# HIERARCHICAL STRUCTURES IN THE AGGREGATION OF PREMIUM RISK FOR INSURANCE UNDERWRITING


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**Abstract**

In the valuation of the Solvency II Capital Requirement, the correct appraisal of risk interdependencies acquires particular importance. These interdependencies refer to the recognition of risk diversification in the aggregation process. There are different levels of aggregation and hence different kind of diversification: the first level aggregates the stand-alone line of business, the second level regards the aggregation of different kind of risks like market and underwriting, a third level could be the aggregation of different entity in a group.

Solvency II allows companies to capture these diversification effects but the identification of a proper method can represent a delicate issue. In fact, while Internal Models permit to obtain the portfolio multivariate distribution in independence case, only the use of copula functions can consent to obtain the same multivariate distribution under dependence assumptions.

However the choice of the copula and the parameter estimation could be very problematic when only few data are available. So it could be useful to find a closed formula based on Internal Models independence results with the aim to obtain the Capital Requirement under dependence assumption too. QIS Aggregation Formula represents a first simple way to do that, but it could underestimate the diversification effect.

In this paper we present an alternative method, based on the idea (proposed by Sandstrom) to correct QIS Aggregation Formula with proper calibration factors and opportunely extended with the aim to consider very skewed distribution too.

In the last part we compare the Capital Requirements obtained, for only premium risk, applying the aggregation formula to different non-life multi-line insurers, with the results derived by elliptical copulas and Hierarchical Archimedean Copulas.



**Keywords:** Aggregation and dependency in non-life insurance, Premium Risk, Internal Model, Hierarchical archimedean copulas, reinsurance treaties

## Introduction

The new solvency regulations allow companies to include diversification benefits in the valuation of risk-based capital. So it appears necessary to find the correct way to describe the diversification effect on the Capital Requirement obtained by either Standard Formula and Internal Model.
The total risk of a portfolio could be determined only fixing the rules for aggregating the various risks and describing the dependency among them.
Focusing on the Capital Requirement for only Premium Risk, it is important to decide the way to aggregate single LoB capital charge.
There are different methods of modelling dependence and many papers show that diversification benefits depend on the model chosen[1].
In a simulation approach with an Internal Model, copula functions are the easiest way to describe it; however the problem is to choose the copula and to determine the Kendall Tau.
In fact, the lack of observed data produces many difficulties in identifying the correct copula, especially in calibrating the dependence for extreme events.
Furthermore in n-dimension Archimedean Copulas have the known disadvantage to represent the dependence structure only with few parameters (an only one in many cases). To describe the complexity of the observed dependence, it could be necessary to use Hierarchical Copulas, causing many practical problems to identify the kind and the sort of the structure.
An alternative could be to aggregate single Lob capital charges using a proper aggregation formula that takes in consideration the dependence between the different LoBs.
QIS aggregation formula could be a useful method to include it in the valuation.
However when applied to single LoB Internal Models results, this formula gives only an approximate estimation of diversification effect under independence assumption and must be scaled[2].
Furthermore Sandstrom shows[3] that this formula doesn't provide a correct calibration for skewness assuming that the underlying distributions are Gaussians. It's known that aggregate claims are usually positive skewed distributed.
In the next we analyse Sandstrom aggregation formula, based on normal power approximation, and we show how that formula can have some limitations when the skewness of single LoBs is very high.
It's proposed an alternative approach, useful only to aggregate Internal Model results without using copula functions. This approach could be compared with the results of a multivariate distribution of aggregate claim amount based on the use of elliptical copulas and with other distributions obtained with different hierarchical structures.
These different approaches will be applied with the aim to compare different Capital Requirements obtained with an Internal Model for only premium risk.

[1] See for example at this regard: "Required Capital in Non-Life Insurance: the new scenario according to Solvency II", Savelli & Clemente, 2008
[2] See at this regard "Modelling Aggregate Non-Life Underwriting Risk: Standard Formula vs Internal Model", – revised version November 2008, submitted at Special Issue GIIA 2008, Savelli & Clemente
[3] See "Solvency II: Calibration for skewness", Arne Sandstrom, SAJ, 2007

## The Collective Model

In the next sections we will evaluate the Capital Requirement obtained using different aggregation methods. The Capital, needed for only premium risk, has been determined with a Collective Risk Simulation Model where the aggregate claims amount follows a compound mixed process[4].
Premium risk is here derived only by claims arising in the next one-year time horizon: i.e. the risk that next-year premiums are lower than expenses (in this model deterministic and equal to expenses loading) plus volume of incurred losses for the claims (including both paid amounts during the period and provisions made at the end of year).
Below it will be presented briefly the features of different multi-line insurers analysed and the main results obtained under independence assumptions.
For these analyses four different non-life insurance companies are regarded (their figures are summed up in Figure 1), all of them having different dimension and/or different claim size coefficient of variability (CV).
Furthermore all insurers underwrite business in the same 5 lines of business (Accident, Motor Damages, Property, Motor Third-Party Liability and General Third Party Liability) with the same weight on the gross written premiums volume (rather similar to the actual proportion in the Italian insurance market):

- LoB 1: Accident: 10%
- LoB 2: Motor Damages : 10%
- LoB 3: Property: 15%.
- LoB4 : MTPL: 55%
- LOB5: GTPL: 10%

Consequently, the examined companies have the following initial total gross premium volume (without regarding the increase by approximately 5% in the forthcoming year, relevant for our risk capital evaluation):

- Company **OMEGA**: 1.000 millions of Euros
- Company **TAU**: 500 millions of Euros
- Company **TAU HIGH**: 500 millions of Euros
- Company **EPSILON**: 100 millions of Euros.

As we can see from Figure 1 companies TAU and TAU HIGH have the same volume of premiums (50% of Company OMEGA) but they differ for the claim size CV $c_Z$ (standard deviation/mean), higher than 50% for the Company TAU HIGH. Finally, Company EPSILON has the same parameters of insurers OMEGA and TAU but it has a largely minor dimension (1/10 of OMEGA):

[4] For main hypotheses and for exact moments see for example "Risk-based capital requirements for property and liability insurers according to different reinsurance strategies and the effect on profitability", Havning & Savelli (2005)

**Figure 1: Parameters for premium and claims**

| | LoBs | $n_0$ | σ(q) | g | $m_0$ | $c_z$ | i | λ | exp |
|---|---|---|---|---|---|---|---|---|---|
| **OMEGA** | LoB1 | **17,374** | 14.0% | 1.9% | 3,200 | 3 | 3% | 22.40% | 31.95% |
| | LoB2 | **18,515** | 28.9% | 1.9% | 2,500 | 2 | 3% | 64.25% | 23.98% |
| | LoB3 | **16,580** | 11.2% | 1.9% | 6,000 | 8 | 3% | 6.28% | 29.51% |
| | LoB4 | **111,316** | 8.7% | 1.9% | 4,000 | 4 | 3% | 1.88% | 17.52% |
| | LoB5 | **7,721** | 13.9% | 1.9% | 10,000 | 12 | 3% | -7.03% | 28.22% |
| **TAU** | LoB1 | **8,687** | 14.0% | 1.9% | 3,200 | 3 | 3% | 22.40% | 31.95% |
| | LoB2 | **9,258** | 28.9% | 1.9% | 2,500 | 2 | 3% | 64.25% | 23.98% |
| | LoB3 | **8,290** | 11.2% | 1.9% | 6,000 | 8 | 3% | 6.28% | 29.51% |
| | LoB4 | **55,658** | 8.7% | 1.9% | 4,000 | 4 | 3% | 1.88% | 17.52% |
| | LoB5 | **3,861** | 13.9% | 1.9% | 10,000 | 12 | 3% | -7.03% | 28.22% |
| **TAUHIGH** | LoB1 | 8,687 | 14.0% | 1.9% | 3,200 | **4.5** | 3% | 22.40% | 31.95% |
| | LoB2 | 9,258 | 28.9% | 1.9% | 2,500 | **3** | 3% | 64.25% | 23.98% |
| | LoB3 | 8,290 | 11.2% | 1.9% | 6,000 | **12** | 3% | 6.28% | 29.51% |
| | LoB4 | 55,658 | 8.7% | 1.9% | 4,000 | **6** | 3% | 1.88% | 17.52% |
| | LoB5 | 3,861 | 13.9% | 1.9% | 10,000 | **18** | 3% | -7.03% | 28.22% |
| **EPSILON** | LoB1 | **1,737** | 14.0% | 1.9% | 3,200 | 3 | 3% | 22.40% | 31.95% |
| | LoB2 | **1,852** | 28.9% | 1.9% | 2,500 | 2 | 3% | 64.25% | 23.98% |
| | LoB3 | **1,658** | 11.2% | 1.9% | 6,000 | 8 | 3% | 6.28% | 29.51% |
| | LoB4 | **11,132** | 8.7% | 1.9% | 4,000 | 4 | 3% | 1.88% | 17.52% |
| | LoB5 | **773** | 13.9% | 1.9% | 10,000 | 12 | 3% | -7.03% | 28.22% |

