The use of multi-year internal models for management decisions in multi-year risk management

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Abstract

Increasing natural catastrophes, recent dynamics in capital markets, and fundamental changes in regulatory requirements (Solvency II for European Union member countries) have placed increasing challenges on management strategy in insurance companies. As a result many companies are developing modern management techniques such as *value and risk-based management*.

While in the Solvency II framework the time horizon is one year, the strategic risk-return profile of the insurer should be set according to multi-year calculations. In this context multi-year internal models are essential. In the actual literature several questions concerning the use of internal models in a multi-year management context are not answered to date. The aim of this paper is to give a helpful contribution to this emerging and important field of research. So in this paper we present a multi-year model approach to quantify the risk-return situation of one calendar year or several years by assuming a multi-year risk-capital concept that can be used as a general condition in strategic corporate management. This multi-year approach can help management to answer the essential question: How many years of catastrophe risks or extreme developments at the capital markets can the company economically withstand at a certain confidence level without needing external capital sources?

We give an example of strategic risk-adjusted performance management in practice. The aim of this paper is not only grounded in academic research, but also of high importance for insurance practice. The study wants to give a realistic and helpful idea of management processes in order to define a suitable balance between reinsurance cover and asset allocation. We close with the presentation of the iterative strategic management process in order to demonstrate and encourage the use of internal models in strategic corporate management as a basis for decision-making.

Keywords: Multi-year internal models, multi-year risk capital, Solvency II, parameter risk, economic value added, return on risk adjusted capital, value and risk-based management

Contents

1. Introduction

Increasing natural catastrophes, recent dynamics in capital markets, and fundamental changes in regulatory requirements (Solvency II for European Union member countries) have placed increasing challenges on management strategy in insurance companies. Suitable structure in insurance portfolio together with asset allocation matched to insurance cash flows has become a major task for management directed towards maximum return in relation to the risk taken for capital invested. As a result many companies are developing modern management techniques such as *value and risk-based management*.

Internal risk models can play an important part in supporting management decisions in a risk-return-oriented strategy. Increasing transparency in the risk situation, identification of high-risk factors, and identification of segments that generate or decrease shareholder value are essential for generating a strategic value and risk-based management approach aimed towards a long-term and sustainable increase in shareholder value. Discussions at European level in the context of Solvency II have also emphasised the importance of internal risk models in supporting management decision-making processes. These models should also be used in a company's "Own Risk and Solvency Assessment" (ORSA).[1]

While in the Solvency II framework the time horizon is one year, the strategic risk-return profile of the insurer should be set according to multi-year calculations. In this context multi-year internal models are essential. In the actual literature several questions concerning the use of internal models in a multi-year management context are not answered to date.[2] The aim of this paper is to give a helpful contribution to this emerging and important field of research. So in this paper we present a multi-year model approach to quantify the risk-return situation of one calendar year or several years by assuming a multi-year risk-capital concept that can be used as a general condition in strategic corporate management. This multi-year approach can help management to answer the essential question: How many years of catastrophe risks or extreme developments at the capital markets can the company economically withstand at a certain confidence level without needing external capital sources? So multi-year risk

[1] See *CEIOPS* (2008).
[2] In *Diers* (2008b) a model approach of a multi-year model is presented.

quantification can serve as a helpful management technique in the new context of ORSA (Section 2).

Only high-quality internal models optimally reflecting the risk situation facing the company allow insurers to assess the level of risk capital required according to the corporate risk structure. This importantly involves measuring and evaluating all relevant risks the insurers are exposed to. Prediction risk is a major risk that needs to be quantified using internal models, which can be divided into parameter risk and process risk. While the standard formula (Solvency II) and internal models both usually take account of process and parameter risks in modelling reserve risk, parameter risk is often omitted in premium risk, thus only taking process risk into account, although there have been discussed some methods of modelling parameter uncertainty in literature.[3] In Section 3 we will be quantifying the effects of including parameter uncertainty in premium risk using example data in order to raise the awareness of the importance of these risks.

It is one of company management's major responsibilities to minimise shortfall probability by adequate underwriting policy, sufficient reinsurance cover and suitable asset allocation strategy. In this context future product development will be oriented towards goals such as required risk-capital and use of diversification potential. Instruments such as deductibles for policyholders in storm insurance aimed towards reducing risk-capital requirement are gaining in importance. Strategies should be selected in such a way as to fulfil the requirements on risk-capital coverage with economic capital while achieving the highest possible return. So for example management has to decide which strategy might improve the risk and return situation of a company if not enough risk capital is available – changing the asset allocation by lowering share quota, lowering risk via introduction of deductibles for the policyholders in storm insurance policies, extending reinsurance cover, or any suitable combination of these or other strategies.

We used a very detailed and fully developed internal risk model to examine the effectiveness of management strategies on corporate performance indicators such as EVA (economic value added) and RoRAC (return on risk-adjusted capital).[4] With this model we try to give a very realistic analysis, because claim distributions in non-life insurance with fat tails (e.g. catastrophe claims) have a special influence on the results of the company and on the diversification effects which we show in this study. We give an example of strategic risk-adjusted performance management in practice. The aim of this paper is not only grounded in academic research, but also of high importance for insurance practice. The study wants to give a realistic and helpful idea of management processes in order to define a suitable balance between reinsurance cover and asset allocation. We close with the presentation of the iterative strategic management process in order to demonstrate and encourage the use of internal models in strategic corporate management as a basis for decision-making.

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[3] See for example *Cairns* (2000).

[4] We used the internal model presented in *Diers* (2007a), which is extended by the modelling aspects described in Sections 2 and 3 of this article.

2. Multi-year model approach

2.1 Results by calendar year

We have based this model design of an internal simulation model in non-life insurance by modelling strategically important insurance segments and asset classes according to economic principles, and simulating the results with reference to the underlying dependencies.[5] An economic result, $EcRes_t$, projection for a future calendar year, t, may be written by the change in economic capital, $EcCap$, within the calendar year considered:[6]

$$(1) \qquad EcRes_t = EcCap_t - EcCap_{t-1}$$

$$= EcResLiab_t + EcResAs_t - O_t - A_t,$$

where:

$EcResLiab_t$ = net insurance result at time t,
$EcResAs_t$ = investment result at time t,
O_t = result from operational risk at time t,
A_t = tax at time t.[7]

The net insurance result $EcResLiab_t$ consists of the underwriting result of the simulated accident year t and the development result of all former accident years up to t. The selected risk measure ρ can now be applied to the random variable $EcRes_1$ in order to determine the one-year risk capital, such as the tail value at risk $TVaR$ at the (1–α) percentile:[8]

$$(2) \qquad \rho(EcRes_1) = TVAR_\alpha(EcRes_1).$$

2.2 Multi-year risk capital

Usually, management will require that extreme risks such as natural catastrophe claims and large claims be viewed from a perspective spanning several years, so that the following question can be answered: How much risk capital does a company currently provide to maintain a certain confidence level to ensure its status as a going concern for another five years, i.e. taking five future underwriting years into account, without needing external capital sources?

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[5] See *Diers* (2007a) for detail. Refer to *Diers* (2008b) for multi-year models covered in this section.

[6] Economic capital is defined as the difference between the market value of assets and liabilities (best estimate plus market value margin). We have taken a somewhat simplified view, ignoring for example other assets and liabilities. We refer to net earnings before dividend payout.

[7] See *Diers* (2007a) for tax modelling.

[8] Tail-Value-at-Risk of a random variable of loss L is defined as $TVaR_\alpha(L) = E[L \mid L \geq VaR_\alpha(L)]$ at confidence level 1-α, $\alpha \in (0,1)$, with value at risk VaR. Tail value at risk is a coherent risk measure for random variables with continuous distribution. We will not be discussing the advantages and disadvantages of risk measures such as value at risk and tail value at risk here, but this discussion is necessary – see for example *Koryciorz* (2004), *Pfeifer* (2004b) and *Rootzén / Klüppelberg* (1999).

To address issues of this nature, we defined a "multi-year" risk-capital concept taking n, $n \in IN$, future years into account and referring to the random *MaxLoss* variable defined as follows:

(3) $$MaxLoss(n) = \underset{1 \le t \le n}{MAXIMUM}\{KumLoss_t\}, \quad \text{where}$$

$$KumLoss_1 = -EcRes_1 \cdot (1 + r_f)^{-1} \qquad \text{and}$$

$$KumLoss_t = KumLoss_{t-1} - EcRes_t \cdot (1 + r_f)^{-t},\ 1 \le t \le n.$$

All of the risks are discounted at a risk-free interest rate of r_f to the beginning of the simulation at t=0 in order to create a uniform view of all of the years simulated. *MaxLoss* represents the maximum of the amount that needs to be covered over the years for each simulation. This amount needs to be provided at t=0 in the simulation path to allow the insurance company to cover all losses incurred over the entire period simulated (n years) without external capital supply in this simulation path.

