25'332	25'459	25'515	25'695	25'741	25'816	25'879	25'919	26'023	26'029	26'126	26'126
2003	14'590	22'337	23'442	24'031	24'665	24'942	25'292	25'380	25'621	25'794	25'868	25'998	26'056	26'239	26'286	26'363	26'427	26'468	26'574	26'580	26'680	26'680
2004	13'976	21'527	22'615	23'242	23'653	23'918	24'254	24'339	24'569	24'736	24'806	24'931	24'986	25'162	25'207	25'281	25'342	25'381	25'483	25'489	25'585	25'585
2005	12'932	20'118	21'309	21'824	22'209	22'459	22'774	22'853	23'070	23'226	23'293	23'410	23'462	23'627	23'669	23'738	23'796	23'832	23'928	23'934	24'023	24'023
2006	12'538	20'357	21'435	21'953	22'341	22'592	22'909	22'989	23'206	23'364	23'431	23'548	23'600	23'767	23'809	23'879	23'937	23'974	24'070	24'076	24'166	24'166
2007	12'888	19'413	20'441	20'935	21'304	21'544	21'846	21'922	22'130	22'280	22'344	22'456	22'506	22'664	22'704	22'771	22'826	22'861	22'953	22'959	23'045	23'045
																						673'153

The next table shows the resulting chain ladder reserves and the corresponding standard-deviation for each accident year and for the total. The ultimate reserve risk calculated with Mack's formula amounts to 13.3%, whereas the one year reserve risk according to formula (59) reduces to 6.6%.

		ultimate (Mack)		one year	
acc. year	Cl-Reserve	CoVa	sqrt mse	Vko_CDRi	sqrt mse
1979-1987	-	-	-	-	-
1988	84	206.4%	174	206.4%	174
1989	86	198.0%	170	30.3%	26
1990	199	129.2%	257	91.0%	181
1991	246	111.7%	275	36.7%	90
1992	326	92.1%	301	32.7%	107
1993	415	78.9%	328	31.0%	129
1994	475	70.8%	336	14.4%	68
1995	687	73.6%	506	54.9%	377
1996	708	70.0%	495	12.9%	91
1997	971	60.6%	589	26.0%	252
1998	1'015	59.7%	605	19.7%	200
1999	1'259	51.4%	647	16.3%	205
2000	1'413	46.8%	662	17.6%	249
2001	1'627	42.8%	696	7.1%	116
2002	1'701	48.4%	823	31.3%	532
2003	2'015	43.0%	867	12.4%	249
2004	2'343	41.2%	964	16.3%	381
2005	2'714	34.4%	933	9.7%	262
2006	3'809	28.6%	1'088	14.7%	558
2007	10'157	13.1%	1'328	7.9%	801
Total	32'250	13.3%	4'294	6.6%	2'131

We have also looked at the claims development results over the last 10 years of the same lob and then applied the estimator (44) . The resulting reserve risk was only 3.7%, which confirms our belief mentioned in Section 4.1, that the true reserve risk is underestimated by (44).

Finally, we should consider an adverse extraordinary scenario and also take it into account when considering the reserve risk. If we assume that claims inflation increases by 3 %-points (additional inflation to the one already existent in the development triangle) and stays on this level for 10 years, then this would create a reserve loss of 4'401 or 13.6% of the reserves. Inflation scenarios would also affect the reserves of other lob, and its impact should be taken into account as a reserve scenario for the total business of a company.

The observations in the development triangles might vary quite a lot due to random fluctuations, in particular for small and medium sized companies. It would therefore be helpful to know an industry wide payment pattern and a technique how to combine the individual pattern with the industry wide pattern to obtain an optimal estimate. Here we just want to mention that such a technique was presented in [4]. The main idea is to use credibility to obtain credibility estimates of the chain ladder factors, which are then a weighted mean between the industry wide factors and the factors obtained from the company's own triangle.

4.4 On the Estimation of the Correlation Matrices

We believe that we should first think about the reasons why risks of different lob or the premium and reserve-risk of the same lob are correlated. The main reasons for such correlations are calendar year effects affecting the different risks simultaneously.

To understand the impact of diagonal factors let us have a closer look at the reserve risk. However, the same considerations could also be applied to the CY-risk.