It is to be pointed out that these insurers are the same used in a previous paper and some crucial parameters as safety loading coefficient (λ), standard deviation of structure variable ($\sigma_{\tilde{q}}$ ) and expenses coefficient (*exp*) are obtained mainly by Italian market Loss Ratios and Combined Ratios[5]. The Capital Requirements for the four companies have been obtained by the Internal Model.
Figure 2 shows the 99.5% RBC ratios obtained for all four companies for each LoB and the aggregate in case of independence.
The Ratios, between the Capital Requirement for only premium risk and the initial gross premiums volume, have been obtained for different confidence levels using a VaR risk measure on a one-year time horizon and without considering the effect of Reinsurance treaties.

**Figure 2: RBC ratio (99.5% - Gross Reinsurance) for 4 different Companies for LoB and Total Business in case of independence (Number of simulations = 1.000.000)**

| LoB | OMEGA | TAU | TAUHIGH | EPSILON |
|---|---|---|---|---|
| Accident | 10.40% | 10.78% | 11.71% | 13.91% |
| Mot. Damages | 12.47% | 12.69% | 12.99% | 13.04% |
| Property | 21.82% | 26.35% | 37.35% | 55.34% |
| MTPL | 18.84% | 18.99% | 19.52% | 20.78% |
| GTPL | 58.39% | 76.51% | 106.53% | 159.08% |
| **Aggregate** | **7.96%** | **8.68%** | **10.53%** | **14.76%** |

[5] For further details on parameters estimation, parameters calibration, parameters impact on Capital Requirement and for Italian market Loss Ratios and Combined Ratios patterns see "Modelling Aggregate Non-Life Underwriting Risk: Standard Formula vs Internal Model" – revised version November 2008, submitted at Special Issue GIIA 2008, Savelli & Clemente

Single LoB RBC ratios reflect the characteristic of the aggregate claims amount distributions.
As expected, for all companies the highest ratio is registered for the line GTPL (58.4% for OMEGA and 159% for the small Company) due mainly to its large claim size CV and to the negative safety loadings. Property Line shows a high ratio too (from 21.8% to 55.3% for the EPSILON), while Lines MTPL and Accident have lower ratios. Motor Damage has a 12.5% ratio, notwithstanding the large safety loading $\lambda$, because of the large standard deviation of q. At this regard, the RBC ratio for this LoB is almost the same for all companies.
The total capital requirement for the whole company OMEGA is then equal to 7.96% of gross premiums in case of independence (almost 77 million of Euro).
As to the Company TAU, having half dimension of the Company OMEGA but identical parameters, for the 99.5% confidence level the capital requirement does not receive a large improvement and the ratio increase to 8.68% under independence assumptions.
Regarding Company TAUHIGH, with the only difference from TAU of the claim size CV (values multiplied for 1.5 for each LoB), the requirement becomes more significant (10.53%).
This ratio is not so far to that one (14.76%) obtained for the smallest company (EPSILON), having the only difference of dimension with companies OMEGA and TAU.
The Ratios, under linear Correlation assumptions (Corr. QIS3 and Corr. QIS2), is obtained aggregating single-Lob Internal Model Capital with QIS aggregation formula, based on QIS correlation matrix and scaled with the aim to capture the different way to consider diversification effect between Internal Model and aggregation formula in independence case[6] (see Figure 3).
For all companies the ratio is increasing by roughly 75% under QIS3 dependence assumptions. The positive effect of aggregation of different lines is very clear, that in case of not full correlation allows the companies a significant saving of required capital. For instance, in case of these two extreme dependencies the required capital ratio is decreasing from 21.8% to 8% of premiums for the largest company and from 38.3% to 14.8% in case of the smallest company.

**Figure 3: RBC Ratio (Gross Reinsurance) for the 4 companies, for different confidence levels and according to different dependence assumptions.**

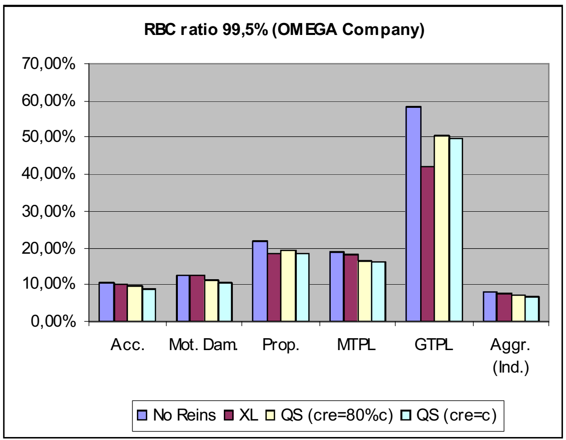
| | | RBC Ratio (Gross Reinsurance) | | |
|---|---|---|---|---|
| | | **99%** | **99.5%** | **99.97%** |
| **OMEGA** | No Corr. | 6.5% | 8.0% | 14.2% |
| | Corr QIS3 | 11.6% | 14.0% | 25.9% |
| | Corr. QIS2 | 9.6% | 11.5% | 19.4% |
| | Full. Corr. | 18.3% | 21.8% | 40.8% |
| **TAU** | No Corr. | 7.1% | 8.7% | 18.8% |
| | Corr. QIS3 | 12.8% | 15.5% | 32.3% |
| | Corr. QIS2 | 10.2% | 12.3% | 24.0% |
| | Full. Corr. | 20.2% | 24.4% | 50.3% |
| **TAUHIGH** | No Corr. | 8.3% | 10.5% | 34.8% |
| | Corr. QIS3 | 14.9% | 18.7% | 50.9% |
| | Corr. QIS2 | 11.6% | 14.4% | 39.6% |
| | Full. Corr. | 23.5% | 29.5% | 74.7% |
| **EPSILON** | No Corr. | 11.2% | 14.8% | 52.0% |
| | Corr. QIS3 | 19.2% | 24.7% | 71.0% |
| | Corr. QIS2 | 14.7% | 18.7% | 56.8% |
| | Full. Corr. | 30.0% | 38.3% | 100.9% |

[6] For further details on the formula used see "Modelling Aggregate Non-Life Underwriting Risk: Standard Formula vs Internal Model", Savelli & Clemente, – revised version November 2008, submitted at Special Issue GIIA 2008, Savelli & Clemente.