The selected risk measure, ρ, can now be applied to the *MaxLoss*: $\Omega \rightarrow$ IR in order to determine the risk-capital requirement, e.g. tail value at risk (TVaR):

(4) $$\rho(MaxLoss(n)) = TVaR_{\alpha n}\,(MaxLoss(n)).$$

The confidence level 1-αn may decrease with increasing values of n. By definition, the multi-year risk capital is always at least as high as the one-year risk capital for values of $\alpha n = \alpha 1 = \alpha$. If the insurance company can cover its multi-year risk capital with its own economic capital at t=0, $EcCap_0$, the following will apply:

(5) $$EcCap_0 \ge \rho(MaxLoss(n)).$$

The company can therefore cover all losses that may be incurred over the simulation period without external capital supply at a probability of more than $1-\alpha n$.[9]

The multi-year risk-capital concept is therefore suitable as a *strict constraint* aimed towards addressing strategic issues in an "Own Risk and Solvency Assessment" (ORSA). Fig. 1 shows the structure of an internal risk model.[10]

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[9] Note that optimum equity level with optimum company division strategy should be determined by shareholder value, see *Gründl / Schmeiser* (2002).

[10] See *Diers* (2007a).

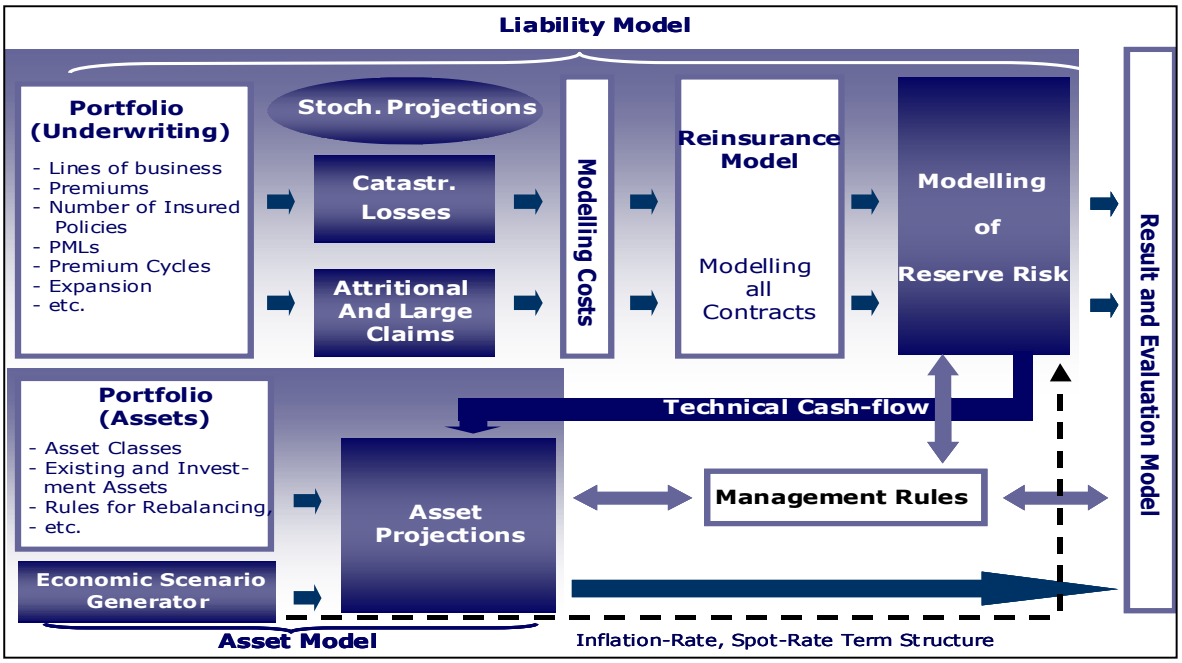

Fig. 1: Structure of an internal DFA risk model

3. Parameter risk in premium risk – A quantitative study

Modelling in internal models is based on a kind of prediction process. That means for example that we try to predict future claim losses based on observations from previous years. This may give rise to different sources of uncertainty: model uncertainty, prediction uncertainty, which can be divided into parameter uncertainty and process uncertainty. Process uncertainty describes the uncertainty from the actual random process. Parameter uncertainty, on the other hand, results from the uncertainty in estimating the parameters from the model.

While modelling reserve risk, for example, process and parameter risk are considered in both the standard formula (Solvency II) and in stochastic modelling in internal models (Mack's model, Bayesian methods, bootstrap methods etc.). By contrast in premium risk parameter risk is often ignored, leaving only the process risk modelled.

Some methods of modelling parameter uncertainty have been covered in the literature (see for example *Cairns* (2000) and *Mata* (2000)). We will be referring to two of these in the following – bootstrap and Bayesian, as described by *Borowicz/Norman* (2006a) – in modelling premium risk and give a quantification of parameter risk for one line of business. In the case study (Section 4) we use a fully developed internal risk model of an example company in order to quantify the effect of parameter uncertainty in insurance results for the example company. Our aim is raise the awareness of the importance of these risks.

Sources of parameter risk

This section will begin with a brief explanation of parameter risk and its origin. In internal models parametric distributions are fitted to historical claims severity and frequency distributions for prediction of future claims. Assume Y as random variable and consider y as

realisations of *Y*. Assume that the distribution class is already known, that is, that the distribution of *Y* has been fully specified except for an unknown parameter. If this class is referred to as $\Gamma = \{F_\theta : \theta \in \Re^d\}$, element F_{θ_0} remains to be determined in the distribution family as corresponds to the distribution of *Y*. Since parameter θ_0 is unknown, it will have to be estimated from the finite set of observations. Traditionally, the parameter is estimated as a deterministic value, for example using the method of moments or maximum likelihood estimation. But if you take two finite samples from the identical distribution, the maximum likelihood estimates for the parameters will differ. So the estimator itself is only a realisation of a random variable. This results in parameter risk.

Modelling parameter risk using bootstrap methods

First we use the bootstrap method to quantify parameter risk. The bootstrap method is based on the idea of "sampling with replacement" from the original sample, which is known in discrete probability theory in connection with basic urn models. The assumption is that the observations are independent and identically distributed.

Our aim is to explain the parameter risk calculation using an example for large claim modelling.[11] We start with modelling claim numbers of large claims. Fig. 2 shows an example record for ultimate claim numbers of large claims from the last n=10 years in an example segment.[12]

Year	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007
Number of Large Claims	2	5	4	6	7	4	6	9	6	3

Fig. 2: Claim frequencies observed

Applying the bootstrap method we obtain a data set $y^s = (y_1^s, ..., y_n^s)$ by sampling with replacement from the observed data. Repeating this procedure can create a multitude of data sets (e.g. 10,000 scenarios *s*).

For modelling the number of claims here we use the Poisson distribution as the process distribution.[13] Parameter λ^s of the Poisson distribution is estimated from each simulated data record y^s. This approach yields the empirical bootstrap distribution (see Fig. 3).

For generating predictions for claim numbers we have to include process risk. So we draw a single claim number z^s from a Poisson distribution with parameter λ^s for each scenario *s* (e.g. 10,000) and obtain the empirical claim frequency distribution including parameter and process uncertainty (see Fig. 4).

Applying the bootstrap process is highly "time-intensive." Another disadvantage is that the

[11] See also *McLennan/Murphy* (2004).
[12] €250,000 has been set as threshold for large claims.
[13] We will desist from using statistical methods for selecting a suitable model for modelling claim frequencies here (e.g. Poisson or negative binomial).

parameters can only be simulated within a limited range set by observations, because we sample from the observed data.

Modelling parameter risk using a Bayesian approach

One alternative in modelling parameter risk is the use of a Bayesian approach as described in *Borowicz/Norman* (2006a).