Using the same notation as in Section 3.1.2 we find

$$\widetilde{R_i} = Y_i C_i \tag{60}$$

with

$$\begin{aligned} E\left[Y_i\right] &= 1, \\ \operatorname{Var}\left(Y_i\right) &= \tau_i^2. \end{aligned}$$

Assume that (60) reflects the situation before calendar year effect and that the reserves risk of different lob are independent, that is

$$Cov\left(Y_i, Y_j\right) = 0 \text{ for } i \neq j.$$

Add now a diagonal effect Δ independent of $\widetilde{R_i}$ with $E\left[\Delta\right] = 1$ affecting different lob simultaneously (e.g. claims inflation) and denote by

$$\begin{aligned} \widetilde{R_i}^* &= R_i C_i \Delta, \\ C_i^{PY*} &= \widetilde{R_i}^* - R_i, \end{aligned}$$

the a posteriori reserve, respectively the a posteriori claim amount PY after calendar year effect. Then

$$\begin{aligned}
\mathrm{Var}\left(\widetilde{R_i}^*\right) &= R_i^2\left(E\left[Y_i^2\right]E\left[\Delta^2\right]-1\right)\\
&= R_i^2\left\{\left(E\left[Y_i^2\right]-1\right)\left(1+E\left[\Delta^2\right]-1\right)+E\left[\Delta^2\right]-1\right\}\\
&= R_i^2\left\{\tau_i^2+\sigma_\Delta^2+\tau_i^2\sigma_\Delta^2\right\}\\
&\simeq R_i^2\left\{\tau_i^2+\sigma_\Delta^2\right\},
\end{aligned}$$

where in the last equation we have assumed that $\tau_i^2\sigma_\Delta^2 << \tau_i^2+\sigma_\Delta^2$ and where

$$\sigma_\Delta^2 = \mathrm{Var}\left(\Delta\right).$$

Under the assumption that the calendar year effect is also effective for lob j we obtain

$$\begin{aligned}
Cov\left(\widetilde{R_i}^*,\widetilde{R_j}^*\right) &= Cov\left(C_i^{PY^*},C_j^{PY^*}\right)\\
&= R_iR_j\mathrm{Var}\left(\Delta\right),
\end{aligned}$$

$$\begin{aligned}
\mathrm{Corr}\left(\widetilde{R_i}^*,\widetilde{R_j}^*\right) &= \mathrm{Corr}\left(C_i^{PY^*},C_j^{PY^*}\right)\\
&= \frac{\sigma_\Delta^2}{\sqrt{\tau_i^2+\sigma_\Delta^2}\sqrt{\tau_j^2+\sigma_\Delta^2}}.
\end{aligned} \tag{61}$$

(61) is an intuitive and handy formula, which gives quite a good qualitative insight: correlation induced by a calendar year effect becomes smaller the smaller σ_Δ^2 is compared to τ_i^2 and τ_j^2. As an example consider claims inflation. Assume that the yearly standard deviation for claims inflation is 1% and that the reserve risks before the calendar year effect "inflation" for lob i and j are 3%. Then we obtain from (61) that

$$\mathrm{Corr}\left(C_i^{PY^*},C_j^{PY^*}\right) = 11\%.$$

If we increase the calendar year inflation to 3%, the correlation would be 50%. This shows for instance, that the correlation induced by varying inflation is bigger in countries with high inflation than in countries with low inflation. The same considerations can also be applied to other calendar year effects and might help experts to assess the correlation matrices.

Appendices

A Details on the SST

A.1 Lines of Business

The standard SST models splits the business into the following lines of business:

lob	description
1	motor liability
2	motor hull
3	property
4	general liability
5	workers compensation (UVG)
6	corporate accident without UVG
7	corporate health
8	individual health
9	marine
10	aviation
11	credit and surety
12	legal protection
13	others

A.2 Standard Parameters

The supervision authority provided the following standard default parameters for the SST 2008, which could be used by the companies, if they did not have accurate estimates from their own data.