In order to show some relevant impact of the reinsurance management on the solvency profile of the insurer and to analyse the effect of different aggregation methods in net reinsurance scenario too, the next alternative reinsurance strategies are here firstly taken into account for the Large (OMEGA) and Small Insurers (EPSILON):

- a Quota Share treaty for each LoB with retention quota a=85% and a fixed commission (applied to the ceded premium) equal to the 80% of expenses coefficient (not depending on the loss ratio of the year);
- a Quota Share treaty for each LoB with retention quota a=85% and a fixed commission (applied to the ceded premium) equal to the expenses coefficient (not depending on the loss ratio of the year);
- an Excess of Loss treaty with a claim retention limit $M_{t,LoB} = E(Z_{t,LoB}) + 15 * \sigma(Z_{t,LoB})$ [7]
- a Combined Quota Share and XL covers: more precisely a quota-share treaty for Accident, Motor Damages and Property with retention quota a=85% and a fixed commission equal to the 80% of expenses coefficient and an Excess of Loss treaty with retention limit $M_{t,LoB}$ for MTPL and GTPL

**Figure 4: 99.5% RBC Ratio aggregated (independence assumption) and for single LoB for OMEGA according to different Reinsurance treaties**




| **RBC ratio 99.5%** | No Reins. | QS ($c^{re}$=80%c) | QS ($c^{re}$=c) | XL | Mixed (XL,QS) |
|---|---|---|---|---|---|
| Accident | 10.4% | 9.9% | 8.8% | 10.3% | 9.9% |
| Mot. Damage | 12.5% | 11.4% | 10.6% | 12.5% | 11.4% |
| Property | 21.8% | 19.5% | 18.5% | 18.5% | 19.5% |
| MTPL | 18.8% | 16.6% | 16.0% | 18.3% | 18.3% |
| GTPL | 58.4% | 50.5% | 49.6% | 41.9% | 41.9% |
| Aggregate (Indep.) | **8.0%** | **7.5%** | **6.8%** | **7.6%** | **7.9%** |

Figure 4 shows the main results obtained, under independence assumptions for the Big Insurer (OMEGA). Quota Share treaty (a=85%), with reinsurer commissions equal to the quota expenses afforded by insurer, gives the lowest RBC ratio (6.77%) obtaining a decreasing of 15% respect to "Non Reinsurance case".

[7] The safety loading $\lambda^{RE}$ applied by the reinsurer on risk premium for the XL treaty, has been determinated for each LoB using the next formula:

$$\lambda^{RE} = \frac{\lambda \cdot E(X) - \lambda^{M} \cdot E(X^{NET})}{E(X) - E(X^{NET})} \cdot (1 + \Delta^{RE})$$

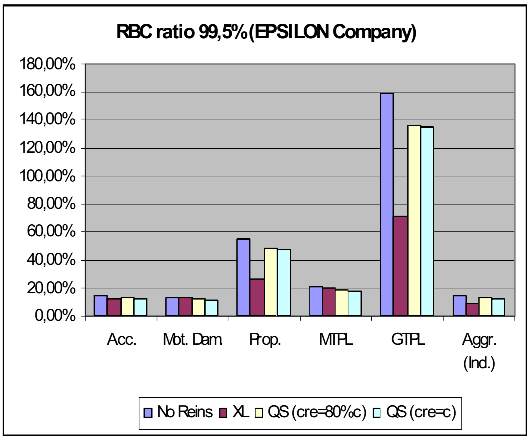
where:
- $E(X^{NET})$ is the expected aggregate claim amount with a claim-size retention limit $M_t$
- $\Delta^{RE}$ is a coefficient (equal to 50%)
- $\lambda^{M}$ is the theoretical safety loading with an aggregate claim amount with a claim size retention limit $M_t$:

$$\lambda^{M} = \lambda \frac{Cv(X^{NET})}{Cv(X)}$$

For GTPL Line the insurer safety loading ($\lambda$) is negative. We have assumed that the reinsurer asks a positive safety loading $\lambda^{RE}$, obtained with the above formula considering the absolute value of $\lambda$

How expected the Quota-Share with lower commissions provides higher aggregate RBC ratio.
LoBs with high skewness of aggregate claims (Like Property and GTPL) show a higher reduction with XL.
Quota-Share plays almost in the same way for the Small Company too: in case of commissions equal to the expenses coefficient the RBC ratio is reduced of roughly 15%.
XL treaty reduces the heavy tail of aggregate claim amount (with a simulated skewness of 0.30 against 3.68) and gives the lowest RBC ratio for Epsilon Company (see Figure 5).
It is worth to emphasize how XL has a high impact on aggregate claims variability with a more flat scale of capital requirements according to company size.

**Figure 5: 99.5% RBC Ratio aggregated (independence assumption) and for single LoB for EPSILON according to different Reinsurance treaties**




| RBC ratio 99.5% | No Reins. | QS ($c^{re}$=80%c) | QS ($c^{re}$=c) | XL | Mixed (XL,QS) |
|---|---|---|---|---|---|
| Accident | 13.9% | 12.8% | 11.8% | 12.5% | 12.8% |
| Mot. Damage | 13.0% | 11.8% | 11.1% | 12.9% | 11.8% |
| Property | 55.3% | 48.0% | 47.0% | 26.7% | 48.0% |
| MTPL | 20.8% | 18.2% | 17.7% | 19.3% | 19.3% |
| GTPL | 159.1% | 136.1% | 135.2% | 71.5% | 71.5% |
| Aggregate (Indep.) | **14.8%** | **13.3%** | **12.5%** | **9.2%** | **11.0%** |

## 2. Factor Based Aggregation

The QIS Standard formula for the overall Capital Requirement is determined by a modular approach, where the single capital charges are aggregated using a proper aggregation formula based on a linear correlation approach.
So the Solvency Capital Requirement is obtained joining the single Capital Charges ($CC_i$=$VaR_i$-$P_i$) with a correlation matrix ($Corr_{i,j}$):

$$SCR = \sqrt{\sum_{i=1}^{L}\sum_{j=1}^{L} Corr_{i,j} \cdot CC_i \cdot CC_j} - \sum_{i=1}^{L} \lambda_i P_i \qquad (1)$$

This approach assumes that the underlying distributions are Gaussians and it doesn't provide a correct calibration for skewness[8]. So as already mentioned, it could happen that Standard Formula considers less than Internal Model the diversification effect[9] under independence assumption. In fact, Internal Model determines the Capital Requirement on the Aggregate Claims distribution, while QIS derives it, joining single-line Capital Charges by an approximation formula.

[8] See at this regard "Solvency II: Calibration for skewness" Arne Sandstrom, SAJ, 2007
[9] See at this regard: "Modelling Aggregate Non-Life Underwriting Risk: Standard Formula vs Internal Model", Savelli & Clemente. For instance, for a Big Multi-Line Insurer with five LoBs, regarding the target confidence level of 99,50% the Standard Aggregation Formula obtains a required ratio of 8.54% instead of 7,96% by Internal Model

Sandstrom shows that one way to tackle the problem with skewed distributions is to use a Cornish-Fisher expansion to transform the quantile and the tail expectations of the skewed distribution in a standard normal distribution.
The correct quantile, obtained with a VaR risk measure, is:

$$C_{SCR}^2 = \sum_{i=1}^{L} f_{V,i}^2 C_i^2 + 2\sum_{i=1}^{L}\sum_{j\neq i} \rho_{ij} f_{V,i} C_i f_{V,j} C_j \qquad (2)$$

where $C_i$ is the capital Charge of the LoB i at confidence level $\alpha$. It's obtained without considering safety loading and could be written as a function of the standard deviation multiplier k:

$$CC_i^{\alpha} = VaR_i^{\alpha} - P_i = k_i^{\alpha}\sigma_i$$

The calibration factor, for the i-th risk charge and with a VaR risk measure[10], obtained by Sandstrom is:

$$f_{V,i} = \frac{6z_{\alpha} + \gamma_{SCR}(z_{\alpha}^2 - 1)}{6z_{\alpha} + \gamma_i(z_{\alpha}^2 - 1)}$$

where $z_{\alpha}$ is the $\alpha$ percentile of the standard normal distribution, $\gamma_{SCR}$ and $\gamma_i$ are the skewness of the multivariate distribution and of the i-th line.
This interesting approach usually gives a good approximation only if the distribution of single-Lob is not very skewed, but it can have some problems with high variable LoBs like GTPL or Property.