Consider y as observations from random variable Y with unknown parameters. Let $\theta = (\theta_1,...,\theta_m)$ be the random parameter vector of the distribution of Y. With some initial knowledge about the distribution of the unknown parameters $p(\theta)$, namely the likelihood and the prior distribution, one can produce a posterior distribution $p(\theta \mid y)$ using Bayes' Theorem:

(6) $$p(\theta \mid y) = \text{constant} \cdot \text{likelihood} \cdot \text{prior}.$$

Under uniform prior distributions for parameter vector θ the posterior distribution (based on observations y) is simply proportional to the likelihood:[14]

(7) $$p(\theta \mid y) \propto L(\theta \mid y).$$

We return to the example segment with the observed claim numbers $y = (y_1, \ldots, y_{10})$ from Fig. 2. Assume that as above the Poisson distribution is the underlying process distribution. In this case, the posterior distribution of the parameter $\theta = \lambda$ is exactly solvable. With uniform prior it can be shown that the density of the corresponding posterior distribution can be represented using the density from gamma distribution:[15]

(8) $$p(\theta \mid y) = f(\theta;\beta;\gamma) = \frac{\theta^{\beta-1}\exp(-\frac{\theta}{\gamma})}{\Gamma(\beta)\gamma^{\beta}}, \quad \text{where } \Gamma(x) = \int_0^{\infty} t^{x-1}e^{-t}dt,$$

with parameters

$$\beta = \sum_{i=1}^{n} y_i + 1 \quad \text{and} \quad \gamma = \frac{1}{n}.$$

For other choices of frequency distribution, it may not be possible to write the posterior distribution in such a form. In these cases one can use Gibbs/ARMS sampling methods.[16]

Fig. 3 shows a comparison for the percentile graphs of the parameters simulated using the two methods, bootstrap and Bayesian. The Bayesian approach leads to a greater volatility in the parameters.

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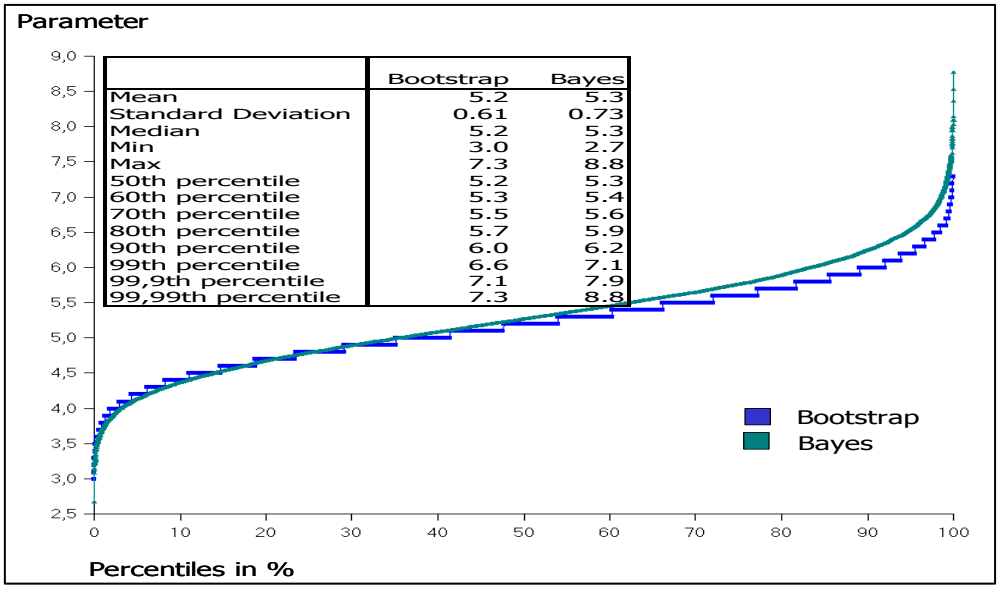
[14] See *Borowicz/Norman* (2006a).
[15] See *Borowicz/Norman* (2006a).
[16] See *Gilks/Richardson/Spiegelhalter* (1995) and *Gilks/Best/Tan* (1994).



	Bootstrap	Bayes
Mean	5.2	5.3
Standard Deviation	0.61	0.73
Median	5.2	5.3
Min	3.0	2.7
Max	7.3	8.8
50th percentile	5.2	5.3
60th percentile	5.3	5.4
70th percentile	5.5	5.6
80th percentile	5.7	5.9
90th percentile	6.0	6.2
99th percentile	6.6	7.1
99,9th percentile	7.1	7.9
99,99th percentile	7.3	8.8

Fig. 3: Percentile graphs for the parameters simulated in large claim frequencies (Bootstrap and Bayesian approach)

In the next step we model the predictive distribution for the number of large claims for quantifying prediction risk. The predictive distribution is simulated by first simulating the Poisson parameter λ^s from the gamma distribution in (8) per simulation s, and then for each simulation s we draw a single number z^s from a Poisson distribution with parameter λ^s. So we obtain the empirical claim frequency distribution including parameter and process uncertainty. In this case the underlying Poisson model allows prediction distribution representation as negative binomial distribution with the following parameters:

$$m = \sum_{i=1}^{n} y_i + 1 \qquad \text{and} \qquad q = \frac{1}{n+1}.$$ [17]

Fig. 4 shows some of the percentiles from the simulated large claim frequencies including and excluding parameter uncertainty (bootstrap and Bayes). Greater volatility in large claim frequencies emerges where parameter uncertainty is included.

	Process Risk	Prediction Risk (Bootstrap)	Prediction Risk (Bayes)
Mean	5.2	5.2	5.3
Standard Deviation	2.27	2.35	2.44
Median	5.0	5.0	5.0
Min	0.0	0.0	0.0
Max	16.0	17.0	17.0
50th percentile	5.0	5.0	5.0
60th percentile	6.0	6.0	6.0
70th percentile	6.0	6.0	6.0
80th percentile	7.0	7.0	7.0
90th percentile	8.0	8.0	9.0
99th percentile	11.0	11.0	12.0
99,9th percentile	13.0	14.0	15.0
99,99th percentile	16.0	17.0	17.0

Fig. 4: Percentiles for process risk and prediction risk (bootstrap and Bayesian approach)

[17] The convention is such that the expected value of $NegBin(m,q)$ is $mq/(1-q)$ and the variance is $mq/(1-q)^2$.

After including parameter uncertainty in modelling large claim frequencies, the next step is to include this in modelling large claim severity. We have sixty-five large claims for the last ten years as observations (ultimates, suitably indexed). Assume that a translated gamma distribution G*amma*(β,γ) has been fitted to large claims observed using the usual statistical methods, with parameters β = 1.18 and γ = 263,998.6 estimated using the maximum likelihood method (threshold €250,000).

Using the log-likelihood function of the gamma distribution and the sampling methods described in *Gilks/Richardson/Spiegelhalter* (1995), the parameter distribution (Fig. 5) can be simulated as shown in *Borowicz/Norman* (2006a), where the negative dependencies between parameters β and γ remain included as shown in Fig. 6.[18]

