International ASTIN Colloquium
Den Haag, May 2013

Speakers‘ Corner

How to use a wrong tariff

Michael Fackler
Independent Actuary
Munich (Germany)

International ASTIN Colloquium
Den Haag, May 2013

More precisely:

How to apply **an** arguably **far too expensive tariff** **to an** arguably **very good commercial risk**

Michael Fackler
Independent Actuary
Munich (Germany)

The situation

- Commercial risk or portfolio, consisting of a number of single "units"
- Line of business working with premium rates (e.g. Property, Marine, PA)
- Bordero available, providing per unit risk characteristics and size (SI and/or MPL, etc.)
- Tariff available, yielding gross premiums per unit
- Very heterogeneous risk, from many smaller to a few very large units, say between € 10000 and 10 mln

The problem

- Total tariff premium seems far too high, loss experience is far lower
- Loss experience is such scarce that experience rating is impossible (say 2 losses in 15 years)

What shall we do? It must be great business!

The method

- Estimate frequency* from loss record (experience)
- Estimate severity from bordero and tariff (exposure)

Basic assumption: The tariff arguably discriminates fairly well between good and bad units, i.e. should be roughly correct up to an (unknown) factor.

* If zero refer to M. Fackler: “Rating without data“, ASTIN Colloquium Helsinki, 2009

Variables describing unit i

V_i size (SI, insured value, MPL, ...)

G_i gross premium

g_i gross premium rate G_i/V_i

R_i net premium

f_i frequency

L_i average loss, $R_i = f_i L_i$

l_i average loss degree L_i/V_i

q_i average loss ratio R_i/G_i, $R_i = q_i G_i$, $f_i l_i = q_i g_i$

w_i weight $w_i := f_i/g_i = q_i/l_i$

Abbreviations

Sum $V_+ = \Sigma V_i$

Average $\overline{V} = V_+/n$

Weighted Average with weights x_i $\overline{V}^{x_i} := \dfrac{\sum x_i V_i}{\sum x_i}$

and analogously for the other variables

Squeezing the available data

- known: V_i, G_i, g_i
- to determine: R_+
- (somewhat) observable: f_+

Smooth heterogeneity: split average loss into a "constant" and a "variable" component

$$L_i = {}^{c}L_i + {}^{v}L_i = {}^{c}L_i + {}^{v}l_i\, V_i$$

- ${}^{c}L_i$ is not much related to V_i, e.g. expenses
- ${}^{v}L_i$ is related to V_i, i.e. ${}^{v}l_i$ may not vary much

Calculation

$$R_+ = \sum f_i\,{}^cL_i + \sum f_i\,{}^v l_i V_i \;\le {}^cL_{\max} \sum f_i + {}^v l_{\max} \sum f_i V_i =$$

$$= f_+\left({}^cL_{\max} + {}^v l_{\max} \sum f_i V_i \Big/ \sum f_i\right) = f_+\left({}^cL_{\max} + {}^v l_{\max} \overline{V}^{f_i}\right)$$

- f_+ can be estimated
- ${}^cL_{max}$ and ${}^vl_{max}$ are unknown but fair (prudent) estimates should be possible, say €50000 and 30%.

$\overline{V}^{f_i}$ is totally unknown: a *frequency-weighted average*

Can we estimate it, at least find an upper bound?

Comparing averages

First attempt:

$$\frac{\overline{V}^{f_i}}{\overline{V}} = \frac{\sum f_i V_i}{\sum f_i} \frac{\sum 1}{\sum V_i} = \frac{\sum f_i V_i}{\sum V_i} \frac{\sum 1}{\sum f_i} = \frac{\bar{f}^{V_i}}{\bar{f}}$$

Numerator large, denominator unclear – does not help

Do we know another average? Yes, we do!

Comparing weighted averages

$$\overline{V}^{g_i} := \frac{\sum g_i V_i}{\sum g_i} = \frac{\sum G_i}{\sum g_i}$$

This is the *gross-premium-rate-weighted average*. Now

$$\frac{\overline{V}^{f_i}}{\overline{V}^{g_i}} = \frac{\sum f_i V_i}{\sum f_i} \frac{\sum g_i}{\sum g_i V_i} = \frac{\sum w_i G_i}{\sum w_i g_i} \frac{\sum g_i}{\sum G_i} = \frac{\sum w_i G_i}{\sum G_i} \frac{\sum g_i}{\sum w_i g_i} = \frac{\overline{w}^{G_i}}{\overline{w}^{g_i}}$$

... which looks promising, although the w_i are unknown

Analyzing the weights

Recall $w_i = f_i/g_i = q_i/l_i$

f_i, g_i vary a lot, however,

q_i varies much less, might be larger for large units,

$l_i = {}^{c}L_i/V_i + {}^{v}l_i \approx {}^{v}l_i$ for large units

At least if V_i is a MPL (or alike) then ${}^{v}l_i$ varies few and does not decrease (much) for larger units

➔ w_i should not vary too much, at least for large units

Weighted weights

G_i is much larger for large units

g_i should be larger for large units

The G_i- and the g_i-weighted average of the weights w_i are both dominated by the larger units and should not be too far from each other, thus we might have

$$\frac{\overline{V}^{f_i}}{\overline{V}^{g_i}} \leq C \quad \text{with say } C = 10.$$

Final formula

Altogether we have an upper bound

$$R_{+} \leq f_{+}\left({}^{c}L_{\max} + {}^{v}l_{\max} C \;\; \overline{V}^{\,g_i} \right)$$

empirical

known

expert judgement

The End

Thanks for joining this talk.

Feedback welcome – now or via mail.

michael_fackler@web.de