So we propose another way to adjust the QIS Aggregation Formula based on the empirical multiplier observed by the simulation model in the independence case.
Being $k_i^{\alpha} = \frac{[RBC_i^{\alpha} + \lambda P_i]}{\sigma_i} = \frac{[VaR_i^{\alpha} - P_i]}{\sigma_i}$ the multiplier obtained by the Internal Model for the LoB i

and $k_{IND}^{\alpha} = \frac{\left[RBC_{IND}^{\alpha} + \sum_{i=1}^{L} \lambda P_i\right]}{\sqrt{\sum_{i=1}^{L} (\sigma_i)^2}}$ the aggregate multiplier under independence assumptions, the portfolio Capital Charge under linear correlation hypothesis and with a calibration for skewness could be equal to:

$$C_{SCR}^2 = \sum_{i=1}^{L} g_{V,i}^2 C_i^2 + 2\sum_{i=1}^{L}\sum_{j\neq i} \rho_{ij} g_{V,i} C_i g_{V,j} C_j \qquad (3)$$

where $g_{V,i} = \frac{k_{IND}^{\alpha}}{k_i^{\alpha}}$.

[10] Sandstrom proposes a calibration factor based on the Cornish-Fisher expansion too for a TailVar measure: $f_{TV,i} = \frac{6 + \gamma_{SCR}(z_{\alpha}^3 - 1)}{6 + \gamma_i(z_{\alpha}^3 - 1)}$.

Figure 6 compares the RBC Ratios obtained under independence assumption for the four companies, using different aggregation method. The Capital Requirement obtained by the aggregate claims distribution produced by the Internal Model ($IM^{IND}$) is compared to the three previous formulas based on the aggregation of the single-LoB capital charge.
As already mentioned, QIS3 Aggregation Formula ($IM^{IND,QIS}$ see Formula 1) presents higher RBC ratios than Internal Model for all Companies and confidence levels describing only approximately the diversification effect. Differences are more pronounced for smaller or high variable LoBs or for the highest confidence level.
The formula based on the calibration factor, proposed by Sandstrom ($IM^{IND,SAND}$ see Formula 2), shows a very good approximation for the big Company characterized by a low positive skewness (roughly 0.3), but it produces a Capital Requirement overestimate for very skewed distributions (the exact skewness is roughly 2 for TAU HIGH and 5.2 for EPSILON)
A formula based on the empirical multiplier ($IM^{IND,MULT}$ see Formula 3) leads, as was obvious in independence case, to RBC Ratio almost coinciding with Internal Model results.

**Figure 6: RBC Ratio (Gross Reinsurance) under independence assumption for the 4 companies, for different confidence levels and according to different aggregation methods.**

| | | RBC Ratio (Gross Reinsurance) | | |
|---|---|---|---|---|
| | | 99% | 99.5% | 99.97% |
| OMEGA | $IM^{IND}$ | 6.51% | 7.96% | 14.21% |
| | $IM^{IND,QIS}$ | 6.87% | 8.54% | 17.84% |
| | $IM^{IND,SAND}$ | 6.57% | 8.09% | 13.98% |
| | $IM^{IND,MULT}$ | 6.50% | 7.95% | 14.19% |
| TAU | $IM^{IND}$ | 7.06% | 8.68% | 18.82% |
| | $IM^{IND,QIS}$ | 7.57% | 9.59% | 23.24% |
| | $IM^{IND,SAND}$ | 7.70% | 9.54% | 17.10% |
| | $IM^{IND,MULT}$ | 7.06% | 8.69% | 18.83% |
| TAUHIGH | $IM^{IND}$ | 8.32% | 10.53% | 34.79% |
| | $IM^{IND,QIS}$ | 9.04% | 11.97% | 38.72% |
| | $IM^{IND,SAND}$ | 12.25% | 15.35% | 29.32% |
| | $IM^{IND,MULT}$ | 8.30% | 10.52% | 34.75% |
| EPSILON | $IM^{IND}$ | 11.21% | 14.76% | 51.97% |
| | $IM^{IND,QIS}$ | 12.34% | 16.83% | 56.51% |
| | $IM^{IND,SAND}$ | 25.00% | 31.68% | 64.22% |
| | $IM^{IND,MULT}$ | 11.21% | 14.77% | 51.99% |

Analyzing the Net Requirements for OMEGA Company, always under independence assumptions (Figure 7), it is noted that for the quota-share treaty the differences between the aggregation methods are similar to “No Reinsurance case”.
The aggregate claim amount distribution, net of quota share reinsurance, has in fact the same skewness: so the QIS aggregation Formula produces an identical overestimation of the Capital Requirement.
XL treaty reduces the tail of aggregate claim amount distribution and leads to nearer Requirements between the different aggregation methods (for the 99.97% confidence level too).
Both Formulas, based on Normal Power Approximation or Empirical Multiplier, show in all cases a good approximation to Internal Model results.

**Figure 7: RBC Ratio (Net Reinsurance) under independence assumption for OMEGA, for different confidence levels and according to different aggregation methods.**

| | | RBC ratio (Gross and Net Reinsurance) | | |
|---|---|---|---|---|
| | | 99% | 99.50% | 99.97% |
| No Reins. | $IM^{IND}$ | 6.51% | 7.96% | 14.21% |
| | $IM^{IND,QIS}$ | 6.87% | 8.54% | 17.84% |
| | $IM^{IND,SAND}$ | 6.57% | 8.09% | 13.98% |
| | $IM^{IND,MULT}$ | 6.50% | 7.95% | 14.19% |
| QS ($c^{re}$=80% c) | $IM^{IND}$ | 6.23% | 7.46% | 12.76% |
| | $IM^{IND,QIS}$ | 6.55% | 7.96% | 15.87% |
| | $IM^{IND,SAND}$ | 6.29% | 7.59% | 12.58% |
| | $IM^{IND,MULT}$ | 6.21% | 7.43% | 12.71% |
| XL | $IM^{IND}$ | 6.25% | 7.57% | 12.33% |
| | $IM^{IND,QIS}$ | 6.53% | 7.90% | 12.87% |
| | $IM^{IND,SAND}$ | 6.24% | 7.53% | 12.14% |
| | $IM^{IND,MULT}$ | 6.24% | 7.56% | 12.32% |

EPSILON Capital Requirements (Figure 8) confirm and emphasize the above comments.
Sandstrom formula results shows very well the different ways of reducing skewness between the two treaties.
Capital Requirements, obtained with Sandstrom formula in the cases of Quota-Share and No Reinsurance, are much higher than Internal Model results for the high skewed claims amount distribution.
XL treaty, reducing the skewness, leads to similar RBC ratios between the two methods.

**Figure 8: RBC Ratio (Net Reinsurance) under independence assumption for EPSILON, for different confidence levels and according to different aggregation methods**

| | | RBC ratio (Gross and Net Reinsurance) | | |
|---|---|---|---|---|
| | | 99% | 99.50% | 99.97% |
| No Reins. | $IM^{IND}$ | 11.21% | 14.76% | 51.97% |
| | $IM^{IND,QIS}$ | 12.34% | 16.83% | 56.51% |
| | $IM^{IND,SAND}$ | 25.00% | 31.68% | 64.22% |
| | $IM^{IND,MULT}$ | 11.21% | 14.77% | 51.99% |
| QS ($c^{re}$=80% c) | $IM^{IND}$ | 10.24% | 13.28% | 44.91% |
| | $IM^{IND,QIS}$ | 11.20% | 15.02% | 48.73% |
| | $IM^{IND,SAND}$ | 21.96% | 27.63% | 55.27% |
| | $IM^{IND,MULT}$ | 10.25% | 13.29% | 44.94% |
| XL | $IM^{IND}$ | 7.56% | 9.15% | 12.75% |
| | $IM^{IND,QIS}$ | 8.43% | 10.08% | 15.69% |
| | $IM^{IND,SAND}$ | 7.85% | 9.33% | 14.30% |
| | $IM^{IND,MULT}$ | 7.56% | 9.15% | 12.74% |

All these different approaches must be analyzed with the aim to be used under linear correlation assumptions. At this regard, Figure 9 shows an interesting comparison between the same aggregation methods based on the use of the QIS3 correlation coefficient.
The simulation results ($IM^{DEP,GAUSS}$), obtained by a multivariate distribution with a Gaussian copula, are compared with closed formulas based on the aggregation of single-Lob capital charge obtained by Internal Model under independence assumption.
As already mentioned in previous papers, the QIS aggregation formula ($IM^{DEP,QIS}$), properly scaled, gives results not so far from the Gaussian copula.
The RBC ratios seem to be higher than the copula for the high variable and the small companies in the highest confidence level
RBC ratios obtained with the calibration factor ($IM^{DEP,SAND}$) are, like in the independence case, too high for the same companies with very skewed distributions.
While the calibration based on the empirical multiplier, obtained by the independence results, leads to RBC ratio similar to the Gaussian Copula. The differences should probably be brought to the effect of dependency on portfolio skewness. Indeed, this effect is neglected by the use of empirical multiplier obtained by the independence results.