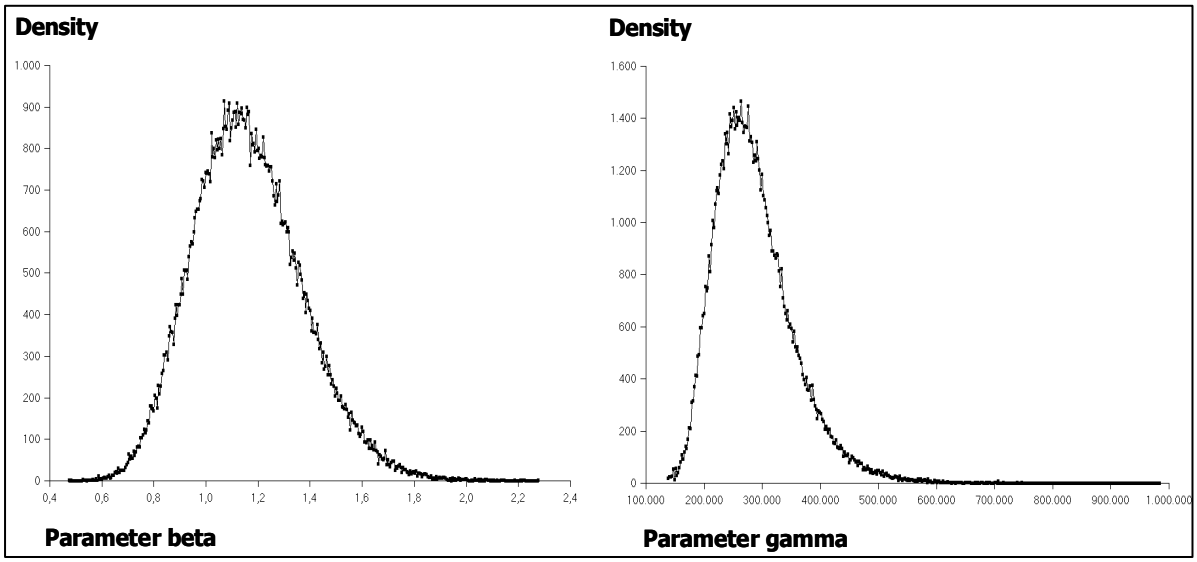


Fig. 5: Simulated frequency distributions for the two parameters, beta and gamma

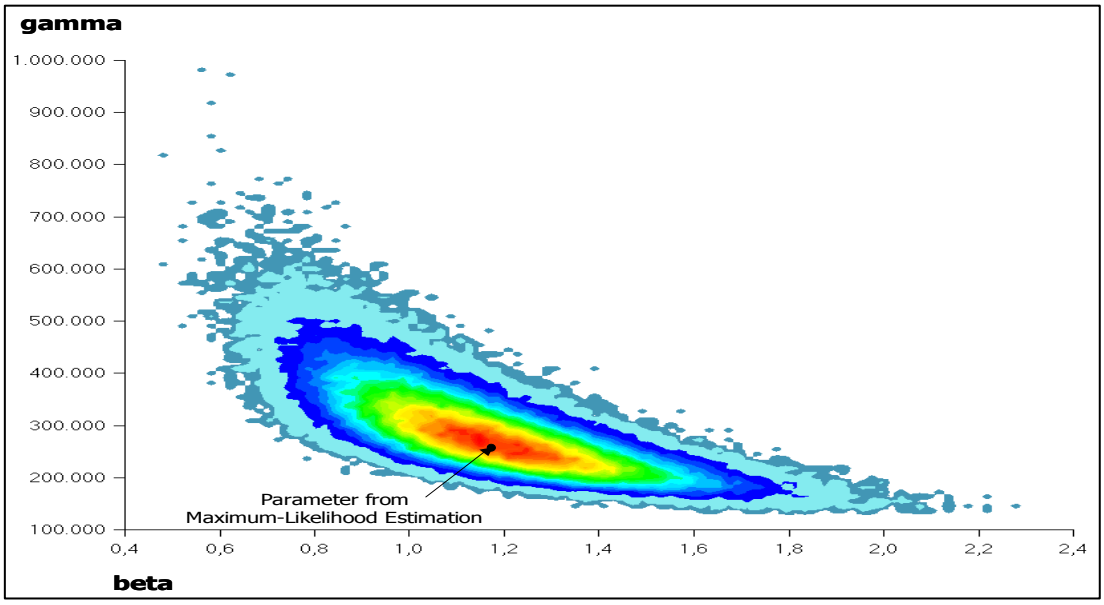
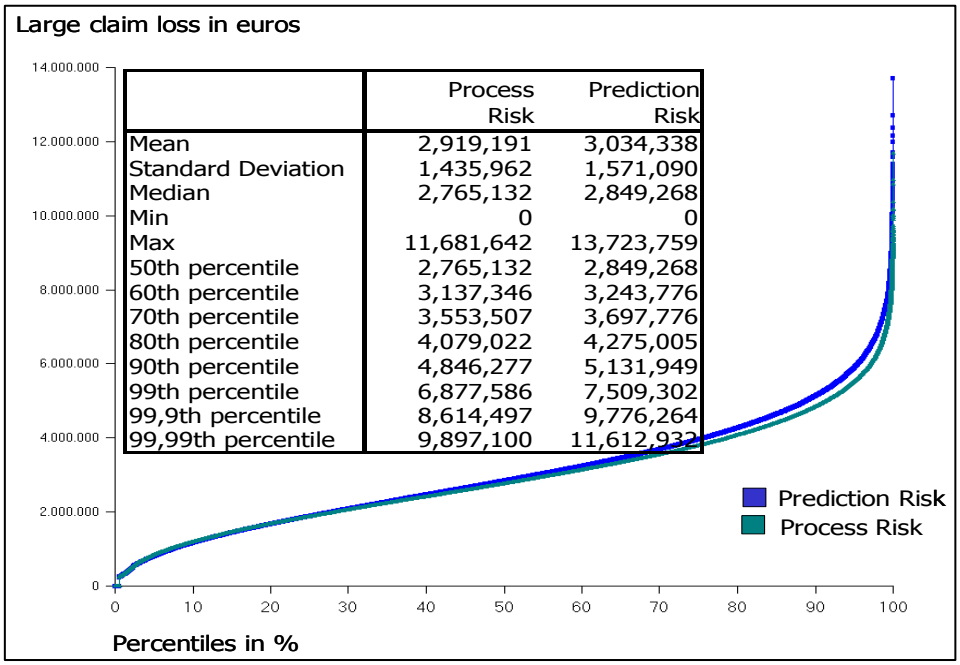


Fig. 6: Dependency structure between the parameters simulated (beta and gamma) and parameter estimated using the maximum-likelihood method

[18] See *Borowicz/Norman* (2005).

After separately modelling large claim frequency und large claim severity (both including parameter uncertainty), the next step is to simulate large claim losses (Fig. 7), assuming that the claim severities are independent and identically distributed random variables and that they are independent of claim number, thus fulfilling the condition of a collective model.[19]



	Process Risk	Prediction Risk
Mean	2,919,191	3,034,338
Standard Deviation	1,435,962	1,571,090
Median	2,765,132	2,849,268
Min	0	0
Max	11,681,642	13,723,759
50th percentile	2,765,132	2,849,268
60th percentile	3,137,346	3,243,776
70th percentile	3,553,507	3,697,776
80th percentile	4,079,022	4,275,005
90th percentile	4,846,277	5,131,949
99th percentile	6,877,586	7,509,302
99,9th percentile	8,614,497	9,776,264
99,99th percentile	9,897,100	11,612,932

Fig. 7: Percentile graph of large claim losses in the example segment including parameter risk (prediction risk) and exclusive parameter risk (process risk) (100,000 simulations)

The final step is to quantify the risk-capital requirement for large claim loss in the example segment including and excluding the parameter risk. Assume a TVaR as risk measure at confidence level at 99.8%.[20] While risk-capital requirement amounts to €5.8 million for process risk, including parameter risk we need a risk capital of €6.7 million.

The extent of parameter uncertainty depends strongly on the data volume observed. Few observations will usually lead to substantially higher parameter uncertainty compared to observed data based on a long history with lots of observations.

To quantify these effects the following example is based on a segment with only five large claims as observed data instead of sixty-five in the above example. We have also applied gamma distribution in this example for modelling claim severities. Fig. 8 shows the percentile graphs for large claim severity including and excluding parameter uncertainty compared to historical data.

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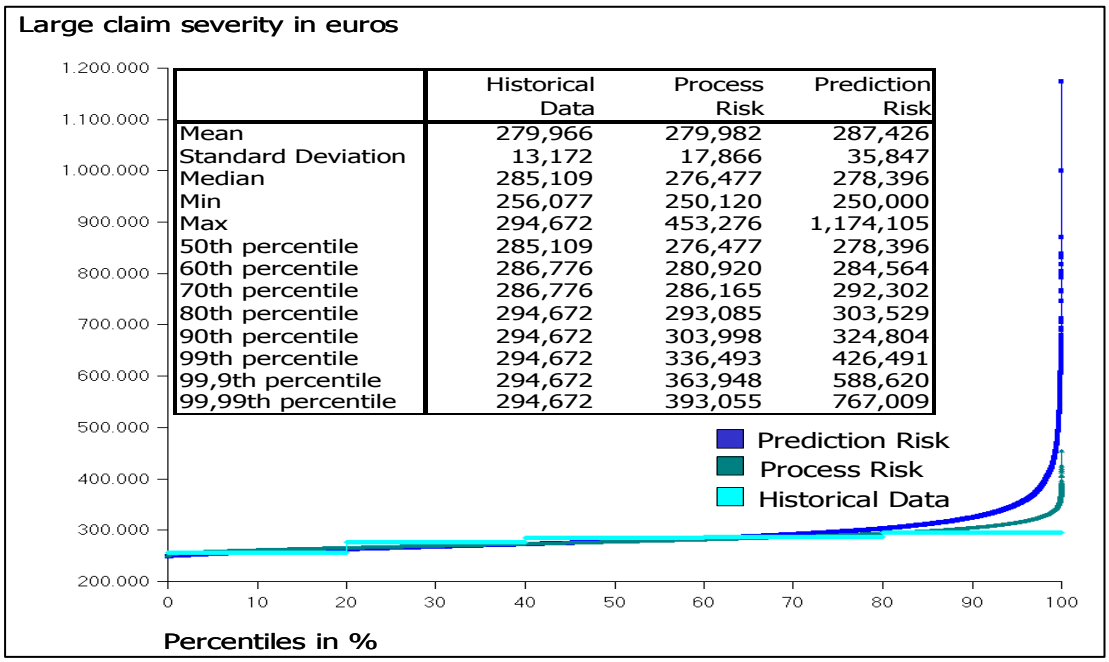
[19] See *Diers* (2007a) for modelling large claim losses following the collective model.

