Time-Consistent and Market-Consistent Actuarial Valuations

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Motivation

- Standard actuarial premium principles usually consider “static” premium calculation:
  - What is price today of insurance contract with payoff at time $T$?
- Actuarial premium principles typically “ignore” financial markets
- Financial pricing considers “dynamic” pricing problem:
  - How does price evolve over time until time $T$?
- Financial pricing typically “ignores” unhedgeable risks
- Examples:
  - Pricing very long-dated cash flows $T \sim 30 – 100$ years
  - Pricing long-dated options $T > 5$ years
  - Pricing pension & insurance liabilities
  - Pricing employee stock-options
In this joint paper with Mitja Stadje: [Pelsser and Stadje, 2013] we want to combine

1. Time-Consistent pricing operators, see [Jobert and Rogers, 2008]
2. Market-Consistent pricing operators, see [Malamud et al., 2008]

We will be interested in continuous-time limits of these discrete algorithms for different actuarial premium principles:

1. Variance Principle
2. Mean Value Principle
3. Standard-Deviation Principle
Content of This Talk

1 Pure Insurance Risk
   - Diffusion Model for Insurance Risk
   - Variance Principle (→ exponential indiff. pricing)
   - Standard-Dev. Principle (→ $E[]$ under new measure)
   - Cost-of-Capital Principle (→ St.Dev price)
   - Davis Price, see [Davis, 1997]
     - St.Dev is “small perturbation” of Variance price

2 Financial & Insurance Risk
   - Diffusion Model for Financial Risk
   - Market-Consistent Pricing
   - Variance Principle
   - Numerical Illustration

3 Conclusions
Consider unhedgeable insurance process $y$:

$$dy = a(t, y) \, dt + b(t, y) \, dW$$

To keep math simple, concentrate on diffusion setting

Discretisation scheme as binomial tree:

$$y(t + \Delta t) = y(t) + a\Delta t + \begin{cases} +b\sqrt{\Delta t} & \text{with prob. } \frac{1}{2} \\ -b\sqrt{\Delta t} & \text{with prob. } \frac{1}{2} \end{cases}$$
Time Consistency

- Time Consistent price $\pi(t, y)$ satisfies property
  \[\pi[f(y(T))|t, y] = \pi[\pi[f(y(T))|s, y(s)]|t, y] \quad \forall t < s < T\]
- Price of today of holding claim until $T$ is the same as buying claim half-way at time $s$ for price $\pi(s, y(s))$
- “Semi-group property”
- Similar idea as “tower property” of conditional expectation
Variance Principle

- Actuarial Variance Principle $\Pi^\nu$:
  \[ \Pi^\nu_t[f(y(T))] = \mathbb{E}_t[f(y(T))] + \frac{1}{2} \alpha \text{Var}_t[f(y(T))] \]

- $\alpha$ is Absolute Risk Aversion

- Apply $\Pi^\nu$ to one binomial time-step to obtain price $\pi^\nu$:
  \[ \pi^\nu(t, y(t)) = \mathbb{E}_t[\pi^\nu(t + \Delta t, y(t + \Delta t))] + \frac{1}{2} \alpha \text{Var}_t[\pi^\nu(t + \Delta t, y(t + \Delta t))] \]

- Note: we omit discounting for now
Pricing PDE

- Assume $\pi^v(t, y)$ admits Taylor approximation in $y$
- Evaluate Var.Princ. for binomial step & take limit for $\Delta t \to 0$
  - Same as derivation of Feynman-Kaç, but for $E[] + \frac{1}{2} \alpha \text{Var}[]$
- This leads to pde for $\pi^v$:
  \[
  \pi^v_t + a\pi^v_y + \frac{1}{2} b^2 \pi^v_{yy} + \frac{1}{2} \alpha (b\pi^v_y)^2 = 0
  \]
- Note, non-linear term = “local unhedgeable variance” $b^2 (\pi^v_y)^2$
- Find general solution to this non-linear pde via log-transform:
  \[
  \pi^v(t, y) = \frac{1}{\alpha} \ln E_t \left[ e^{\alpha f(y(T))} \big| y(t) = y \right].
  \]
- Exponential indifference price, see [Henderson, 2002] or [Musiela and Zariphopoulou, 2004]
Include Discounting