**Figure 9: RBC Ratio (Gross Reinsurance) under linear correlation assumption (QIS3 correlation matrix) for the 4 companies, for different confidence levels and according to different aggregation methods.**

| | | RBC Ratio (Gross Reinsurance) | | |
|---|---|---|---|---|
| | | **99%** | **99.5%** | **99.97%** |
| **OMEGA** | $IM^{DEP,GAUSS}$ | 11.37% | 13.54% | 24.50% |
| | $IM^{DEP,QIS}$ | 11.63% | 13.96% | 25.87% |
| | $IM^{DEP,SAND}$ | 10.30% | 12.30% | 20.73% |
| | $IM^{DEP,MULT}$ | 11.10% | 13.11% | 21.76% |
| **TAU** | $IM^{DEP,GAUSS}$ | 12.38% | 14.93% | 30.20% |
| | $IM^{DEP,QIS}$ | 12.75% | 15.53% | 32.32% |
| | $IM^{DEP,SAND}$ | 11.73% | 14.17% | 25.08% |
| | $IM^{DEP,MULT}$ | 12.09% | 14.37% | 28.60% |
| **TAUHIGH** | $IM^{DEP,GAUSS}$ | 14.39% | 17.89% | 45.22% |
| | $IM^{DEP,QIS}$ | 14.89% | 18.69% | 50.86% |
| | $IM^{DEP,SAND}$ | 17.37% | 21.49% | 41.71% |
| | $IM^{DEP,MULT}$ | 14.05% | 17.19% | 51.59% |
| **EPSILON** | $IM^{DEP,GAUSS}$ | 18.64% | 23.76% | 65.55% |
| | $IM^{DEP,QIS}$ | 19.23% | 24.73% | 70.96% |
| | $IM^{DEP,SAND}$ | 34.00% | 42.89% | 88.62% |
| | $IM^{DEP,MULT}$ | 18.26% | 23.33% | 76.34% |

Also the valuation, carried out net of reinsurance, confirms how much observed previously (see Figure 10).
The quota-share turns out the more convenient treaty for OMEGA also in dependency and, not reducing the skewness of the aggregate claim amount, it produces analogous differences between the various methods of aggregation.
In the event of a treaty XL the tail reduction concurs to obtain nearer requirements between Gaussian copulas and closed formulas.
For the highest confidence level the scaled aggregation formula ($IM^{DIP,QIS}$) leads to higher requirement than elliptical copulas.
While empirical multipliers seem to have a good approximation to copulas. Being the multiplier a simulation result, the formula doesn't show always the same differences regarding copulas, especially for the 99.97% confidence level.

**Figure 10: RBC Ratio (Net of Reinsurance) under linear correlation assumption (QIS3 correlation matrix) for the OMEGA company.**

| | | RBC ratio (Gross and Net Reinsurance) 99% | 99.50% | 99.97% |
|---|---|---|---|---|
| No Reins. | $IM^{DEP,GAUSS}$ | 11.37% | 13.54% | 24.50% |
| | $IM^{DEP,QIS}$ | 11.63% | 13.96% | 25.87% |
| | $IM^{DEP,SAND}$ | 10.30% | 12.30% | 20.73% |
| | $IM^{DEP,MULT}$ | 11.10% | 13.11% | 21.76% |
| QS ($c^{re}$=80% c) | $IM^{DEP,GAUSS}$ | 10.45% | 12.43% | 21.77% |
| | $IM^{DEP,QIS}$ | 10.59% | 12.56% | 22.68% |
| | $IM^{DEP,SAND}$ | 9.46% | 11.16% | 18.32% |
| | $IM^{DEP,MULT}$ | 10.08% | 11.78% | 19.09% |
| XL | $IM^{DEP,GAUSS}$ | 11.07% | 12.48% | 18.77% |
| | $IM^{DEP,QIS}$ | 10.70% | 12.61% | 19.38% |
| | $IM^{DEP,SAND}$ | 10.32% | 12.10% | 18.33% |
| | $IM^{DEP,MULT}$ | 10.32% | 12.12% | 18.62% |

How expected, it's confirmed, also in dependency assumption, a meaningful reduction for the small company in the event of a treaty XL (Figure 11).
Such reduction turns out most remarkable on the highest confidence level.
Using Sandstrom aggregation formula, the high skewness leads to overestimate the capital requirement both in gross reinsurance case and with a quota-share treaty.
As already mentioned, reducing the tail with an XL, the formula appears aligned to the others.
Empirical multipliers show, in this case too, RBC ratios near to Gaussian copulas.

**Figure 11: RBC Ratio (Net of Reinsurance) under linear correlation assumption (QIS3 correlation matrix) for the EPSILON company.**

| | | RBC ratio (Gross and Net Reinsurance) 99% | 99.50% | 99.97% |
|---|---|---|---|---|
| No Reins. | $IM^{DEP,GAUSS}$ | 18.64% | 23.76% | 65.55% |
| | $IM^{DEP,QIS}$ | 19.23% | 24.73% | 70.96% |
| | $IM^{DEP,SAND}$ | 34.00% | 42.89% | 88.62% |
| | $IM^{DEP,MULT}$ | 18.26% | 23.33% | 76.34% |
| QS ($c^{re}$=80% c) | $IM^{DEP,GAUSS}$ | 16.52% | 21.06% | 59.81% |
| | $IM^{DEP,QIS}$ | 17.06% | 21.74% | 61.03% |
| | $IM^{DEP,SAND}$ | 29.60% | 37.16% | 76.00% |
| | $IM^{DEP,MULT}$ | 16.26% | 20.58% | 65.69% |
| XL | $IM^{DEP,GAUSS}$ | 13.06% | 15.28% | 22.77% |
| | $IM^{DEP,QIS}$ | 13.41% | 15.73% | 22.62% |
| | $IM^{DEP,SAND}$ | 12.85% | 14.91% | 21.79% |
| | $IM^{DEP,MULT}$ | 12.45% | 14.68% | 19.74% |

