Background

• Risk based capital proposals, e.g. EU Solvency II and USA SMI rely on stochastic models.
  • VaR@99.5% and TVaR@99%
• There are many stochastic loss reserve models that claim to predict the distribution of ultimate losses.

How good are these models?

• This presentation describes tests of the predictions of currently popular stochastic loss reserve models on real data from 50 insurers in each of four lines of insurances.
• It proposes two new models that improve the predictions.
The CAS Loss Reserve Database
Created by Meyers and Shi
With Permission of American NAIC

• Schedule P (Data from Parts 1-4) for several US Insurers
  • Private Passenger Auto
  • Commercial Auto
  • Workers’ Compensation
  • General Liability
  • Product Liability
  • Medical Malpractice (Claims Made)

• Available on CAS Website
  http://www.casact.org/research/index.cfm?fa=loss_reserves_data
Notation

- $w =$ Accident Year $w = 1, ..., 10$
- $d =$ Development Year $d = 1, ..., 10$
- $C_{w,d} =$ Cumulative (either incurred or paid) loss
- $I_{w,d} =$ Incremental paid loss $= C_{w,d} - C_{w-1,d}$
### Illustrative Insurer – Incurred Losses

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### Illustrative Insurer – Paid Losses

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Criteria for a “Good” Stochastic Loss Reserve Model

• Using the upper triangle “training” data, predict the distribution of the outcomes in the lower triangle
  • Can be observations from individual (AY, Lag) cells or sums of observations in different (AY, Lag) cells.

• Using the predictive distributions, find the percentiles of the outcome data.

• The percentiles should be uniformly distributed.
  • Histograms
  • Test with PP Plots/KS tests
    • Plot Expected vs Predicted Percentiles
    • KS 95% critical values = 19.2 for \( n = 50 \) and 9.6 for \( n = 200 \)
Illustrative Tests of Uniformity
Data Used in Study

- Insurers listed in Meyers – Summer 2012 e-Forum
- 50 Insurers from four lines of business
  - Commercial Auto
  - Personal Auto
  - Workers’ Compensation
  - Other Liability
- Both paid and incurred losses
Test of Mack Model on Incurred Data

Conclusion – The Mack model predicts tails that are too light.
Conclusion – The Mack model is biased upward.
Test of Bootstrap ODP on Paid Data

Conclusion – The Bootstrap ODP model is biased upward.
Possible Responses to the model failures

• The “Black Swans” got us again!
  • We do the best we can in building our models, but the real world keeps throwing curve balls at us.
  • Every few years, the world gives us a unique “black swan” event.

• Build a better model.
  • Use a model, or data, that sees the “black swans.”
Bayesian MCMC Models

• Use R and JAGS (Just Another Gibbs Sampler) packages
• Get a sample of 10,000 parameter sets from the posterior distribution of the model
• Use the parameter sets to get 10,000 simulated outcomes
• Calculate summary statistics of the simulated outcomes
  • Mean
  • Standard deviation
  • Percentile of the actual outcome
The Correlated Chain Ladder (CCL) Model

- \( \mu_{1,d} = \alpha_1 + \beta_d \)
- \( C_{1,d} \sim \text{lognormal}(\mu_{1,d}, \sigma_d) \)
- \( \mu_{w,d} = \alpha_w + \beta_d + \rho \cdot (\log(C_{w-1,d}) - \mu_{w-1,d}) \) for \( w = 2, \ldots, 10 \)
- \( C_{w,d} \sim \text{lognormal}(\mu_{w,d}, \sigma_d) \)
- \( \rho \sim U(-1,1) \)
- \( \alpha_w \) and \( \beta_d \) are widely distributed. \( \beta_1 = 0 \).

- \( \sigma_d = \sum_{i=d}^{10} a_i \) \( a_i \sim U(0,1) \) Forces \( \sigma_d \) to decrease as \( d \) increases

- Estimate distribution of \( \sum_{w=1}^{10} C_{w,10} \)
The Correlated Chain Ladder Model Predicts Distributions with Thicker Tails

• Chain ladder applies factors to last *fixed* observation
• CCL uses *uncertain* “level” parameters for each accident year.
  \[ \text{Var} \left[ C_{w,d} \right] = E_{\alpha_w} \left[ \text{Var} \left[ C_{w,d} \mid \alpha_w \right] \right] + \text{Var}_{\alpha_w} \left[ E \left[ C_{w,d} \mid \alpha_w \right] \right] \]
• Mack uses point estimations of parameters
• CCL uses Bayesian estimation to get a posterior distribution of parameters
• Mack assumes independence between accident years
• CCL allows for correlation between accident years
  \[ \text{Corr}[\log(C_{w-1,d}), \log(C_{w,d})] = \rho \]
Posterior Distribution of $\rho$ for Illustrative Insurer
Generally Positive Posterior Means of $\rho$
Predicting the Distribution of Outcomes

• Use JAGS (Just Another Gibbs Sampler) software to produce a sample of 10,000 \{\alpha_w\}, \{\beta_d\}, \{\sigma_d\} and \{\rho\} from the posterior distribution.