- We should include discounting into our pricing
- Absolute Risk Aversion $\alpha$ is not “unit-free”, but has unit $1/\€$
  - This conveniently compensates the unit $(\€)^2$ of $\text{Var}[]$...
- Therefore, “$\alpha$-today” is different than “$\alpha$-tomorrow”
- Relative Risk Aversion $\gamma$ is unit-free
- Express ARA relative to “benchmark wealth” $X_0 e^{rT}$
- Explicit notation: $\alpha \rightarrow \gamma/X_0 e^{rT}$ leads to pde:

$$\begin{align*}
\pi^v_t + a\pi^v_y + \frac{1}{2} b^2 \pi^v_{yy} + \frac{1}{2} \frac{\gamma}{X_0 e^{rt}} (b\pi^v_y)^2 - r\pi^v &= 0 \\
\pi^v(t, y) &= \frac{X_0 e^{rt}}{\gamma} \ln \mathbb{E} \left[ e^{\frac{\gamma}{X_0 e^{rT}} f(y(T))} \bigg| y(t) = y \right]
\end{align*}$$

- Note: express all prices in discounted terms
Backward Stochastic Differential Equations

- Pricing PDE:
  \[
  \pi_t^v + a\pi_y^v + \frac{1}{2} b^2 \pi_{yy}^v + \frac{1}{2} \frac{\gamma}{X_0 e^{rt}} (b\pi_y^v)^2 - r\pi^v = 0
  \]

- This non-linear PDE represents the solution to a so-called BSDE for the triplet of processes \((y_t, Y_t, Z_t)\)
  \[
  \begin{align*}
  dy_t &= a(t, y_t) \, dt + b(t, y_t) \, dW_t \\
  dY_t &= -g(t, y_t, Y_t, Z_t) \, dt + Z_t \, dW_t \\
  Y_T &= f(y(T)),
  \end{align*}
  \]

- with “generator” \(g(t, y, Y, Z) = \frac{1}{2} \frac{\gamma}{X_0 e^{rt}} Z^2 - rY\).

- Recent literature studies uniqueness & existence of solutions to BSDE’s, see [El Karoui et al., 1997]

- Via BSDE’s we can study time-consistent pricing operators in a much more general stochastic setting. But we will not pursue this here.
Mean Value Principle

- Generalise to *Mean Value Principle*

\[
\Pi_t^m[f(y(T))] = v^{-1}(E_t[v(f(y(T)))]))
\]

- for any function \( v() \) which is a convex and increasing
  - Exponential pricing is special case with \( v(x) = e^{\alpha x} \)
- Do Taylor-expansion & limit \( \Delta t \to 0 \):

\[
\pi_t^{mf} + a\pi_y^{mf} + \frac{1}{2} b^2 \pi_{yy}^{mf} + \frac{1}{2} \frac{v''(\pi^{mf})}{v'(\pi^{mf})}(b\pi_y^{mf})^2 = 0
\]

- Note: \( \pi^{mf}(t, y) := \pi^m(t, y)/e^{rt} \) is price expressed in discounted terms
- Interpretation as generalised Variance Principle with “local risk aversion” term: \( v''()/v'() \)
Standard-Deviation Principle

- Actuarial Standard-Deviation Principle:
  \[ \Pi^s_t[f(y(T))] = \mathbb{E}_t[f(y(T))] + \beta \sqrt{\text{Var}_t[f(y(T))]} \]

- Pay attention to “time-scales”:
  - Expectation scales with \( \Delta t \)
  - St.Dev. scales with \( \sqrt{\Delta t} \)

- Thus, we should take \( \beta \sqrt{\Delta t} \) to get well-defined limit
  - Note: \( \beta \) has unit \( 1/\sqrt{\text{time}} \)
Pricing PDE

- Do Taylor-expansion & limit $\Delta t \to 0$:
  \[
  \pi_t^s + a\pi_y^s + \frac{1}{2} b^2 \pi_{yy}^s + \beta \sqrt{(b\pi_y^s)^2 - r\pi^s} = 0
  \]

- Again, non-linear pde. But if $\pi^s$ is monotone in $y$ then
  \[
  \pi_t^s + (a \pm \beta b)\pi_y^s + \frac{1}{2} b^2 \pi_{yy}^s - r\pi^s = 0
  \]
  \[
  \pi^s(t, y) = \mathbb{E}_t^S [f(y(T)) | y(t) = y]
  \]