## 3. Hierarchical Archimedean Copulas

Recently, copulas have emerged as a powerful tool to create more flexible and more realistic multivariate distributions in finance
Owing to the increase in popularity of copulas to measure dependent risks, generating multivariate copulas has become a very crucial exercise. Current methods for generating multivariate Archimedean copulas could become a very difficult task as the number of dimension increases.
Some papers presents an algorithm for generating multivariate exchangeable Archimedean copulas based on a multivariate extension of a bivariate result[11]:

$$C(u_1, u_2, ..., u_n) = \phi^{-1}(\phi(u_1) + \phi(u_2) + ... + \phi(u_n))$$

where $\phi$ denotes the generator of the multivariate copula $C$.
This class of copulas has one generating function and one parameter that characterizes the dependence structure of the joint distribution function: this aspect could be extremely restrictive in a higher-dimensional case.
A hierarchical Archimedean[12] copula joins two or more ordinary bivariate or higher-dimensional Archimedean copulas by another Archimedean copula.
It is notationally and computationally demanding, but conceptually simple, to build a multi-level hierarchy of Archimedean copulas.
The basic idea is to build a hierarchy of Archimedean copulas, where at each level it's aggregated the Archimedean copulas from the previous level ending at the top level with a joint distribution function.
A simple generalisation of the multivariate Archimedean Copulas, analyzed by some authors[13], is referred as fully nested, since a higher dimensional copula is obtained by adding one dimension step by step: the multivariate copula is obtained choosing the initial pair of marginals and the marginal added step by step.
The n-dimensional copula requires n-1 generator:

$$C(u_1, u_2, ...u_n) = \phi_{n-1}{}^{-1}\{\phi_{n-1} \circ \phi_{n-2}^{-1}[...(\phi_2 \circ \phi_1^{-1}(\phi_1(u_1) + \phi_1(u_2)) + \phi_2(u_3))] + \phi_{n-1}(u_n)\}$$

where $\phi_{n-1} \circ \phi_{n-2}^{-1}$ is a composite function between the generator of two different levels.

In a more general case, Hierarchical Archimedean Copulas are determinate having $n_l$ distinct distributions ($j=1,2,\dots n_l$) at every level of the hierarchy $l=0,1,\dots,L$, that are obtained joining previous level distributions with a bivariate Archimedean copulas:

$$\begin{aligned} C_{l,j} &= C_{l,j}(C_{l-1,h}(u_1, u_2), C_{l-1,h+1}(u_1, u_2)) = \\ &\phi_{l,j}{}^{-1}(\phi_{l,j}\ o\ \phi_{l-1,h}^{-1}(\phi_{l-1,h}(u_1), \phi_{l-1,h}(u_1) + \phi_{l,j}\ o\ \phi_{l-1,h+1}^{-1}(\phi_{l-1,h+1}(u_2), \phi_{l-1,h+1}(u_2)) \end{aligned}.$$

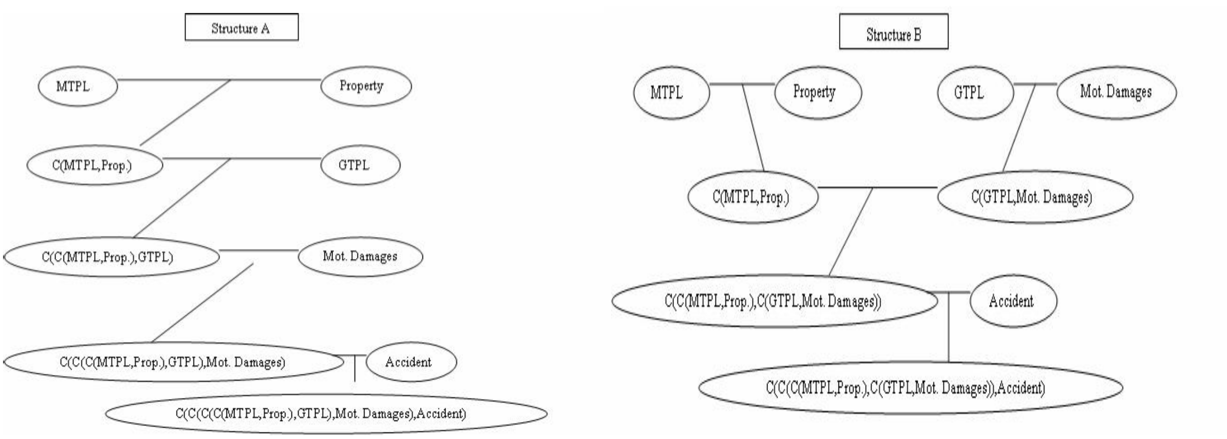
where $\phi_{l,j}$ denotes the generator of the copula $C_{l,j}$ and $\phi_{l,j}\ o\ \phi_{l-1,h+1}^{-1}$ are composite functions that group the copula $C_{l,j}$ to the copula $C_{l-1,h}$.

[11] See for example "Simulating Exchangeable Multivariate Archimedean Copulas and its Applications", Wu, Valdez, Sherris, 2006
[12] See "Hierarchical Archimedean Copulas", Savu and Trede, 2006
[13] See "Modelling dependence with Copulas and Applications to Risk Management", Embrecht, Lindskog, McNeil, 2003 e "Sampling from Archimedean Copulas", Whelan, 2004

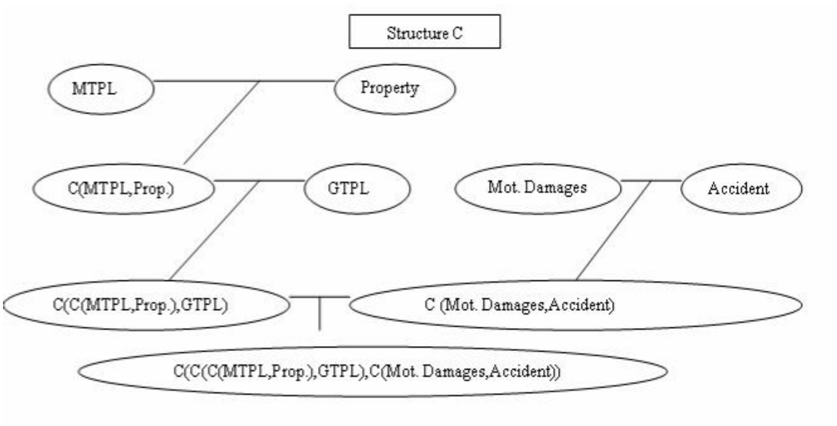
So for example if we have four different marginals, the resulting Archimedean copula can have the following analytical form:
$C_{2,1} = C_{2,1}(C_{1,1}(u_1,u_2), C_{1,2}(u_3,u_4))$.
This kind of aggregation is referred as partial nested hierarchical Archimedean copula.
Some authors show that a number of conditions must be satisfied to ensure that the resulting structure is a hierarchy and in order to the hierarchical Archimedean Copulas to be a proper cumulative distribution function some other conditions must be satisfied.
While only some few simple assumptions are sufficient to obtain a hierarchy, more stringent conditions could be necessary to arrive to a cumulative distribution function.
First of all, the inverse generator function $\phi_{l,j}^{-1}$ must be completely monotone, and secondly we must have some assumption on the composite functions $\phi_{l,j} \; o \; \phi_{l-1,h+1}^{-1}$ .
Embrecht et al.[14] shows that in case of fully nested copulas the degree of dependence has to be greatest for the most deeply nested copulas. The conditions will be satisfied if $\theta_{l+1,i} < \theta_{l,j}$ for all l,j,i: copula parameter and dependence reduce with increasing level.

In the next we will use various hierarchical structures with the aim to obtain the overall Capital Requirement under different assumptions.
We assume that every structure allows combining the lines in the same order, which marginals are sorted in decreasing way according to the 99.5% percentile value.
So MTPL and Property will be joined at the first level and then GTPL, Motor Damages and Accident will be added in this order through various aggregation trees.
The three different structures, figured out in the next, are the following ones:
- a fully nested copula (choosing the initial pair of marginals and then adding a marginal step by step) (In the next Structure A);
- a partial nested copula where two couples of marginal are joined with two copulas and then combined together. At the top level, the fifth LoB is added (Structure B);
- a partial nested copula where the first two levels are the same of a fully nested, while the last two lines are joined before together and then with the others (Structure C).
Finally we use the same copula function for all the aggregations within a structure.

**Figure 12: Different Hierarchical Tree (Structure A,B,C) with LoBs sorted in decreasing way by 99.5% percentile**




[14] See "Modelling dependence with Copulas and Applications to Risk Management", Embrecht, Lindskog, McNeil, 2003 e See "Sampling Nested Archimedean Copulas", McNeil, 2007, preprint




For a proper comparison with previous results, copula parameters have been obtained using the QIS3 correlation matrix. These parameters have been determined, considering the various hierarchical structures and the different LoBs order, by a formula based on the linear correlation coefficient[15].