• For each member of the sample
  • \mu_1 = \alpha_1 + \beta_{10}
  • For \( w = 2 \) to \( 10 \)
    • \( C_{w,10} = \) random lognormal \((\alpha_w + \beta_{10} + \rho(\log(C_{w-1,10}) - \mu_{w-1})), \sigma_d)\)

• Calculate \( \sum_{w=1}^{10} C_{w,10} \)

• Calculate summary statistics, e.g. \( E \left[ \sum_{w=1}^{10} C_{w,10} \right] \) and \( Var \left[ \sum_{w=1}^{10} C_{w,10} \right] \)
Results for the Illustrative Incurred Data

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<th>CV</th>
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Note the increase in the standard error of CCL over Mack.
Compare SDs for All 200 Triangles
Test of Mack Model on Incurred Data

Conclusion – The Mack model predicts tails that are too light.
Conclusion – CCL model percentiles lie within KS statistical bounds.
Improvement with Incurred Data

• Accomplished by “pumping up” the variance of Mack model.

What About Paid Data?

• Start by looking at CCL model on cumulative paid data.
Conclusion – The Bootstrap ODP model is biased upward.
Test of CCL on Paid Data

Conclusion – Roughly the same performance a bootstrapping and Mack
How Do We Correct the Bias?

• Look at models with payment year trend.
  • Ben Zehnwirth has been championing these for years.
• Payment year trend does not make sense with cumulative data!
  • Settled claims are unaffected by trend.
• Recurring problem with incremental data – Negatives!
  • We need a skewed distribution that has support over the entire real line.
The Lognormal-Normal (ln-n) Mixture

\[ X \sim \text{Normal}(Z, \delta), \quad Z \sim \text{Lognormal}(\mu, \sigma) \]
The Correlated Incremental Trend (CIT) Model

- $\mu_{w,d} = \alpha_w + \beta_d + \tau \cdot (w + d - 1)$
- $Z_{w,d} \sim \text{lognormal}(\mu_{w,d}, \sigma_d)$ subject to $\sigma_1 < \sigma_2 < \ldots < \sigma_{10}$
- $I_{1,d} \sim \text{normal}(Z_{1,d}, \delta)$
- $I_{w,d} \sim \text{normal}(Z_{w,d} + \rho \cdot (I_{w-1,d} - Z_{w-1,d}) \cdot e^\tau, \delta)$

- Estimate the distribution of $\sum_{w=1}^{10} C_{w,10}$

- “Sensible” priors on $\alpha_w, \sigma_d$, and $\tau$. $\beta_1 = 0$
  - Needed to control $\sigma_d$
  - Interaction between $\tau, \alpha_w$ and $\beta_d$. 
## CIT Model for Illustrative Insurer

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Posterior Distribution of $\mu$ and $\tau$ for Illustrative Insurer

Should we allow $\rho$ in the model?

Predominantly negative trends
Posterior Mean $\rho$ for All Insurers On Paid Data
Posterior Mean $\rho$ for All Insurers On Incurred Data
Posterior Mean $\tau$ for All Insurers
Test of Bootstrap ODP on Paid Data

Conclusion – The Bootstrap ODP model is biased upward.
Test of CIT with $\rho = 0$ on Paid Data

Conclusion – Overall improvement but look at Personal Auto
Test of CIT on Paid Data

Conclusion – CIT model percentiles are an improvement but do not lie within the KS bounds.
• Mack underpredicts the variability of outcomes with incurred data.
• Both Mack and Bootstrap ODP are biased high with paid data.
• Bayesian MCMC models
  • Easily modified to produce new models.
  • Easily implemented to produce predictive distributions of outcomes.
• CCL model improves significantly on predictions with incurred data.
  • Important feature – Correlation between accident years
• CIT models improves somewhat on predictions with paid data.
  • Important features – Payment year trend and correlation between accident years
• Shortcoming – Study needs to be repeated on different time periods.

Summary
Purpose of CAS Loss Reserve Database

- Large scale testing of model predictions.