[20] This study's aim does not extend to discussing the advantages and disadvantages of the TVaR risk value. See for example the critical comments of Rootzén/Klüppelberg (1999) and *Pfeifer* (2004b) for reference to disaster risks. We define risk capital requirement on the random variable E(L)-L, where L represents the random variable of large claim loss with expectation value E(L). Value at risk at a high confidence level of 1-α is defined at the (1–α) quantile for the F_L loss distribution of loss L: $VaR_\alpha(L) := Q_{1-\alpha}(L) = \inf\left[x \in IR : F_L(x) \geq 1-\alpha\right]$, with real numbers IR. Tail value at risk is defined as: $TVaR_\alpha(L) = E[L \mid L \geq VaR_\alpha(L)] = VaR_\alpha(L) + E[L - VaR_\alpha(L) \mid L \geq VaR_\alpha(L)]$, where E refers to the expectation value, which is conditional in this case.

The percentile graph for large claim severity including parameter uncertainty (prediction risk) shows substantially higher values in the tail due to the low observation number compared to where parameter uncertainty is ignored (Fig. 8). This means that ignoring the parameter risk can lead to a substantial underestimation of the risk situation, especially in segments with only few observed data.



	Historical Data	Process Risk	Prediction Risk
Mean	279,966	279,982	287,426
Standard Deviation	13,172	17,866	35,847
Median	285,109	276,477	278,396
Min	256,077	250,120	250,000
Max	294,672	453,276	1,174,105
50th percentile	285,109	276,477	278,396
60th percentile	286,776	280,920	284,564
70th percentile	286,776	286,165	292,302
80th percentile	294,672	293,085	303,529
90th percentile	294,672	303,998	324,804
99th percentile	294,672	336,493	426,491
99,9th percentile	294,672	363,948	588,620
99,99th percentile	294,672	393,055	767,009

Fig. 8: Percentile graphs for large claim severity in a segment with a very sparse claim history (100,000 simulations)

These examples also show that ignoring parameter risk – even in a history of sixty-five claims – can lead to a sizable underestimation in risk-capital requirement. However, note that we have modelled large claims from one line of business, and that these effects may occur in modelling attritional, large and catastrophe claims from all lines of business modelled. In addition a parameter uncertainty in dependency structures should be considered.[21]

To conclude, we have demonstrated that parameter risk in premium risk may have a substantial effect on the company's risk situation, and therefore constitutes a risk that should be taken into account to a sufficient degree.

4. Case study – Strategic management decisions in risk management

Risk-adjusted performance strategy aims towards maximising the risk-return situation with constraints such as complying with risk-capital requirements as well as accounting regulations and supervisory regulations.[22] A host of restrictions such as cross-selling, cross-cancellation and price-sales function effects makes solving this optimisation process difficult; we therefore refer to improvement in risk-return situation rather than optimisation in the whole company.

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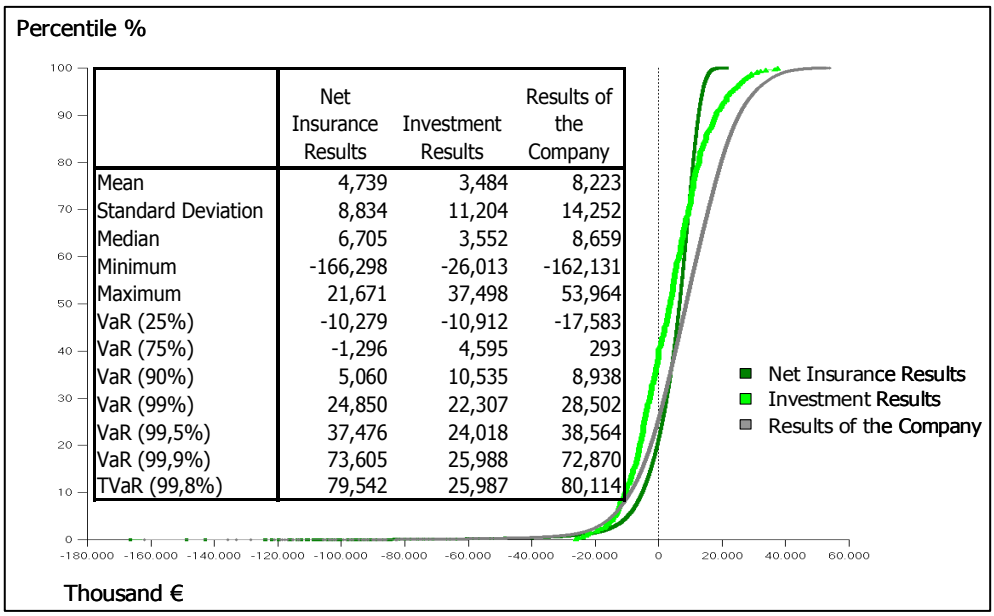
[21] See *Borowicz/Norman* (2006b). This views premium risk in an example. Parameter uncertainty can be found in a similar form in other risk categories

[22] Maximising risk-adjusted performance indicators and maximising company value are only compatible under certain circumstances; cf. *Gründl/Schmeiser* (2002).

The aim of the following simulation study is to examine the effect of various management strategies on performance indicators in order to identify those strategies with the most positive effect. Here we use a detailed and fully developed internal risk model from an example company as reference. See *Diers* (2007a) for the mathematical description of the internal model used here. We additionally modelled parameter uncertainty in premium risk using the Bayesian approach (see Section 3).

Our example company mainly underwrites insurance policies with private and low industrial businesses. Management has based its strategic decisions on a five-year planning period. We have assumed that no major change in underwriting and reinsurance policy or asset allocation will take place during this period, and an annual extension of the business (except storm) of 2% in the number of policies. In the first simulated year t=1 our company has hedging strategies for investment results, in the following years no hedging strategies are provided. We have not included any multi-year management rules in order to make the results easier to follow.[23] Our assumption in this model is that the claims – including those from natural catastrophes – will be independent in future accident years (concerning the different simulated future accident years). Fig. 9 shows the simulated distributions of net insurance results, investment results and the total results of the example company.



	Net Insurance Results	Investment Results	Results of the Company
Mean	4,739	3,484	8,223
Standard Deviation	8,834	11,204	14,252
Median	6,705	3,552	8,659
Minimum	-166,298	-26,013	-162,131
Maximum	21,671	37,498	53,964
VaR (25%)	-10,279	-10,912	-17,583
VaR (75%)	-1,296	4,595	293
VaR (90%)	5,060	10,535	8,938
VaR (99%)	24,850	22,307	28,502
VaR (99,5%)	37,476	24,018	38,564
VaR (99,9%)	73,605	25,988	72,870
TVaR (99,8%)	79,542	25,987	80,114

Fig. 9: Simulated distribution of net insurance results (after reinsurance), investment results and total

Using TVaR at confidence level 99.8% as risk measure, risk capital for net insurance results amounts to 80.1 Mio. € (see Fig. 9), without considering parameter uncertainty it is 73.9 Mio. €. So in the following we include parameter uncertainty in premium risk.

The company possesses economic capital of €100 million at t=0, and the market value of liabilities (including market value margin) amounts to €200 million. The risk-return indicators shown in Fig. 10 have been calculated according to current strategy. We have calculated the one-year and the five-year risk capital using *TVaR* as risk measure at an internal confidence level of 99.8% in the following.

[23] Except the necessary management strategies according to the asset model such as asset allocation, rebalancing, sales priorities etc.