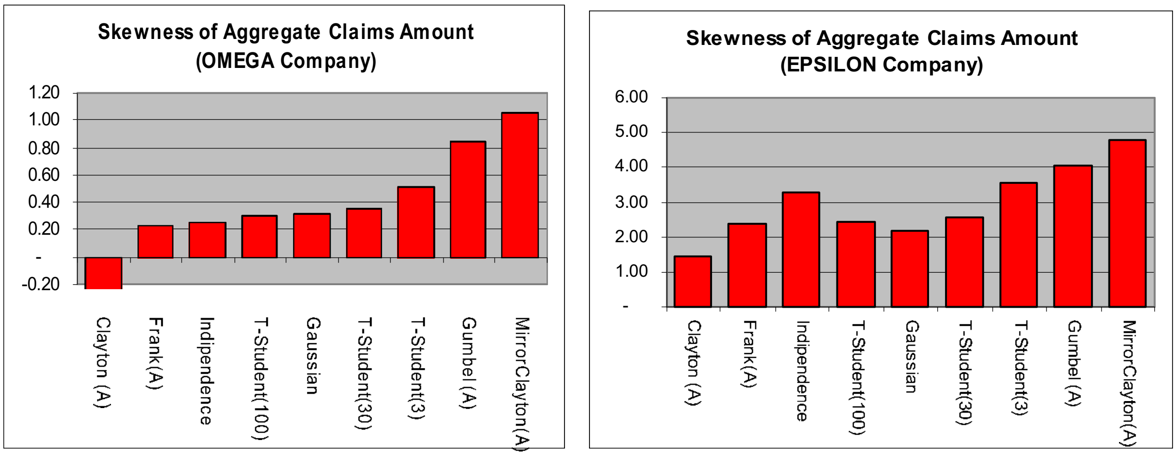
Figure 13 compares the simulated skewness of aggregate claims amount obtained under independence assumption and by both elliptical and fully nested (structure A) Archimedean Copulas.
While all dependence structure leads to a higher variability coefficient of the overall distribution, the skewness has a various behavior according to the copula chosen.
Elliptical copulas shows for the Big Company a portfolio distribution skewed as well as that observed under independence assumption. Only the t-student with few degree of freedom (three d.g.f.) has a higher skewness (0.51 against 0.25 under independence).
The other results confirms the tail dependency of the different functions: clayton copula, in the standard version, produces a fat right tail and leads for OMEGA Company to a negative skewness, while Gumbel and "Mirror Clayton"[16] show higher skewed distributions, more than three times that in the independence case.
For the small Company we have an analogous increase of the variability coefficient, while it could be observed a lower increasing of skewness with copulas with high left tail dependency.
Copula Clayton confirms the reduction of skewness but in this case skewed lines don't consent to obtain an overall distribution negative skewed.

[15] It could be proved that the correlation coefficient between the multivariate distribution of the sum of I LoB $X_i$ and the multivariate distribution of J LoB $X_j$, where every element of $X_i$ is different from every $X_j$, is equal to:

$$\rho(\sum_{i=1}^{I} X_i, \sum_{j=1}^{J} X_j) = \frac{\sum_{i=1}^{I}\sum_{j=1}^{J} \rho(X_i, X_j)\sigma(X_i)\sigma(X_j)}{2\sigma(\sum_{i=1}^{I} X_i)\sigma(\sum_{j=1}^{J} X_j)} \; with \; \forall i \neq \forall j$$

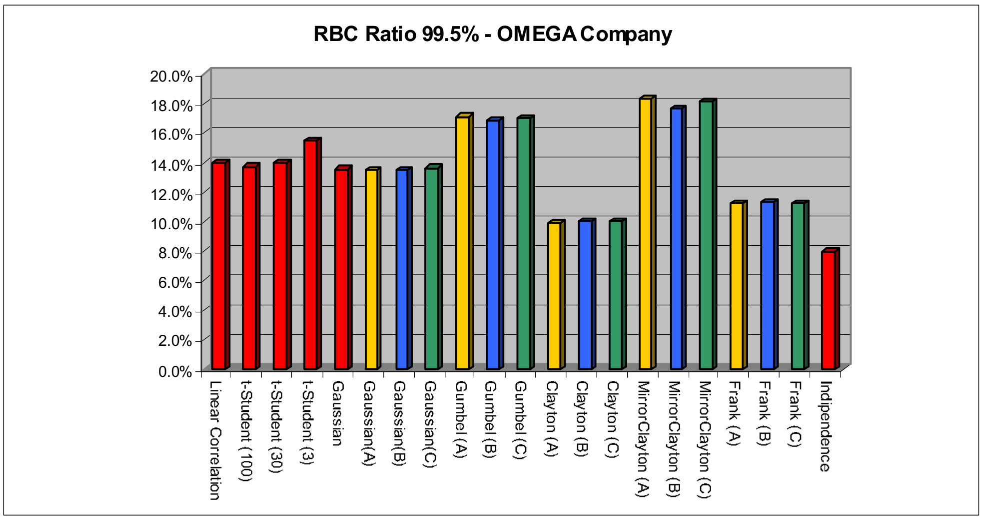
[16] The marginal distributions obtained by this copula, named Mirror Clayton, are determined as $(u_1, u_2) = (1 - (u_1^{Clayton}, u_2^{Clayton}))$, where $u_1^{Clayton}$ and $u_2^{Clayton}$ are the marginals simulated by a Clayton Copula.

**Figure 13: Simulated skewness of Aggregate Claims Amount (Gross Reinsurance) for OMEGA and EPSILON Companies according to different dependence assumptions**




The Capital Requirements of OMEGA Company are obviously influenced by the different values of portfolio variability coefficient and skewness.
Linear Correlation RBC ratio, obtained with the scaled QIS aggregation formula (See IM$^{DIP,QIS}$), appears similar to the capital needed with an elliptical copula.
The Fully Nested copulas (Structure A) leads to very different Capital Requirements with the lowest value obtained with a Clayton and the highest with the mirror Clayton (9.9% and 18.32% against the 7.96% under independence).
It seems that the different structures have an impact on the capital requirement only when tail is very fat. The B Structure shows a lower tail dependency and lower Capital Requirements (roughly reduced of 3-4%) for copulas with fat upper tail.
Finally it's worth to emphasize that elliptical copulas give the same requirement using the multivariate copulas or a hierarchical tree (see Gaussian results in Figure 14)

**Figure 14: RBC Ratio (99.5%) OMEGA Company (Gross Reinsurance) with Different Hierarchical Tree**




It could be observed similar results for the small Company too. Elliptical Copulas leads in this case, for the lower skewness, to lower Capital Requirement than Linear Correlation.

Fully nested Gumbel Copula and Mirror Clayton have again a double RBC than independence, while Clayton Copula shows an increase of the requirement (almost 25%) analogous to OMEGA Company.

