Premium indexing in lifelong health insurance
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Outline

Introduction

Indexing for medical inflation
   Notations and assumptions
   Indexing at time $t = 1$
   Indexing at time $t = 2, 3, \ldots$

Application of the indexing mechanism
   Relationships between $j_t^{[V]}, j_t^{[P]}$ and $j_t^{[B]}$
   Possible procedures

Numerical illustration
   Technical basis
   Optimal $\alpha$ as a function of the age at entry
   Optimal $\alpha$ for a given portfolio of new entrants

Conclusions
Introduction
Research background

Complementary health insurances:

▶ outside social security system
▶ occupational or non-occupational.

Belgian legislation:

▶ lifelong contract,
▶ fixed technical basis,
▶ premium adaptation only with fixed medical index.

Medical inflation is impossible to predict
→ ongoing monitoring
Indexing for medical inflation

Notations and assumptions

Notation:

- $x$: age at policy issue;
- $t$: time since policy issue;
- $(t)$: time at which quantities are valuated.

Assumptions:

- level premiums,
- non-transferrable reserves,
- apart from inflation, all assumptions made in the technical basis are met.
Health insurance contract:

- APV of the benefits:

\[ B_x^{(0)} = \sum_{k=0}^{\infty} c_{x+k}^{(0)} v(0, k) k p_x. \]

- Non-exit probability:

\[ k p_{x+t} = \exp \left( - \int_0^k (\mu_{x+t+s} + \lambda_{x+t+s}) \, ds \right), \]

\[ = \left( 1 - k q_{x+t}^{[d]} \right) \left( 1 - k q_{x+t}^{[w]} \right). \]
Level premium:

\[
\pi_x^{(0)} = \frac{B_x^{(0)}}{\bar{a}_x}, \text{ where } \bar{a}_x = \sum_{k=0}^{\infty} v(0, k) \ k p_x.
\]

Initial reserve:

\[
V_0^{(0)} = B_x^{(0)} - \pi_x^{(0)} \bar{a}_x = 0.
\]
Indexing for medical inflation

time $t=1$

- Reserves:
  - Assets (retrospective reserve):
    \[ V^{(0)}_1 = \left[ \pi^{(0)}_x - c^{(0)}_x \right] \left[ v(0, 1) \right] p_x^{-1}. \]
  - Liabilities (prospective reserve):
    \[ V^{(0)}_1 = B^{(0)}_{x+1} - \pi^{(0)}_x \ddot{a}_{x+1}, \]
    \[ B^{(0)}_{x+1} = \sum_{k=0}^{\infty} c^{(0)}_{x+1+k} v(1, 1+k) p_{x+1}, \]
    \[ \ddot{a}_{x+1} = \sum_{k=0}^{\infty} v(1, 1+k) p_{x+1}. \]
Medical inflation during $[0, 1]$:

$$B_{x+1}^{(1)} = (1 + j_1^{[B]})B_{x+1}^{(0)}.$$  

Inequality:

Assets $\neq$ Liabilities

$$V_1^{(0)} \neq (1 + j_1^{[B]})B_{x+1}^{(0)} - \pi_x^{(0)}\ddot{a}_{x+1}.$$
Restore actuarial equivalence:

- premium increase, \( j_1^{[P]} \);
- reserve increase, \( j_1^{[V]} \);

Following Pitacco (1990):

\[
(1 + j_1^{[V]}) V_1^{(0)} = (1 + j_1^{[B]}) B_{x+1}^{(0)} - (1 + j_1^{[P]}) \pi_x^{(0)} \ddot{a}_{x+1},
\]

or, equivalently,

\[
V_1^{(1)} = B_{x+1}^{(1)} - \pi_x^{(1)} \ddot{a}_{x+1}.
\]
Indexing for medical inflation
time $t=2,3,\ldots$

- Reserves:
  - Assets (retrospective reserve):
    $$V_{t}^{(t-1)} = \left[ V_{t-1}^{(t-1)} + \pi_{x}^{(t-1)} - c_{x}^{(t-1)} \right] \left[ v(t-1, t) \rho_{x+t-1} \right]^{-1}.$$
  - Liabilities (prospective reserve):
    $$V_{t}^{(t-1)} = B_{x+t}^{(t-1)} - \pi_{x}^{(t-1)} \ddot{a}_{x+t},$$
    $$B_{x+t}^{(t-1)} = \sum_{k=0}^{\infty} c_{x+t+k}^{(t-1)} v(t, t+k) \rho_{x+t},$$
    $$c_{x+t+k}^{(t-1)} = c_{x+t+k}^{(0)} \prod_{h=1}^{t-1} \left( 1 + j_{h}^{[B]} \right),$$
    $$\pi_{x}^{(t-1)} = \pi_{x}^{(0)} \prod_{h=1}^{t-1} \left( 1 + j_{h}^{[P]} \right).$$
• Medical inflation during $[t - 1, t]$:

$$B_{x+t}^{(t)} = (1 + j_t^{[B]}) B_{x+t}^{(t-1)}.$$

• Inequality:

$$V_{t}^{(t-1)} \neq (1 + j_t^{[B]}) B_{x+t}^{(t-1)} - \pi_{x}^{(t-1)} \dot{a}_{x+t}.$$
Restore actuarial equivalence:

\[(1 + j_t^{[V]}) V_t^{(t-1)} = (1 + j_t^{[B]}) B_{x+t}^{(t-1)} - (1 + j_t^{[P]}) \pi_x^{(t-1)} \ddot{a}_{x+1},\]

or, equivalently,

\[V_t^{(t)} = B_{x+t}^{(t)} - \pi_x^{(t)} \ddot{a}_{x+t}.\]
Application of the indexing mechanism