In the following, we have taken the one-year *return on risk-adjusted capital* as risk-adjusted performance indicator as follows:

$$RoRAC = \frac{Expected\ \mathrm{Re}\,sult}{RiskCapital} = \frac{E(Ec\mathrm{Re}\,s_1)}{\rho(-Ec\mathrm{Re}\,s_1)} \tag{9}$$

The risk-adjusted *economic value added* is defined as:

$$EVA\_ra = E(EcRes_1) - r_{Cap} \cdot \rho(-EcRes_1), \tag{10}$$

while setting $r_{Cap} = 0.1$.[24]

RoRAC is only useful as an indicator for a positive expected result and risk capital. In this case, the statements $RoRAC > r_{Cap}$ and $EVA\_ra > 0$ are equivalent.

The company shows a slightly positive *EVA_ra* value of €0.2 million, which means that the company is just covering its capital cost rate taken to be 10% here. The one-year risk capital of €80.1 million is covered by economic capital of €100 million; the coverage rate is 125%.[25] However, the company has an internal capital requirement to survive for five future years at the confidence level given above without needing external capital sources. Thus for the actual strategy, the company will need economic capital resources at a level of €113 million that means €13 million more than the actual economic capital of the company (t=0). Strategies need to be defined in order to lower the five-year risk capital for condition $EcCap_0 \geq \rho(MaxLoss(5))$ to be fulfilled. The strategy to be selected should be the strategy that yields the highest *EVA_ra* given this condition.

In Thousand Euros			Net Insurance Results	Investment Results	Results of the Company	Diversification Benefit
Actual Strategy	Return	Mean	4,739	3,484	8,223	
						25,415
	Risk	Risk Capital	79,542	25,987	80,114	24,1%
	Risk-Adjusted Performance Measures	*RoRAC*	6.0%	13.4%	10.3%	
		EVA_ra			212	
	Risk Capital (5 Years)				113,021	
	Shortfall	One Year	$P(KumLoss(1) > EcCap_0)$		0.024%	
		Five Years	$P(\mathrm{Max}Loss(5) > EcCap_0)$		0.15%	

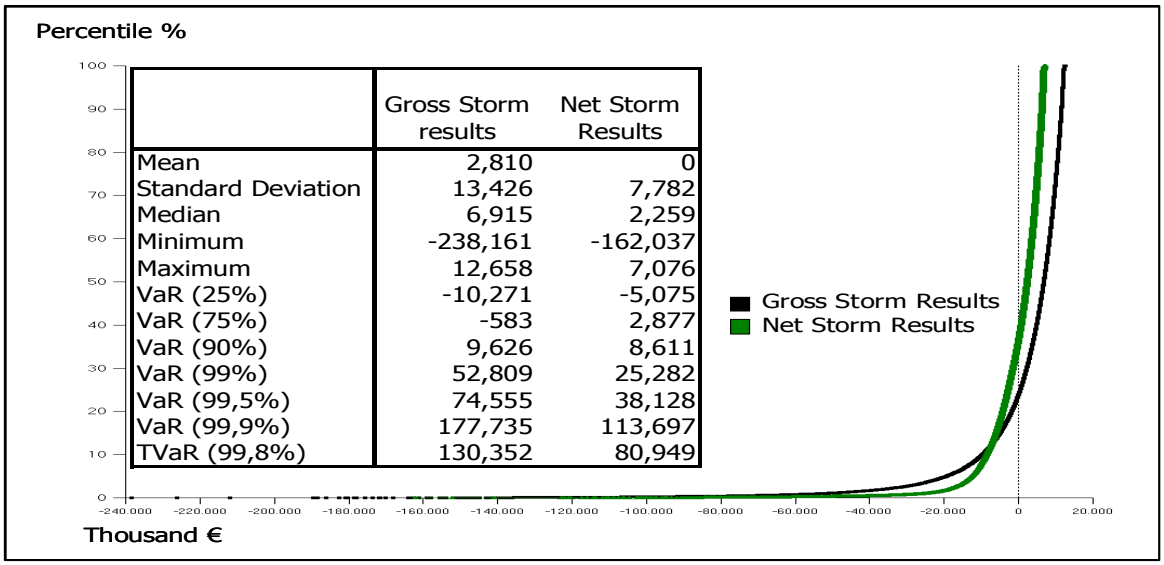
Fig. 10: Risk and return indicators for the company

Since storm is a high-risk factor for the company, the effects of various reinsurance strategies for the storm segment and, alternatively, the introduction of deductibles on the risk-return situation of the company should be quantified. The company currently has a reinsurance quota

[24] We have set capital costs of 10% and only refer to this for risk capital, resulting in a risk-adjusted *EVA_ra*. The non-risk interest rate referring to economic capital and liabilities is deducted from the investment results; the non-risk interest rate provides a benchmark for capital investors. Non-risk interest on liabilities is added to the insurance result; non-risk interest on economic capital is entered in a separate position, which we will not be considered here; see *Diers* (2008b).

[25] To make matters easier, we have omitted operational risks.

share agreement of 20% in the storm segment, followed by an event XL (excess-of-loss contract per event) at a priority of €10 million and reinsurers recoveries limited to €20 million with regard to storm events (liability limit).[26] The agreement includes one reinstatement. Fig. 11 shows the simulated distribution for the underwriting results before and after reinsurance.



	Gross Storm results	Net Storm Results
Mean	2,810	0
Standard Deviation	13,426	7,782
Median	6,915	2,259
Minimum	-238,161	-162,037
Maximum	12,658	7,076
VaR (25%)	-10,271	-5,075
VaR (75%)	-583	2,877
VaR (90%)	9,626	8,611
VaR (99%)	52,809	25,282
VaR (99,5%)	74,555	38,128
VaR (99,9%)	177,735	113,697
TVaR (99,8%)	130,352	80,949

Fig. 11: Distribution function for underwriting storm results (gross and net) in the accident year

The following will examine three strategies of increased reinsurance cover. In all strategies we have the actual quota share agreement of 20% in the storm segment, followed by an event XL at a priority of €10 million with alternative liability limits for the reinsurer of €40 million (Strategy 1), €80 million (Strategy 2), and €125 million (Strategy 3). The reinsurance premiums for the alternative reinsurance agreements have been calculated using technical pricing.[27] As a further alternative, introduction of deductibles of €250 (Strategy 4) and €500 (Strategy 5) in the storm segment are considered while completely desisting from reinsurance protection. Fig. 14 shows the effects of Strategies 1 – 5 on the company's risk-return situation.

Strategies 1 to 3 test liability limit in the event excess-of-loss agreement at various levels. Gradually increasing the reinsurance level slightly increases the *RoRAC* for net insurance results compared to the original scenario (Fig. 9 and 10) from 6.0% in the actual strategy up to 6.3% in Strategy 3. Here our company benefits from diversification effects of the reinsurer. This effect together with the increase in diversification effects (related to risk capital) concerning net insurance results and investment results lead to a gradually increase in total *RoRAC* from 10.3% to 14.1%. *EVA_ra* also gradually increases from Strategy 1 to 3. The increase of diversification effects (between net insurance results and investment results) are based on the declining dominance of storm risks in risk capital.

Figure 12 shows how catastrophe claims (here storm) dominates the tail of company's results. While the worst net insurance results occur together with the worst results for the company, the other results are relatively independent, showing an extremely high tail dependency. This

[26] This means that the reinsured has to incur €10 million before any reinsurance recovery can be made. Priority and liability limits have been reduced by the quota given.

[27] We used tail value at risk pricing with a confidence level of 99.8% and a required risk-adjusted return of 5.5% on the risk capital for the reinsurer (plus costs). The return expectation of the reinsurer has been set to this level (and not to 10%) for the sake of simplicity in taking diversification in the reinsurer's risk capital into account, as the return expected by the reinsurer refers to the undiversified risk capital. See for example *Diers* (2007a).

is caused by the domination of storm risks in the tail of the net insurance results and company's results, which will lead to a domination of storm risks in the company's risk capital. Figure 13 shows the effects of Strategy 3 where the dominance of storm risks is declining leading to an increasing diversification effect between net insurance results and investment results. This example clarifies the importance of the use of diversification benefit in management strategy.