standard default parameters for CY-risks			standard default parameters for PY-risks		
lob	σ_{par}	CoVa (claim size)	lob	τ_{par}	τ_{fluct}
1	3.50%	7.00	1	3.5%	2.5%
2	3.50%	2.50	2	3.5%	20.0%
3	5.00%	5.00	3	3.0%	15.0%
4	3.50%	8.00	4	3.5%	4.0%
5	3.50%	7.50	5	2.0%	4.0%
6	4.75%	4.50	6	3.0%	5.0%
7	5.75%	2.50	7	3.0%	7.0%
8	5.75%	2.25	8	5.0%	0.0%
9	5.00%	6.50	9	5.0%	25.0%
10	5.00%	2.50	10	5.0%	20.0%
11	5.00%	5.00	11	5.0%	15.0%
12	4.50%	2.25	12	5.0%	0.0%
13	4.50%	5.00	13	5.0%	50.0%

Correlation matrix for CY-year risks

lob	1	2	3	4	5	6	7	8	9	10	11	12	13
1	**1**	0.5	0	0.25	0.25	0.25	0	0	0	0	0	0	0
2	0.5	**1**	0.25	0	0	0	0	0	0	0	0	0	0
3	0	0.25	**1**	0.25	0	0	0	0	0	0	0	0	0
4	0.25	0	0.25	**1**	0	0	0	0	0	0	0	0	0
5	0.25	0	0	0	**1**	0.5	0.5	0	0	0	0	0	0
6	0.25	0	0	0	0.5	**1**	0.5	0	0	0	0	0	0
7	0	0	0	0	0.5	0.5	**1**	0.25	0	0	0	0	0
8	0	0	0	0	0	0	0.25	**1**	0	0	0	0	0
9	0	0	0	0	0	0	0	0	**1**	0	0	0	0
10	0	0	0	0	0	0	0	0	0	**1**	0	0	0
11	0	0	0	0	0	0	0	0	0	0	**1**	0	0
12	0	0	0	0	0	0	0	0	0	0	0	**1**	0
13	0	0	0	0	0	0	0	0	0	0	0	0	**1**

big claims CY

lob	Pareto parameter
1	2.50
2	1.85
3	1.40
4	1.80
5	2.00
6	2.00
7	3.00
8	3.00
9	1.50
10	
11	0.75
12	
13	1.50

A.3 Derivation of the Variance of the Normal Claim Amount

We consider a specific lob i. For simplicity reasons we drop the index i and we write S instead of $C^{CY,n}$. As in Section 3.1.1 we also consider

$$X = \frac{S}{\widetilde{P}},$$

where $\widetilde{P} = E[S]$ is the pure risk premium. For any random variable Z, the coefficient of variation is defined by

$$CoVa(Z) = \frac{\sqrt{\operatorname{Var}(Z)}}{E[Z]}.$$

Note that $\operatorname{Var}(X) = CoVa(S)$.

From model assumptions 3.1 follows that, on the condition given $\boldsymbol{\Theta}$,

$$S = \sum_{\nu=1}^{N} Y^{(\nu)},$$

where

$$\begin{aligned} N &\sim Poi(w\lambda\Theta_1), \\ \Theta_1 &= \text{random variable with } E[\Theta_1] = 1, \end{aligned}$$

and where the claim severities $Y^{(\nu)}$, $\nu = 1, 2, \ldots, N$, are independent and have the same distribution as $\Theta_2 Y$ with $E[Y] = \mu$. Hence we have

$$\begin{aligned} E\left[Y^{(\nu)}\middle|\,\boldsymbol{\Theta}\right] &= \Theta_2\mu, \\ \operatorname{Var}\left(Y^{(\nu)}\middle|\,\boldsymbol{\Theta}\right) &= \Theta_2^2 \operatorname{Var}(Y), \\ \Theta_2 &= \text{random variable with } E[\Theta_2] = 1\ . \end{aligned}$$

We denote by $\nu = w \cdot \lambda$ the a priori expected number of claims. Since Θ_1 and Θ_2 are independent we have

$$\begin{aligned} E[S|\,\boldsymbol{\Theta}] &= w\lambda\mu\Theta_1\Theta_2\ , \\ \operatorname{Var}(S|\,\boldsymbol{\Theta}) &= w\lambda\Theta_1 \cdot \left[\Theta_2^2 \cdot \left(\operatorname{Var}(Y) + \mu^2\right)\right]\ , \\ E[X|\,\boldsymbol{\Theta}] &= \Theta_1\Theta_2 && (62) \\ \operatorname{Var}(X|\,\boldsymbol{\Theta}) &= \frac{1}{w\lambda} \cdot \Theta_1\Theta_2^2\left(CoVa^2(Y) + 1\right), && (63) \end{aligned}$$