- “Upwind” drift-adjustment into direction of risk
Cost-of-Capital Principle

- Cost-of-Capital principle, popular by practitioners
  - Used in QIS5-study conducted by EIOPA
- Idea: hold buffer-capital against unhedgeable risks. Borrow from shareholders by giving “excess return” $\delta$
- Define buffer via Value-at-Risk measure:

$$\Pi^c_t[f(y(T))] = \mathbb{E}_t[f(y(T))] + \delta \text{VaR}_{q,t} [f(y(T)) - \mathbb{E}_t[f(y(T))]].$$
Scaling & PDE

- Again, pay attention to “time-scaling”:
  - First, scale VaR back to *per annum* basis with $1/\sqrt{\Delta t}$
  - Then, $\delta$ is like interest rate, so multiply with $\Delta t$
  - Net scaling: $\delta \Delta t / \sqrt{\Delta t} = \delta \sqrt{\Delta t}$.

- Limit: for small $\Delta t$ the VaR behaves as $\Phi^{-1}(q) \times \text{St.Dev.}$ Hence, limiting pde is same as $\pi^s$ but with $\beta = \Phi^{-1}(q)\delta$.

- Conclusion: In the limit for $\Delta t \to 0$, CoC pricing is the same as st.dev. pricing (*for a diffusion process!*).
The variance price $\pi^v$ is “hard” to calculate, the st.dev. price $\pi^s$ is “easy” to calculate.

Can we make a connection between these two concepts?

“Yes, we can!” using small perturbation expansion.

Consider existing insurance portfolio with price $\pi^v(t,y)$, now add “small” position with price $\varepsilon \pi^D(t,y)$. Subst. into pde:

\[
\begin{align*}
\left( \pi^v_t + \varepsilon \pi^D_t \right) + a(\pi^v_y + \varepsilon \pi^D_y) + \frac{1}{2} b^2 (\pi^v_{yy} + \varepsilon \pi^D_{yy}) + \\
\frac{1}{2} \frac{\gamma}{X_0 e^{rt}} b^2 \left( (\pi^v_y)^2 + 2\varepsilon \pi^v_y \pi^D_y + \varepsilon^2 (\pi^D_y)^2 \right) - r(\pi^v + \varepsilon \pi^D) &= 0
\end{align*}
\]

$\pi^v()$ solves the pde, cancel $\pi^v$-terms.
Pricing PDE

- Simplify pde, and divide by $\varepsilon$:

$$
\pi_t^D + a\pi_y^D + \frac{1}{2} b^2 \pi_{yy}^D + \frac{1}{2} \frac{\gamma}{X_0 e^{rt}} b^2 \left(2\pi_y^v \pi_y^D + \varepsilon (\pi_y^D)^2\right) - r\pi^D = 0
$$

- Approximation: ignore “small” $\varepsilon$-term

$$
\pi_t^D + \left(a + \frac{\gamma}{X_0 e^{rt}} b^2 \pi_y^v\right) \pi_y^D + \frac{1}{2} b^2 \pi_{yy}^D + r\pi^D = 0
$$

$$
\pi^D(t, y) = \mathbb{E}^D_t [f(y(T))|y(t) = y]
$$

- Davis price $\pi^D$ is defined only “relative” to existing price $\pi^v$ of insurance portfolio

- Note, drift-adjustment of st.dev. price scales with $b$
Investigate environment with financial risk that can be traded (and hedged!) in financial market and non-traded insurance risk. Model financial risk as [Black and Scholes, 1973] economy. Model return process $x_t = \ln S_t$ under real-world measure $\mathbb{P}$:

$$dx = \left( \mu(t, x) - \frac{1}{2} \sigma^2(t, x) \right) dt + \sigma(t, x) dW_f$$

Binomial time-step:

$$x(t + \Delta t) = x(t) + \left( \mu - \frac{1}{2} \sigma^2 \right) \Delta t + \begin{cases} + \sigma \sqrt{\Delta t} & \text{with } \mathbb{P}\text{-prob. } \frac{1}{2} \\ - \sigma \sqrt{\Delta t} & \text{with } \mathbb{P}\text{-prob. } \frac{1}{2} \end{cases}$$
No-arbitrage pricing