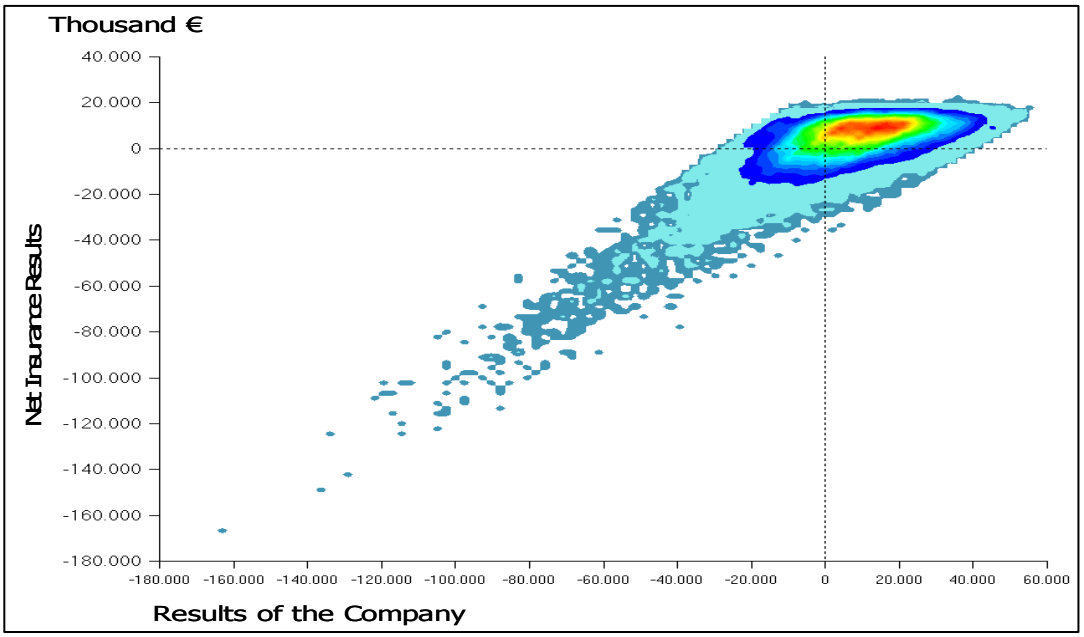


Fig. 12 Simulated net insurance results versus results of the company (actual strategy)

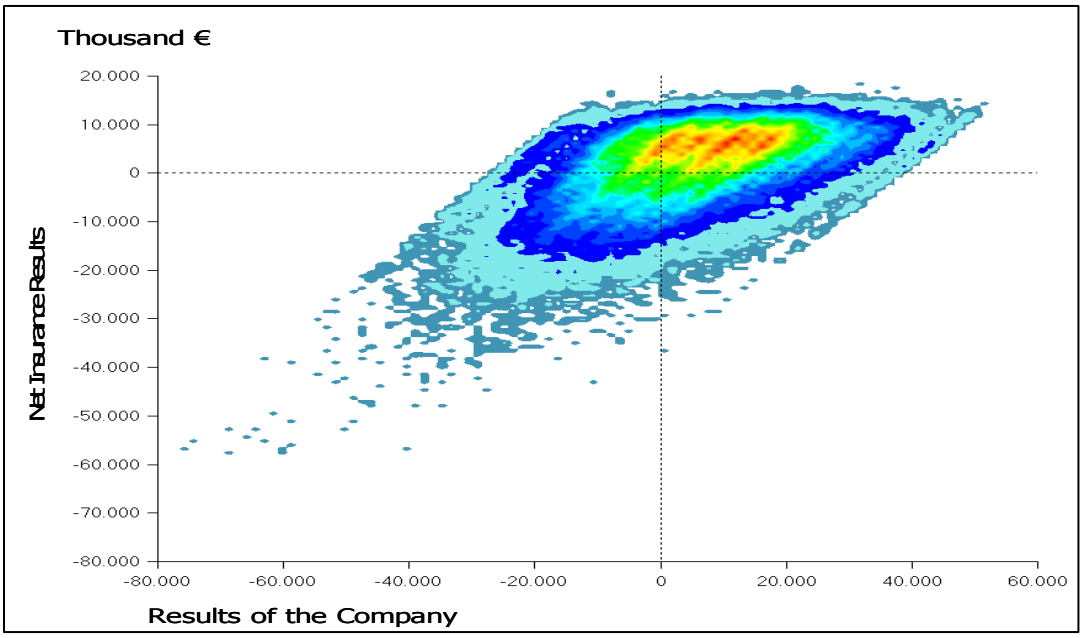


Fig. 13 Simulated insurance results in storm versus results of the company (Strategy 3)

The five-year risk-capital requirement is fulfilled in Strategies 1 to 3 since the requirement is less than €100 million in each case. The one-year and five-year shortfall probability – that is, the probability that the economic capital resources will fall short of the losses to be covered at

any time during the individual year or the five-year period – significantly decreases in Strategies 1 to 3 compared to the original scenario.

In Thousand Euros			Net Insurance Results	Investment Results	Results of the Company	Diversification Benefit
Strategy 1	Return	Mean	3,684	3,484	7,168	
	Risk	Risk Capital	60,399	25,987	61,019	25,367
	Risk-Adjusted Performance Measures	*RoRAC*	6.1%	13.4%	11.7%	29%
		EVA_ra			1,066	
	Risk Capital (5 Years)				94,200	
	Shortfall	One Year	$P(KumLoss(1) > EcCap_0)$		0.004%	
		Five Years	$P(MaxLoss(5) > EcCap_0)$		0.05%	
Strategy 2	Return	Mean	2,459	3,484	5,943	
	Risk	Risk Capital	39,661	25,987	43,918	21,729
	Risk-Adjusted Performance Measures	*RoRAC*	6.2%	13.4%	13.5%	33%
		EVA_ra			1,551	
	Risk Capital (5 Years)				77,513	
	Shortfall	One Year	$P(KumLoss(1) > EcCap_0)$		0.000%	
		Five Years	$P(MaxLoss(5) > EcCap_0)$		0.010%	
Strategy 3	Return	Mean	2,084	3,484	5,568	
	Risk	Risk Capital	33,346	25,987	39,516	19,817
	Risk-Adjusted Performance Measures	*RoRAC*	6.3%	13.4%	14.1%	33%
		EVA_ra			1,616	
	Risk Capital (5 Years)				75,146	
	Shortfall	One Year	$P(KumLoss(1) > EcCap_0)$		0.000%	
		Five Years	$P(MaxLoss(5) > EcCap_0)$		0.005%	
Strategy 4	Return	Mean	6,763	3,484	10,246	
	Risk	Risk Capital	94,700	25,987	95,297	25,390
	Risk-Adjusted Performance Measures	*RoRAC*	7.1%	13.4%	10.8%	21%
		EVA_ra			717	
	Risk Capital (5 Years)				127,213	
	Shortfall	One Year	$P(KumLoss(1) > EcCap_0)$		0.047%	
		Five Years	$P(MaxLoss(5) > EcCap_0)$		0.290%	
Strategy 5	Return	Mean	6,237	3,484	9,721	
	Risk	Risk Capital	69,692	25,987	70,302	25,377
	Risk-Adjusted Performance Measures	*RoRAC*	8.9%	13.4%	13.8%	27%
		EVA_ra			2,690	
	Risk Capital (5 Years)				96,374	
	Shortfall	One Year	$P(KumLoss(1) > EcCap_0)$		0.0096%	
		Five Years	$P(MaxLoss(5) > EcCap_0)$		0.053%	

Fig. 14: Risk-return indicators of the company depending on various insurance strategies

Strategies 4 and 5 test the effect of introduction of deductibles at €250 (Strategy 4) and €500 (Strategy 5) in storm insurance, where the reduction in claims and regulation costs due to the deductibles is passed on to the client by premium adjustments. Moreover in this case the storm segment completely desists from reinsurance protection. Introducing deductibles results in decreasing the one-year risk-capital requirement for insurance results – dominated by storm risk – by 27% with €250 deductibles and 47% with €500 deductibles.[28] This significant reduction in risk capital is a result of the enormous number of small claims that occur due to storm events.[29] While *RoRAC* in storm insurance results before and after introduction of deductibles remains virtually unchanged, the total insurance *RoRAC* substantially increases in

[28] The company has a risk capital requirement for gross insurance results before deductibles of €130.6 million.

[29] We have omitted here the effects of cancellation and cross cancellation by clients, in the case that they do not accept deductibles.

Strategies 5 and 6 compared to the gross insurance *RoRAC* at 5.8%.[30] This results from increasing diversification effects within the insurance business (related to insurance risk capital) because of the reduced domination of storm risks in risk capital.