and

$$\begin{aligned}
E[S] &= w \cdot \lambda \cdot \mu \ , \\
\mathrm{Var}\left(Y^{(\nu)}\right) &= E\left[\Theta_2^2\right] \ \mathrm{Var}\left(Y\right) + \mathrm{Var}\left(\Theta_2^2\right) \ \mu^2 \\
&= E\left[\Theta_2^2\right] \left(\mathrm{Var}\left(Y\right) + \mu^2\right) - \mu^2, \\
\mathrm{Var}[S] &= E\left[\mathrm{Var}\left(S | \boldsymbol{\Theta}\right)\right] + \mathrm{Var}\left(E\left[S | \boldsymbol{\Theta}\right]\right) \\
&= w \cdot \lambda \cdot E\left[\Theta_2^2\right] \cdot \left(\mathrm{Var}\left(Y\right) + \mu^2\right) + \left(w\lambda\mu\right)^2 \cdot \mathrm{Var}\left[\Theta_1 \cdot \Theta_2\right] \\
&= w \cdot \lambda \cdot \left(\mathrm{Var}\left(Y^{(\nu)}\right) + \mu^2\right) + \left(w\lambda\mu\right)^2 \cdot \mathrm{Var}\left[\Theta_1 \cdot \Theta_2\right]. \qquad (64)
\end{aligned}$$

Analogously we obtain by conditioning on Θ_1

$$\begin{aligned}
\mathrm{Var}\left[\Theta_1 \cdot \Theta_2\right] &= E\left[\Theta_1^2\right] \cdot \mathrm{Var}\left(\Theta_2\right) + \mathrm{Var}\left(\Theta_1\right) \\
&= \left(\mathrm{Var}\left(\Theta_1\right) + 1\right) \mathrm{Var}\left(\Theta_2\right) + \mathrm{Var}\left(\Theta_1\right),
\end{aligned}$$

which, inserted into (64) , yields

$$\begin{aligned}
\mathrm{Var}[S] &= \left(w\lambda\mu\right)^2 \left(\mathrm{Var}\left(\Theta_1\right) + \mathrm{Var}\left[\Theta_2\right] + \mathrm{Var}\left(\Theta_1\right) \cdot \mathrm{Var}\left[\Theta_2\right]\right) + w \cdot \lambda \cdot \left[\sigma_Y^2 + \mu^2\right], \\
\mathrm{Var}[X] &= \underbrace{\mathrm{Var}\left[\Theta_1\right] + \mathrm{Var}\left[\Theta_2\right] + \mathrm{Var}\left[\Theta_1\right] \cdot \mathrm{Var}\left[\Theta_2\right]}_{\text{parameter risk}},
\end{aligned}$$

$$\begin{aligned}
CoVa^2(S) &= \underbrace{\mathrm{Var}\left[\Theta_1\right] + \mathrm{Var}\left[\Theta_2\right] + \mathrm{Var}\left[\Theta_1\right] \cdot \mathrm{Var}\left[\Theta_2\right]}_{\text{parameter risk}} \\
&\quad + \underbrace{\frac{1}{w\lambda}\left(CoVa^2\left(Y^{(\nu)}\right) + 1\right)}_{\text{random risk}} \qquad (65) \\
&\simeq \underbrace{\mathrm{Var}\left[\Theta_1\right] + \mathrm{Var}\left[\Theta_2\right]}_{\text{parameter risk}} + \underbrace{\frac{1}{w\lambda_0}\left(CoVa^2\left(Y^{(\nu)}\right) + 1\right)}_{\text{random fluctuation risk}}. \qquad (66)
\end{aligned}$$

(66) follows from the fact that usually $\mathrm{Var}[\Theta_1] \cdot \mathrm{Var}[\Theta_2] << \mathrm{Var}[\Theta_1] + \mathrm{Var}[\Theta_2]$.