- BS-economy is arbitrage-free and complete ⇔ unique martingale measure $Q$.
- No-arbitrage pricing operator for financial derivative $F(x(T))$:
  $$
  \pi_Q(t, x) = e^{-r(T-t)}E^Q_t[F(x(T))]
  $$
- Binomial step for $x$ under measure $Q$:
  $$
  x(t + \Delta t) = x(t) + (\mu - \frac{1}{2}\sigma^2)\Delta t +
  \begin{cases}
    +\sigma\sqrt{\Delta t} & \text{with } Q\text{-prob. } \frac{1}{2} \left(1 - \frac{\mu - r}{\sigma} \sqrt{\Delta t}\right) \\
    -\sigma\sqrt{\Delta t} & \text{with } Q\text{-prob. } \frac{1}{2} \left(1 + \frac{\mu - r}{\sigma} \sqrt{\Delta t}\right)
  \end{cases}
  $$
- Quantity $(\mu - r)/\sigma$ is Radon-Nikodym exponent of $dQ/dP$
- Quantity $(\mu - r)/\sigma$ is also known as market-price of financial risk.
Joint discretisation for processes $x$ and $y$ using “quadrinominal” tree with correlation $\rho$ under measure $\mathbb{P}$:

<table>
<thead>
<tr>
<th>State:</th>
<th>$y + \Delta y$</th>
<th>$y - \Delta y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x + \Delta x$</td>
<td>$\left(\frac{1+\rho}{4}\right)$</td>
<td>$\left(\frac{1-\rho}{4}\right)$</td>
</tr>
<tr>
<td>$x - \Delta x$</td>
<td>$\left(\frac{1-\rho}{4}\right)$</td>
<td>$\left(\frac{1+\rho}{4}\right)$</td>
</tr>
</tbody>
</table>

Positive correlation increases probability of joint “++” or “−−” co-movement.
We are looking for *market-consistent* pricing operators, see e.g. [Malamud et al., 2008]

**Definition**

A *pricing operator* \( \pi() \) is *market-consistent* if for any financial derivative \( F(x(T)) \) and any other claim \( G(t, x, y) \) we have

\[
\pi_{F+G}(t, x, y) = e^{-r(T-t)} \mathbb{E}^Q_t [F(x(T))] + \pi_G(t, x, y).
\]

**Observation:** generalised notion of “translation invariance” for all financial risks
MC Variance Pricing

- **Intuition:** construct MC pricing in two steps using conditional expectations. For full results we refer to [Pelsser and Stadje, 2013].
- **First:** condition on financial risk & use actuarial pricing for “pure insurance” risk

\[
\pi^v(t + \Delta t | x \pm) := \mathbb{E}[\pi^v(t + \Delta t | x \pm] + \frac{1}{2} \gamma X_0 e^{r(t + \Delta t)} \text{Var}[\pi^v(t + \Delta t | x \pm]
\]

- \[
\mathbb{E}[\pi^v(t + \Delta t | x +) = \left(\frac{1 + \rho}{2}\right) \pi^v_{++} + \left(\frac{1 - \rho}{2}\right) \pi^v_{+-}
\]
- \[
\text{Var}[\pi^v(t + \Delta t | x +) = \left(\frac{1 - \rho^2}{4}\right) \left(\pi^v_{++} - \pi^v_{+-}\right)^2
\]
- \[
\mathbb{E}[\pi^v(t + \Delta t | x -) = \left(\frac{1 - \rho}{2}\right) \pi^v_{-+} + \left(\frac{1 + \rho}{2}\right) \pi^v_{--}
\]
- \[
\text{Var}[\pi^v(t + \Delta t | x -) = \left(\frac{1 - \rho^2}{4}\right) \left(\pi^v_{-+} - \pi^v_{--}\right)^2
\]
- For \(\rho = 1\) or \(\rho = -1\) no unhedgeable risk left \(\Rightarrow \text{Var} = 0\)

A. Pelsser (Maastricht U)
Second: use no-arbitrage pricing for “artificial” financial derivative

\[ \pi^v(t, x, y) = e^{-r \Delta t} \mathbb{E}_Q^Q[\pi^v(t + \Delta t|x \pm)] \]

\[ = e^{-r \Delta t} \left( \frac{1}{2} \left( 1 - \frac{\mu - r}{\sigma} \sqrt{\Delta t} \right) \pi^v(t + \Delta t|x+) + \frac{1}{2} \left( 1 + \frac{\mu - r}{\sigma} \sqrt{\Delta t} \right) \pi^v(t + \Delta t|x-) \right) \]