**Figure 15: RBC Ratio (99.5%) EPSILON Company (Gross Reinsurance) with Different Hierarchical Tree**

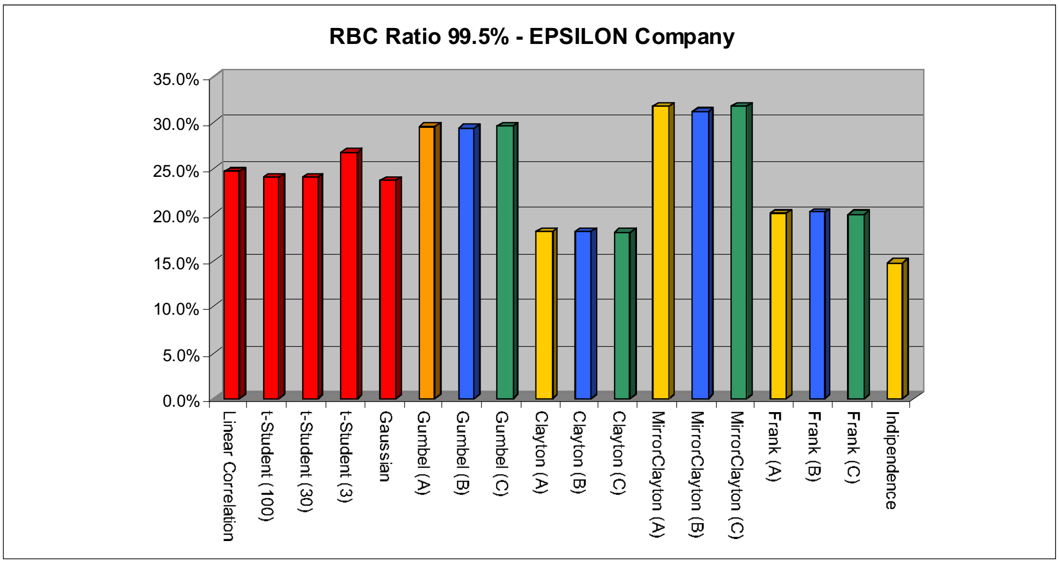
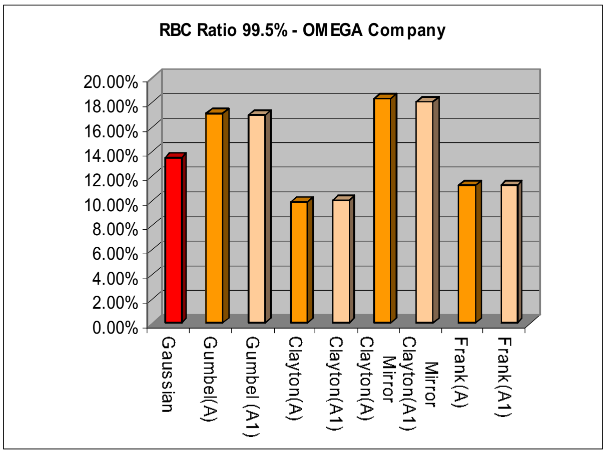
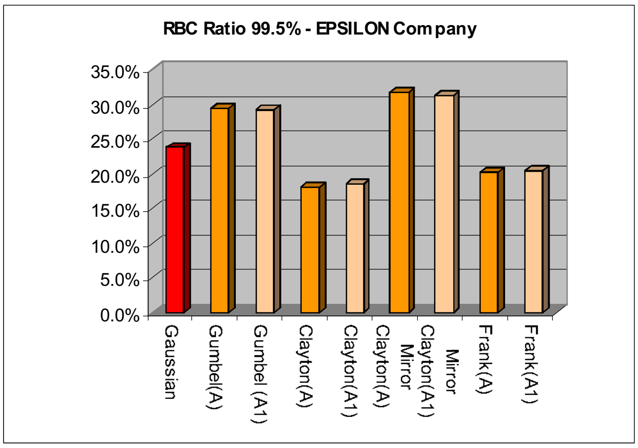



Figure 16 compares the fully nested copulas (Structure A) for both companies obtained sorting the LoBs in a different way.
The above results, obtained where LoBs are sorted in decreasing way by 99.5% percentile, are compared with the same hierarchical tree applied to LoBs sorted in increasing order (see Structure A1 in Figure).
Starting the aggregation from less variable and skewed LoBs, we obtain a Capital Requirement less sensitive to the copula tail dependency. So the 99.5% RBC ratio, determined by the new ordering, is lower for copulas with fat left tail (as Gumbel and Mirror Clayton) and higher for Clayton and Frank. However the LoBs ordering does not seem to have a great impact on the Capital needed.
Small Companies are more influenced from this choice, but it's should be emphasized that copula choice and parameters calibration have a higher impact on the Requirements for both companies.

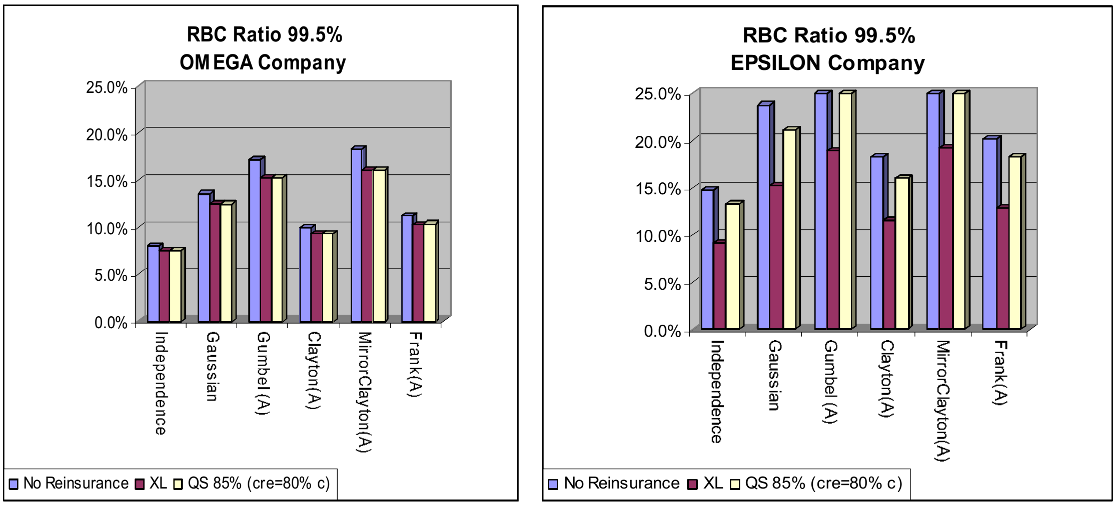
**Figure 16: RBC Ratio (99.5%) OMEGA and EPSILON Company (Gross Reinsurance) with Different LoB Order**



Finally it could be interesting to analyse the combined impact of different dependency structures and various reinsurance treaties.
It's obviously confirmed the greater convenience on the Capital Requirement of the XL treaty for the small insurer.
It's interesting to emphasize that quota share that appears the more convenient treaty in the independence hypothesis for the OMEGA, shows a higher requirement than XL in the dependency cases. This aspect has had to the ability to XL treaty to reduce skewness in greater measure.
For both Insurers the Requirement increase, in the cases of upper tail dependency, appears reduced under a reinsurance cover.
Finally for the Big Insurer the copula choice can have a greater impact than the reinsurance choice.

**Figure 17: RBC Ratio (99.5%) OMEGA and EPSILON Company according to different Reinsurance treaties and to different dependence assumptions**




## Conclusions

In this paper, a Collective Risk Model is applied with the aim to quantify the Solvency Capital Requirement for the Premium Risk only and to compare different aggregation methods.
Internal Models permit to obtain aggregate multivariate distribution under dependence assumptions only with copula functions.
However the choice of copula function and the parameter estimation could represent a problem when only few data are available
So different aggregations method are here proposed, based on a closed formula applied to single LoB capital charge, obtained by internal models.
These methods represent an extension of QIS Formula and try to go on the basic assumption of Normal distribution of the LoB aggregate claim amount.
The formula, proposed by Sandstrom and based on Normal Power Approximation, can be a good way to solve this problem and to consider the skewness of distribution but empirical results shows how that formula can have some limitations when single LoBs are very skewed.
An alternative way, useful only for Internal Model, could be to correct QIS formula using empirical multipliers obtained by Internal Model under independence assumptions.
Case studies show how this method gives similar results to elliptical copula.
Being the multiplier a simulation result, the formula doesn't show always the same differences regarding copulas, especially for the 99.97% confidence level.
Finally Hierarchical Copulas show how the choice of copula function appears much delicate in the valuation of the overall Capital Requirement.

The kind of hierarchical tree and the LoBs order can have, in some particular cases, an impact on the aggregated RBC ratio too.

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