B Details on Solvency II

B.1 Lines of Business

Currently solvency II distinguishes between the following lines of business:

lob	description
1	motor, third party liability
2	motor, other classes
3	marine, aviation & transport (MAT)
4	fire and other damage to property
5	third-party liability
6	credit and suretyship
7	legal expenses
8	assistance
9	miscellaneous non-life insurance
10	NP reins property
11	NP reins casualty
12	NP reins MAT

B.2 Parameters

The following parameters were used in QUIS IV for solvency II.

standard parameters

lob	CY-risk (premium) σ	PY-risk (reserve) τ	max historical years m_n
1	9%	12%	15
2	9%	7%	5
3	13%	10%	10
4	10%	10%	5
5	13%	15%	15
6	15%	15%	15
7	5%	10%	5
8	8%	10%	5
9	11%	10%	10
10	15%	15%	5
11	15%	15%	15
12	15%	15%	10

credibility weights α_i for σ^2

historical years available

m_n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
5	0	0	0.64	0.72	0.79	-	-	-	-	-	-	-	-	-	-
10	0	0	0	0	0.64	0.69	0.72	0.74	0.76	0.79	-	-	-	-	-
15	0	0	0	0	0	0	0.64	0.67	0.69	0.71	0.73	0.75	0.76	0.78	0.79

correlation matrix for Z-variables (combined premium and reserve risk)

lob	1	2	3	4	5	6	7	8	9	10	11	12
1	1	0.50	0.50	0.25	0.50	0.25	0.50	0.25	0.50	0.25	0.25	0.25
2	0.50	1	0.25	0.25	0.25	0.25	0.50	0.50	0.50	0.25	0.25	0.50
3	0.50	0.25	1	0.25	0.25	0.25	0.25	0.50	0.50	0.25	0.25	0.50
4	0.25	0.25	0.25	1	0.25	0.25	0.25	0.50	0.50	0.50	0.25	0.50
5	0.50	0.25	0.25	0.25	1	0.50	0.50	0.25	0.50	0.25	0.50	0.25
6	0.25	0.25	0.25	0.25	0.50	1	0.50	0.25	0.50	0.25	0.25	0.25
7	0.50	0.50	0.25	0.25	0.50	0.50	1	0.25	0.50	0.25	0.50	0.25
8	0.25	0.50	0.50	0.50	0.25	0.25	0.25	1	0.50	0.50	0.25	0.25
9	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	1	0.25	0.25	0.50
10	0.25	0.25	0.25	0.50	0.25	0.25	0.25	0.50	0.25	1	0.25	0.25
11	0.25	0.25	0.25	0.25	0.50	0.25	0.50	0.25	0.25	0.25	1	0.25
12	0.25	0.50	0.50	0.50	0.25	0.25	0.25	0.25	0.50	0.25	0.25	1

References

[1] Bühlmann, H., Gisler, A. (2005). A Course in Credibility Theory and its Applications. Universitext, Springer Verlag.

[2] Bühlmann, H., De Felice M., Gisler, A., Moriconi F., Wüthrich, M.V. (forthcoming). Recursive Credibility Formula for Chain Ladder Factors and the Claims Development Result. ASTIN Bulletin

[3] Gisler, A., Wüthrich, M.V. (2008). The estimation error in the chain-ladder reserving method: a Bayesian approach. ASTIN Bulletin 36/2, 554-565.

[4] Gisler, A. (2006). Credibility for the chain-ladder reserving method.38/2, 565-600.

[5] Gradenwitz O. (2009). Das Versicherungstechnische Risiko im SST: Parameterschätzung, Reserverisiko und Vergleiche mit Solvency II, Master Thesis, ETH-Zürich.

[6] Hürlimann W. (2008). On the non-life solvency II model. ASTIN Colloquium Manchester.

[7] Luder T. (2005). Swiss solvency test in non-life insurance. ASTIN Colloquium Zürich.

[8] Mack, T. (1993). Distribution-free calculation of the standard error of chain ladder reserve estimates. ASTIN Bulletin 23/2, 213–225.

[9] Merz M., Wüthrich M.V. (2008). Modelling the claims development result for solvency purposes. CAS Forum, Fall 2008, 542-568.

[10] Rytgaard, M. (1990). Estimation in the Pareto Distribution. ASTIN Bulletin, 20, 201-216

Alois Gisler
Appointed Actuary Non-Life
AXA Winterthur Insurance Company
General Guisan-Strasse 40
CH - 8401 Winterthur
Switzerland