Compared to Strategies 1 to 3, Strategy 4 shows substantially lower diversification effects between insurance and investment in relation to total risk capital (21%), leading to a lower *RoRAC* and *EVA_ra* for the whole company. At the higher level of €500 deductibles (Strategy 5) we see the highest *EVA_ra* of all of the strategies presented. While the total *RoRAC* slightly lies below *RoRAC* of Strategy 3 (14.1%) the *EVA_ra* is at €2.7 million the highest of all scenarios. This effect results from the substantially lower one-year risk capital involved in Strategy 3 compared to Strategy 5, which therefore leads to a considerable lower return (in absolute values). Whereas the requirements of management for five-year risk-capital cannot be fulfilled in Strategy 4, in Strategy 5 the five-year capital requirement is fulfilled.

Since Strategies 2 and 3 show a very favourable total *RoRAC* while remaining well below the risk limit for the five-year risk-capital requirement, both of the strategies are combined with an investment strategy in order to increase the total return. So share quota is increased from its current level of 15% to 25% (see Fig. 15).

In Thousand Euros			Net Insurance Results	Investment Results	Results of the Company	Diversifi-cation Benefit
Strategy 7:	Return	Mean	2,459	5,074	7,533	
	Risk	Risk Capital	39,661	34,399	47,991	26,069
Strategy 2 and Asset Strategy	Risk-Adjusted Performance Measures	*RoRAC*	6.2%	14.8%	15.7%	35%
		EVA_ra			2,734	
	Risk Capital (5 Years)				96,621	
	Shortfall	One Year	$P(KumLoss(1) > EcCap_0)$		0.000%	
		Five Years	$P(MaxLoss(5) > EcCap_0)$		0.051%	
Strategy 8:	Return	Mean	2,084	5,074	7,158	
	Risk	Risk Capital	33,346	34,399	46,389	21,357
Strategy 3 and Asset Strategy	Risk-Adjusted Performance Measures	*RoRAC*	6.3%	14.8%	15.4%	32%
		EVA_ra			2,519	
	Risk Capital (5 Years)				95,595	
	Shortfall	One Year	$P(KumLoss(1) > EcCap_0)$		0.000%	
		Five Years	$P(MaxLoss(5) > EcCap_0)$		0.046%	

Fig. 15: Risk-return indicators for the company in Strategies 2 and 3 with an additional increase in share quota (asset strategy)

Increasing the share quota raises the *RoRAC* value in investment results from 13.4% to 14.8%, thus also leading to an increase in the total *RoRAC* in both Strategies 7 und 8. In the same way, *EVA_ra* also increases in the two scenarios. A better result places Strategy 7 (highest *EVA_ra* and *RoRAC* of all strategies). This example shows that a strategy which is "optimal" for one segment is not always "optimal" for the company: Strategies 7 and 8 have the same investment strategies and differing insurance strategies. While Strategy 8 leads to a higher *RoRAC* for net insurance than Strategy 7, Strategy 7 leads to a higher total *RoRAC* of 15.7%.

[30] The company shows a gross insurance *RoRAC* level of 5.8% (expected insurance result: €7.5 million, risk capital: €130.6 million).

Reason for this is the higher diversification effect between net insurance results and investment results in Strategy 7 (35%). So management has to set the right incentives in strategic management of lines of business concerning the situation of the company as a whole where the use of diversification benefits plays an important role.[31]

How would management decide in our case study? The decision should be made between Strategies 5 and 7 with nearly the same *EVA_ra*. Both strategies fulfil the risk-capital requirements. The two strategies show completely different diversification effects between the simulated future years. In contrast to Strategy 5, Strategy 7 shows a one-year risk capital requirement of almost a half of the requirement for five-year risk capital. Reason for this is that we do not have hedging strategies for investment results from simulated year $t>1$. So the increased share quota leads to a high increase in multi-year risk capital. Because of the higher *RoRAC* in Strategy 5 the company achieves nearly the same *EVA_ra* as in Strategy 7 in spite of lower risk capital invested. So the management decision for Strategies 5 or 7 will depend on factors including a further risk limit for the one-year risk capital requirement, which may be exceeded in Strategy 5.

In practice for management decisions additional aspects should be taken into account, for example the policyholders' acceptance of deductibles (marketing aspects). Moreover in our model we used technical pricing methods for calculating reinsurance premiums which does not always represent the real conditions because market premiums can differ very much from technical premiums.

With this case study we wanted to give an idea of the way how internal models can be used as an important base for management decisions in a several year context. The example data should demonstrate the effect of management strategies on key indicators for management such as one- or multi-year risk capital or the one-year risk adjusted return such as *EVA_ra*.

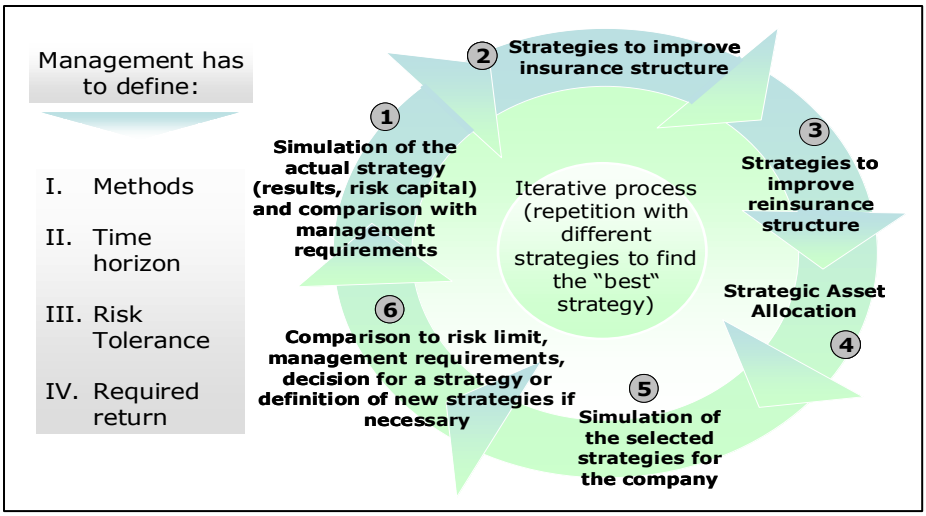


Fig. 16: Iterative process of strategic risk adjusted performance management[32]

In our case study we chose the best strategy with the highest EVA (company) out of a variety of potential strategies, fulfilling the risk capital requirements. Figure 16 shows the underlying

[31] In Diers (2008c) an example for such "right" incentives is given.
[32] See *Diers* (2008b).

iterative process used in our example company in order to encourage the use of internal models as a basis for decision-making in strategic risk-adjusted performance management.

5. Conclusion

Management requires the use of internal models as a base for strategic decisions in modern management techniques such as *value and risk-based management*. Moreover the supervisors will accept internal models for calculation of risk capital for Solvency II purposes only if insurance companies pass the use test (amongst other requirements). Strategic management decisions should be based on time periods spanning several years. In the actual literature several questions concerning the use of internal models in a multi-year management context are not answered to date. This article's aim was to describe a multi-year modelling approach where one-year risks as well as multi-year risks can be quantified and used for multi-year strategic management.

In this context fully developed internal risk models, where the essential risks of the company are taken into account, can play an important part in supporting management decisions in a risk-return-oriented strategy. While in modelling reserve risk usually process and parameter risks are considered, parameter risk is often omitted in premium risk, although there exists some literature on this subject. Thus only process risk is modelled. In this paper we quantified the effects of including parameter uncertainty in premium risk using example data in order to raise the awareness of the importance of these risks.

Management has the important strategic task of minimising shortfall probability by adequate product development, reinsurance protection and asset allocation. Strategies should be selected in such a way as to fulfil the requirements on risk-capital coverage with economic capital while achieving the highest possible return (economic value added, *EVA*). One goal is to ensure effective risk diversification, which is hardly possible without the help of internal models. This study has examined specific management strategies in a five-year time period using a fully developed multi-year risk model. In our example company, on the one hand introducing deductibles of €500 in storm insurance and on the other hand combining increased reinsurance protection and increased share quota led to favourable strategies with respect to risk-adjusted *EVA*.

The strategies and their impact need to be analysed at corporate level at large, where favourable diversification effects have a highly positive effect on total risk capital and *EVA*. In the example portfolio presented in the case study the introduction of deductibles led to growing diversification effects in insurance results and increasing share quota led to growing diversification effects between insurance and investment results.

However, the effect of a strategy on key performance and risk indicators such as *RoRAC* and *EVA* depends largely on current portfolio. In practice further aspects (e.g. marketing) have to be taken into account while deciding about an adequate strategy. So this article's aim was to give an example and an incentive in how to use multi-year internal models in strategic company management as a decision basis. So we close with a general description of the iterative process of risk adjusted management.

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