Do Taylor-expansion & limit \( \Delta t \to 0 \):

\[ \pi_t^v + (r - \frac{1}{2} \sigma^2) \pi_x^v + (a - \rho b \frac{\mu - r}{\sigma}) \pi_y^v + \frac{1}{2} \sigma^2 \pi_{xx}^v + \rho \sigma b \pi_{xy}^v + \frac{1}{2} b^2 \pi_{yy}^v + \frac{\gamma}{X_0 e^{rt}} (1 - \rho^2)(b \pi_y^v)^2 - r \pi^v = 0 \]

Impact on \( x \): “Q-drift” \( (r - \frac{1}{2} \sigma^2) \)

Impact on \( y \): adjusted drift \( (a - \rho b \frac{\mu - r}{\sigma}) \)

Non-linear term for “locally unhedgeable variance” \( (1 - \rho^2)(b \pi_y^v)^2 \)
Pure Insurance Payoff

- Unfortunately, we cannot solve the non-linear pde in general
- Special case: consider pure insurance payoff (and constant MPR $\frac{\mu - r}{\sigma}$), then no (explicit) dependence on $x$

$$\pi^v_t + (a - \rho b \frac{\mu - r}{\sigma}) \pi^v_y + \frac{1}{2} b^2 \pi^v_{yy} + \frac{1}{2} \frac{\gamma}{X_0 e^{rt}} (1 - \rho^2)(b\pi^v_y)^2 - r\pi^v = 0$$

- Note that for $\rho \neq 0$ the pde is different from the “pure insurance” pde
- Via correlation with financial market we can still hedge part of the insurance risk
- Note: incomplete market $\Rightarrow$ no martingale representation, therefore delta-hedge is not $\pi^v_x$
  - In fact: hold $\rho b \pi^v_y / \sigma$ in $x$ as hedge
  - Economic explanation for drift-adjustment in $y$, a kind of “quanto-adjustment”
Pure Insurance Payoff (2)

- General solution via log-transform:
  \[
  \pi^\nu(t, y) = \frac{X_0 e^{rt}}{\gamma (1 - \rho^2)} \ln \mathbb{E}_{\tilde{P}} \left[ e^{\frac{\gamma (1 - \rho^2)}{X_0 e^{rT}} f(y(T))} \bigg| y(t) = y \right]
  \]

- Measure \( \tilde{P} \) induces drift-adjusted process for \( y \)

- See also, [Henderson, 2002] and [Musiela and Zariphopoulou, 2004] who derived this solution in the context of exponential indifference pricing

- We can generalise to Mean Value Principle for any convex function \( \nu() \)
Numerical Illustration

- Consider “unit-linked” insurance contract with payoff:
  \[ y(T)S(T) = y(T)e^x(T) \]
- Numerical calculation in quadrinomial tree with 5 time-steps of 1 year
- “Naive” hedge is to hold \( y(t) \) units of share \( S(t) \)
- In fact: hold \( \pi_x^v + \rho b \pi_y^v / \sigma \) as hedge
- MC Variance hedge also builds additional reserve as “buffer” against unhedgeable risk

![Payoff at T=5](image1.png)

![Fin Delta t=4](image2.png)
Numerical Illustration (2)

- Investigate impact of correlation $\rho$
- Compare $\rho = 0.50$ (left) and $\rho = 0$ (right)

Positive correlation leads to higher delta, as this also hedges part of insurance risk: hold $\pi_X^V + \rho b \pi_Y^V / \sigma$ as hedge
- Price for $\rho = 0.00$ at $t = 0$ is $€26.75$
- Price for $\rho = 0.50$ at $t = 0$ is $€18.79$, due to less unhedgeable risk
- Price for $\rho = 0.99$ at $t = 0$ is $-€1.98$, due to drift-adjustment
MC Standard-Deviation Pricing

- Again, do two-step construction
- First: condition on financial risk & use actuarial pricing for “pure insurance” risk

\[
\pi^s(t + \Delta t|x\pm) := \mathbb{E}[\pi^s(t + \Delta t)|x\pm] + \delta \sqrt{\Delta t} \sqrt{\text{Var}[\pi^s(t + \Delta t)|x\pm]}
\]