Relationships

$$j_t^{[B]} = \left( \frac{V_t^{(t-1)}}{B_x^{(t-1)}} \right) j_t^{[V]} + \left( \frac{\pi_x^{(t-1)} \dot{a}_{x+1}}{B_x^{(t-1)}} \right) j_t^{[P]}.$$ 

Question:

*How to share the additional cost arising from unanticipated benefit inflation between the policy holder ($j_t^{[P]}$) and the insurer ($j_t^{[V]}$)?*
Application of the indexing mechanism

Possible procedures

1. \( j_t^{[V]} = 0 \):
   - advantages:
     - simple and transparent;
     - intuitive;
   - disadvantage:
     - \( j_t^{[P]} \) may fluctuate heavily from year to year.

2. \( j_t^{[P]} = (1 + \alpha)j_t^{[B]} \):
   - \( \alpha = 0 \Rightarrow j_t^{[P]} = j_t^{[B]} = j_t^{[V]} \),
   - \( \alpha > 0 \Rightarrow j_t^{[P]} > j_t^{[B]} > j_t^{[V]} \),
   - \( \alpha < 0 \Rightarrow j_t^{[P]} < j_t^{[B]} < j_t^{[V]} \).
How to determine $\alpha$ such that the contract remains fair?

1. Optimal $\alpha$ for a given age at policy issue:

$$APV_x(\alpha_x^*) = 0, \text{ with } APV_x(\alpha) = \sum_{t=1}^{\infty} j_t [V] V_t^{(t-1)} p_x \nu(0, t).$$

2. Optimal $\alpha$ for a given portfolio of new entrants:

$$APV(\alpha^*) = 0, \text{ with } APV(\alpha) = \sum_{x=x_0}^{\omega-1} n_x \cdot APV_x(\alpha).$$

where $\sum_{x=x_0}^{\omega-1} n_x \cdot \left[ \sum_{t=1}^{\infty} j_t [V] V_t^{(t-1)} p_x \nu(0, t) \right] = 0$ to find the optimal $\alpha$, can be written as an initial equation of equilibrium similar to the one that determines the premiums.
Numerical illustration

Technical basis

- Non-exit probability:

\[ p_y = \left(1 - q_y^{[d]}\right) \left(1 - q_y^{[w]}\right) \]

- Independent survival rates based on the mortality function of the first Heligman-Pollard Law:

\[ \frac{q_y^{[d]}}{1 - q_y^{[d]}} = A(y+B)^C + D e^{-E(\ln y - \ln F)^2} + G H^y, \]

with market parameters:

\[ A = 0.00054, \quad B = 0.017, \quad C = 0.101, \quad D = 0.00013, \]
\[ E = 10.72, \quad F = 18367, \quad G = 1.464 \times 10^{-5} \text{ and } H = 1.11. \]

- Independent lapse rates:

\[ q_y^{[w]} = \begin{cases} 0.1 - 0.002(y - 20), & y = 20, 21, \ldots, 70 \\ 0, & \text{else} \end{cases} \]
Figure: One-year non-exit, survival and lapse probabilities.
interest rate: $i = 2\%$

annual claim amount:

$$c_y^{(0)} = 0.204476472 \times \exp(0.038637y), \quad y \geq 20.$$
Figure: Level premiums $\pi_x^{(0)}$ for different ages.
Figure: Reserves $V_t^{(0)}$ for a person aged 25 at policy issue when $j_t^{[B]} = 0$. 
Numerical illustration
Optimal alpha as a function of the age at entry

1. Calculate $APV_x(\alpha)$ for different values of $\alpha$.
   E.g. take $x = 25$:

   ![Graph showing $APV_{25}(\alpha)$ for different inflation rates]

   **Figure**: $APV_{25}(\alpha)$ when $j_t^{[P]} = (1 + \alpha)j_t^{[B]}$.

2. Select $\alpha_x^*$ for which $APV_x(\alpha_x^*) = 0$. 
Figure: The optimal factor $\alpha^*_x$ as a function of the age at policy issue.
Numerical illustration
Optimal alpha for a given portfolio of new entrants

Figure: Distribution of the age of new entrants.
Figure: $APV(\alpha)$ as a function of $\alpha$ in case $j_t^{[P]} = (1 + \alpha)j_t^{[B]}$. 
Conclusion

- premium indexing mechanisms to restore the actuarial equivalence:

\[ j_t^{[B]} = \left( \frac{V_t^{(t-1)}}{B_x^{(t-1)}} \right) j_t^{[V]} + \left( \frac{\pi_x^{(t-1)} \dot{a}_{x+1}}{B_x^{(t-1)}} \right) j_t^{[P]} \]

1. on policy-per-policy and year-to-year basis with \( j_t^{[V]} \) constant or \( j_t^{[V]} = 0 \).

2. with \( j_t^{[P]} = (1 + \alpha) j_t^{[B]} \) such that the actuarial present value of the future reserve increases equal 0. \( \alpha \) can be chosen:

   2.1 for a given age at policy issue;

   2.2 for a given portfolio of new entrants.
References


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