- Second: do no-arbitrage valuation under \( \mathbb{Q} \)
- This leads to linear pricing pde (if \( \pi^s(t, x, y) \) monotone in \( y \)):

\[
\pi_t^s + (r - \frac{1}{2} \sigma^2) \pi_x^s + \left( a - \rho \frac{\mu - r}{\sigma} \pm \delta \sqrt{1 - \rho^2} b \right) \pi_y^s + \frac{1}{2} \sigma^2 \pi_{xx}^s + \rho \sigma b \pi_{xy}^s + \frac{1}{2} b^2 \pi_{yy}^s - r \pi^s = 0
\]

- Drift adjustment for \( y \) is now combination of “hedge cost” plus “upwind” risk-adjustment \( \pm \delta \sqrt{1 - \rho^2} b \)
MC Davis Price

- We can again consider Davis price, by “small perturbation” expansion
- This leads to pricing pde:

$$\pi_t^D + (r - \frac{1}{2} \sigma^2) \pi_x^D + \left(a - \rho b \frac{\mu - r}{\sigma} + \frac{\gamma}{X_0 e^{rt}} (1 - \rho^2) b^2 \pi_y^v \right) \pi_y^D + \frac{1}{2} \sigma^2 \pi_{xx}^D + \rho \sigma b \pi_{xy}^D + \frac{1}{2} b^2 \pi_{yy}^D - r \pi^D = 0$$

- Drift adjustment for $y$ is now combination of “hedge cost” plus risk-adjustment $\frac{\gamma}{X_0 e^{rt}} (1 - \rho^2) b^2 \pi_y^v$
- Davis price defined relative to existing price $\pi^v(t, x, y)$
- Note, st.dev. pricing depends on $\sqrt{(1 - \rho^2)} b$
Multi-dimensional MC Variance Price

- Vectors \( x \) of asset returns (\( n \)-vector), \( y \) of insurance risks (\( m \)-vector)
  \[
dx = \mu \, dt + \Sigma^{\frac{1}{2}} \cdot dW_f
  
dy = a \, dt + B^{\frac{1}{2}} \cdot dW
\]

- Partitioned covariance matrix \( C \) (\( n + m \)) \( \times \) (\( n + m \))
  - \( P \) is \( n \times m \) matrix of financial & insurance covariances
  \[
  C = \begin{pmatrix}
  \Sigma & P \\
  P' & B
  \end{pmatrix}
  \]

- The market-price of financial risks is an \( n \)-vector \( \Sigma^{-1}(\mu - r) \)

- Multi-dim pricing pde for \( \pi^y \):
  \[
  \pi^y_t + r'\pi^x + \left(a - P'\Sigma^{-1}(\mu - r)\right)'\pi^y + \frac{1}{2} \left(C_{ij}\pi^y_{ij}\right) + \\
  \frac{1}{2} \frac{\gamma}{X_0 e^{rt}} \left(\pi^y'(B - P'\Sigma^{-1}P)\pi^y\right) - r\pi^y = 0
  \]

- Note: \( (B - P'\Sigma^{-1}P) \) is conditional covariance matrix of \( y|\pi^x \)
Multi-dimensional MC StDev Price

Multi-dim pricing pde for $\pi^s$:

$$\pi_t^s + r' \pi_x^s + \left( a - P' \Sigma^{-1}(\mu - r) \right)' \pi_y^s + \frac{1}{2} (C_{ij} \pi_{ij}^s) + \frac{1}{2} \delta \sqrt{\pi_y^s (B - P' \Sigma^{-1} P) \pi_y^s} - r \pi^s = 0$$

Note: unlike 1-dim case, does not simplify to linear pde

Simplification only possible if $(B - P' \Sigma^{-1} P)$ has rank 1 and all $\pi_i^s$ have same sign
Conclusions

1 Pure Insurance Risk
   - Variance Principle ($\rightarrow$ exponential indiff. pricing)
   - Standard-Dev. Principle ($\rightarrow E[]$ under new measure)
   - Cost-of-Capital Principle ($\rightarrow$ St.Dev price)
   - Davis Price: St.Dev. price is “small perturbation” of Variance price

2 Financial & Insurance Risk
   - Market-Consistent Pricing: via two-step conditional expectations
   - MC Variance Principle
   - Numerical Illustration for “unit-linked” contract


References II


